8

Conclusion and Future Work

We first developed a spectral method for the balancedness of a signed graph. The developed spectral method mainly serve the following two purposes (i) distinguishing any two signed graphs based on their degree of unbalance for the given parameters like weights and length of closed walks, (ii) edge prediction of signed graphs. The edge prediction of the developed method is better compared to the state of art method. Also, we conclude that the closed walks of small length are enough to achieve the best accuracy. Considering the importance of eigenvalues of signed graphs we found the spectra of some signed graph with negative cliques. These graphs are induced from a complete graph. Negative of spectra of these graphs gives the spectra of corresponding weakly balanced graph. Next, we use the signed digraph representation of any square matrix to calculate its characteristic and permanent polynomials. In particular, we show that the characteristic and permanent polynomials of any square matrix can be calculated using the characteristic and permanent polynomials of subdigraphs of blocks in its digraph. These subdigraphs give the partition that we call $\mathscr{B}$-partitions. Finally, we give an algorithm to find the $\mathscr{B}$-partitions of digraphs and analyzed the parameterized complexity of matrix determinant and permanent. For a class of matrices, we see that the parametrized complexities beat the state of art complexities.

8.1 FUTURE WORK

We have seen in that there is a spectral criteria to judge whether a signed graph is strongly balanced or not. Unfortunately, a spectral criteria to judge whether a signed graph is weakly balanced or not is still an open problem. Thus, as a future work, we will first work on the following open problem.

Open Problem 8.1. *To find a spectral criteria for weakly balanced signed graph.*

In Chapter 5 we developed the method to calculate the determinant and permanent of square matrix. Computationally, the developed method works fine with the digraphs having several blocks and few cut-vertices which have loops. Whereas, for the matrices whose digraphs have no cut-vertex, computationally this method is not useful. The digraphs having no cut-vertex are called non-separable digraphs. Intuitively, for a non-separable digraph if we can find a vertex whose deletion can lead to several blocks in resulting subdigraph can solve our purpose. Also, in view of discussion in Section 7.2 of Chapter 7, we can calculate the determinant of a square in terms of determinant of small order submatrices. The digraphs corresponding to these small order matrices may have several blocks. For an efficient method, we should prefer a vertex v, in digraph G, such that, the number of blocks in $G \setminus v$ is maximum. This lead us to the following open problem:

Open Problem 8.2. *Let G be a digraph with no cut-vertex. Also, assume $b(G)$ denotes the number of blocks in G. Then find out a vertex v, such that, $b(G \setminus v) \geq b(G \setminus u)$ for all u in G.*

We have checked for several non-separable digraphs, that the vertex having largest degree solves the problem. All the nonseparable digraph having less than six vertices satisfy this. But it

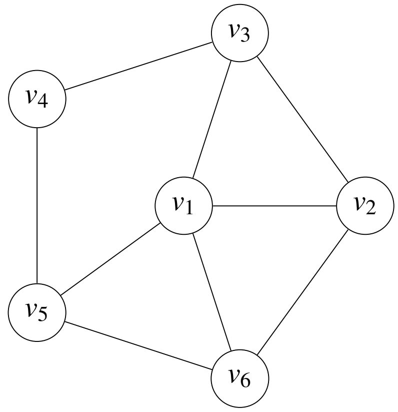

Figure 8.1 : A non separable graph on 6 vertices.

is not true in general for non-separable as there are contradictions. For example in nonseparable given in Figure 8.1, removing vertex v_3 or v_5 gives maximum number of blocks in resulting subdigraph, but v_1 has the maximum degree. Thus, if we solve the above open problem, we can move in direction of a new faster method to calculate the determinant of a given square matrix.

Unfortunately, for the permanent, the above idea seems not working as there are very rare identities for the permanent. This needs some extra attention as we have to try to form some equalities on permanent, and then use them on blocks of digraphs or resulting subdigraphs.

Thus, as a future research work we would like to work on following two problems:

1. Solving the Open problem 8.2. This will guide to the faster method for the determinant of nonseparable digraphs, as well as graphs having blocks of large sizes.
2. Formulating some identities for the permanent of matrices which can use the subdigraphs of blocks for finding the permanent.