Annexure A

Formulae for numerator layout derivation

Mathematical premise: Vector differentiation

Vector differentiation involves finding component-wise derivatives of the dependent variable with respect to independent variable. It is implemented with two conventional strategies (Table A.1): numerator layout and denominator layout [Pedersen et al., 2008]. Here, we present formulae used in for numerator layout strategy, which are used in this thesis. Derivate of $Y \epsilon\ R^m$ (dependent variable) w.r.t $X \epsilon\ R^n$ (independent variable) with numerator layout are shown in Table A.2.

Table A.1 Illustration of vector differentiation with Numerator and Denominator layout.

Numerator layout	Denominator layout
$\frac{dY}{dX} = \begin{bmatrix} \frac{dy_1}{dx_1} & \cdots & \frac{dy_1}{dx_n} \\ . & \cdots & . \\ \frac{dy_m}{dx_1} & \cdots & \frac{dy_1}{dx_1} \end{bmatrix}$	$\frac{dY}{dX} = \begin{bmatrix} \frac{dy_1}{dx_1} & \cdots & \frac{dy_m}{dx_n} \\ . & \cdots & . \\ \frac{dy_1}{dx_n} & \cdots & \frac{dy_m}{dx_n} \end{bmatrix}$

Table A.2 Illustration of different formulae with Numerator layout derivation.

Condition	Expression	Conversion to numerator layout	Derivation
	$\frac{dX}{dX} = I$		$\frac{dX}{dX} = \begin{bmatrix} \frac{dx_1}{dx_1} & \cdots & \frac{dx_1}{dx_n} \\ . & \cdots & . \\ \frac{dx_n}{dx_1} & \cdots & \frac{dx_n}{dx_{n1}} \end{bmatrix}$ $= \begin{bmatrix} 1 & \cdots & 0 \\ . & ..1. & . \\ 0 & \cdots & 1 \end{bmatrix} = I$
A is not function of X	$\frac{dAX}{dX} = A$		$\frac{dAX}{dX} = A\frac{dX}{dX} = AI = A$
A is not function of X	$\frac{dX^T A}{dX} = A^T$	$X^T A = (X^T A)^T = A^T X$	$\frac{dA^T X}{dX} = A^T \frac{dX}{dX} = A^T I = A^T$

A is not function of X	$\frac{dX^T AX}{dX}$ $= X^T(A^T + A)$	$X^T AX = (X^T AX)^T$	$\frac{dX^T AX}{dX} = X^T \frac{dAX}{dX} + X^T \frac{dA^T X}{dX}$ $= X^T(A^T + A)$
a,b are not function of X	$\frac{da^T XX^T b}{dX}$ $= X^T(ba^T + ab^T)$	$a^T X^T Xb = X^T ab^T X$ $(X^T ab^T X)^T = X^T ba^T X$	$\frac{da^T XX^T b}{dX} = X^T ab^T \frac{dX}{dX}$ $+ X^T ba^T \frac{dX}{dX}$ $= X^T(ba^T + ab^T)$

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