7

Conclusion and future scope

Ever since Einstein, Podolsky and Rosen raised the question of completeness of quantum mechanics as a physical theory, entanglement and nonlocality took the centre stage for understanding and analysing the foundations of quantum mechanics and quantum information theory. In the last three decades, quantum entanglement and nonlocal correlations- existing between spatially separated parts of a system- led to many interesting phenomena in quantum computation and information. The progress on the experimental front for realizing entangled systems or communication protocols and validating nonlocality between particles provided additional support to the theory. Moreover, several studies have revealed that nonlocal correlations are not only limited to entangled systems but it can be extended to some separable systems as well. In general, nonlocal correlations associated either with entangled or separable systems offer potential applications in quantum information and computation which are otherwise not possible using classical computation. Therefore, classification and quantification of entanglement and nonlocal correlations in bipartite as well as multiqubit systems are of immense value. Due to the characteristic trait of complexity, the characterization of nonlocality in mixed bipartite or multiqubit systems is much more interesting. In real situations, the analysis of nonlocal correlations becomes even more intricate due to the interaction of entangled systems with the environment. The environmental interactions lead to degradation of entanglement and nonlocal correlations in an underlying quantum system, thereby decreasing the efficiency of the entangled resource enormously. Therefore, in all such cases, one needs to find a mechanism to protect nonlocal correlations and optimize the degree of entanglement and nonlocality in presence of noise. Fortunately, there are several models available to protect entanglement and nonlocality such as entanglement distillation, decoherence free subspace, quantum error correcting codes, quantum zeno effect, and weak measurement and its reversal operations. Hence, the analysis of usefulness of entangled resources- in terms of nonlocal correlations and their efficiencies in quantum information- using these models in presence of noise is of substantial importance.

Therefore, in this thesis, we have analysed the usefulness of two-qubit mixed states under different noisy conditions and applications of weak measurement and its reversal operations. We have shown that nonlocal correlations can be effectively shielded from the effects of noise by performing weak measurements and its reversal operations on individual qubits before and after the distribution of entanglement, respectively. The analytical results obtained for the violation of Bell inequality under the influence of the amplitude-damping noise by finally shared two-qubit states led us to propose an efficient class of two-qubit states which is surprisingly found to be very useful for quantum information processing protocols in comparison to a large set of bipartite pure and mixed states. Our results confirmed that these states although not violating the Bell inequality, are still entangled and have non-zero discord, and can be used as efficient resources. Interestingly, we found that more nonlocal correlations in the initial state do not lead to higher efficiency of the finally shared state, i.e., a non-maximally entangled initial state is found to violate the Bell inequality with a higher degree in comparison to a maximally entangled two-qubit initial state. In case of phase-damping channels, we demonstrated that the violation of Bell-CHSH inequality is independent of the state parameter and weak measurement strength for optimal weak measurement reversal strength, hence, introducing flexibility to start with any initial state instead of starting with maximally entangled states only. For depolarizing noise, we found that the weak measurements can be useful for enhancing the robustness of nonlocal correlations against mild noisy conditions. We further extended our study of two-qubit systems to three and four qubit systems in chapter-3. Precisely, we analysed nonlocal correlations in generalized three and four-qubit GHZ states, three-qubit W states and three-qubit W_n-type states under the influence of noise and weak measurement and its reversal operations. For this, we again established a relation between the maximum expectation value of the Svetlichny operator for a given system, the state parameter, noise parameters, and strength of weak measurement and its reversal operations. The results obtained for different three and four qubit classes clearly suggest that the effect of amplitude-damping noise on multiqubit nonlocality can be suppressed or completely removed depending on the strengths of weak measurement and its reversal operations. Similar to the two-qubit scenario, for amplitude-damping channels, we found that for certain values of weak measurement strengths and range of three-tangle for initially prepared states, the violation of Svetlichny inequality by finally shared state is more if one starts with non-maximally entangled states instead of a maximally entangled GHZ state. For W class of states, the three-qubit Svetlichny inequality is violated only when the sum of concurrences of three bipartite reduced states derived from a W class of state exceeds a certain threshold- the degree of violation clearly depends on the strength of weak measurement operation. In comparison to W states, a larger set of W_n-type states violate the three-qubit Svetlichny inequality with all the states violating the inequality for $\eta = 1$. We have also described the behaviour of nonlocal correlations in GHZ class of states under the influence of phase-damping and depolarising channels. In case of phase-damping channel, similar to the case of bipartite system, we found that the strength of nonlocal correlation remains independent of the initial entanglement under the applications of weak measurement operations. Therefore, after the applications of noise and weak measurement, the expectation value of Svetlichny operator is evaluated to be exactly the same, independent of whether initial state is a maximally or a partially entangled state. However, for the depolarizing channel with mild strength, weak measurements and its reversal operations proved to be advantageous for enhancing the strength of nonlocal correlations in comparison to the adverse impact of depolarizing noise on quantum correlations. We have further shown that the analytical results obtained in all the cases are in excellent agreement with the numerical results obtained for optimizing the expectation value of the Svetlichny operator. In future, it will be interesting to investigate the usefulness of finally shared three-qubit and four-qubit mixed states for quantum information and computation.

