Abstract

Signed graphs are imperative to represent a system where the affinity and aversion co-exist between its entities. In a signed graph, any edge is either positive or negative. A positive edge reflects the existence of affinity between two vertices while a negative edge reflects the existence of aversion. Strong balance is a key concept in signed graphs which gives the notion of stability and structure in it. A cycle in a signed graph is called strongly balanced when it has an even number of negative edges. A signed graph is called strongly balanced graph when all of its cycles are strongly balanced. A slight generalization of strong balance is the concept of weak balance. According to this concept, all the cycles in a signed graph are weakly balanced except those which have only one negative edge. A signed graph is called weakly balanced when all of its cycles are weakly balanced.

An interesting thing about a strongly balanced signed graph is that it can be partitioned into two parts of vertices such that all the edges inside any part are positive and all the edges between these parts are negative. Also, a signed graph is strongly balanced if and only if its eigenvalues are equal to the eigenvalues of its underlying unsigned graph. But what if some of the cycles of signed graphs are not strongly balanced? This gives rise to the concept of degree of unbalance. There are measures of the degree of unbalance with conflicting results. The importance of these measures lies in the fact that they are being used for (*i*) distinguishing two signed graphs based upon their degree of unbalance, (*ii*) the sign prediction of the edges. The eigenvalues of signed graphs can play a major role in defining a measure for the degree of unbalance. In an attempt towards developing a reasonable measure of the degree of unbalance, we first give a measure of the degree of unbalance using eigenvalue of signed graphs. Later we used this measure for sign prediction in signed graphs. We propose a parametrized walk based measure for lack of balance in signed networks inspired by the Katz measure of similarity of two vertices in a network. The proposed measure can be used to distinguish signed social networks on the basis of their degree of lack of balance. Further, the Katz prediction rule is exploited to derive that the cycles of shorter lengths can predict the sign of an edge in certain popular empirical signed social networks better than the longer cycles.

Similar to strongly balanced signed graphs, the vertex set of a weakly balanced graph can be partitioned into more than two parts such that all the edges inside any part are positive and all the edges between any two parts are negative. Finding spectral criteria for the weak balance is still an open problem. However, we have found the characteristic polynomial and hence the spectra of some weakly balanced graphs induced from the complete graph.

There is an interplay between signed digraphs and eigenvalues, determinant, permanent of matrices. Every square matrix can be represented as a weighted signed digraph. The structure of the signed digraph can be exploited to give some interesting matrix theoretical results. Using the blocks in digraphs we define a new partition, that we call as $\mathscr{B}$-partition. The characteristic and permanent polynomial of a matrix can be elegantly calculated using $\mathscr{B}$-partitions of the corresponding digraph. Similarly, the determinant and the permanent of a matrix can be obtained. Later, we analyzed the parametrized complexity of this developed method for the determinant and permanent. For a class of matrices, the complexity of developed method beats the state of art complexities.