The distinction between classical and quantum correlations is laid down in terms of violation of the Bell- or Bell type inequalities for bipartite or multiqubit systems. There are instances where the Bell or Bell type inequalities fail to capture nonlocality in entangled resources. In general, there is no generic method or definition to capture nonclassicality between qubits in bipartite as well as multiqubit systems. For two-qubit systems, although quantum discord describes all types of nonlocal correlations in the system, it is difficult to obtain analytical expressions to evaluate discord due to the optimization procedure involved. A simple description that can be obtained analytically to distinguish the quantum and classical boundary in bipartite and multiqubit systems would always be considered of a great value. Therefore, in chapter-4, we modify the original Bell-CHSH inequality using statistical correlation coefficients representing correlations between qubits. The analysis allowed us to derive a necessary and sufficient condition for the violation of Bell-Cumulant inequality by an arbitrary two-qubit state. We have shown that the modified Bell inequality is violated by all pure states. Our results further confirmed the presence of nonlocal correlations in few bipartite states where the original Bell inequality fails to capture the nonlocality. Moreover, we also derived an analytical expression to estimate the maximum and minimum value of geometric discord using correlation matrix and established a relation between optimum expectation value of the Bell-Cumulant operator and maximum value of geometric discord. The analysis was extended to characterize nonlocality in three-qubit GHZ class using a modified three-qubit Svetlichny inequality. The results obtained in this chapter suggest that correlation coefficients can be used as quantifiers to characterize and analyse the usefulness of bipartite and tripartite systems for quantum communication and information processing.

In order to study the usefulness of nonlocal correlations in context of nonlocal tasks, in chapter-5, we analysed nonlocal correlations in a bipartite entangled system under biased experimental settings, i.e., in a scenario where both Alice and Bob choose their measurements with a certain bias. For this, we estimated maximum classical and quantum bound for a biased nonlocal game and demonstrated that quantum correlations are always a better resource in comparison to classical resources for the whole range of biasing parameters. Moreover, the analysis using fine-grained uncertainty relations indicated range of biasing parameters for which quantum and classical correlations can be distinguished. We further established that bipartite pure and mixed states show similar behaviour in biased as well as unbiased scenario. A problem of particular interest would be to find a mixed state whose behaviour can be differentiated in the two scenarios.

In chapter-6, to analyse the efficiency of partially entangled states in quantum information processing, we proposed and characterized a class of partially entangled four-qubit W-type states for efficient and deterministic quantum information processing. Although we found that the set of states can be used as resources for deterministic information transfer, the efficiency of these states is limited to the availability of experimental set-ups to measure and distinguish four-qubit joint measurements. We, therefore, analysed another scenario where the users in possession of four-qubit W-type states try to establish an optimal bipartite entanglement using standard single and two-qubit measurements only. Using the above strategy, we further compared the efficiencies of three and four qubit W-type states in terms of the shared bipartite entanglement. Interestingly, we found that for certain range of state parameters, four-qubit W-type states can be shown to be better resources in comparison to three-qubit W-type states. Moreover, we have also proposed a way for experimental realization of four-qubit W type states which further increases the importance of results obtained in this study. The study presented in this chapter will be of significant value in situations where users in a protocol have limited access to entanglement in form of a partially entangled state only. Our results will allow one to decide when to use a three or four-qubit W-type state for a particular protocol. Furthermore, the analysis is also generalized for studying the efficiency of generalized N-qubit W-type states for quantum information processing protocols.

We believe that the study presented in this Thesis will be proved beneficial to researchers working in theory as well as experiments. The Thesis also provides prospects for future research for analysing entanglement and correlations using cluster coefficients. We believe that a general description of entangled systems using statistical correlation coefficients may provide a consistent definition of entanglement and nonlocality in bipartite as well as multipartite pure states. In general, we plan to perform detailed studies to map out the behaviour of correlation coefficients and to develop useful quantifiers of entanglement that could perhaps also be generalized to mixed states. In future, we also propose to design a generic Bell-type inequality for bipartite as well as multipartite quantum systems to study the complex nature of multiqubit entanglement. The properties of nonlocal correlations using the modified inequalities under decoherence will be of particular interest. Such studies will be important not only for characterizing the properties and behaviour of bipartite and many-body systems, but also for shedding light on subtle nature of entanglement and nonlocality, and for proposing more efficient information processing and communication protocols. Since, almost all the measures which describe the nonlocality in correlations including discord are very difficult to calculate analytically, except geometric discord. We would further like to address this problem using correlation coefficients to obtain a simple analytical formula to analyse and describe quantum correlations.

…