# 2

# System Model and Review of Literature

In the late 1990s, a practical demonstration of vertical Bell laboratory layered space-time architecture (V-BLAST) multi-antenna wireless system by Bell Labs [Foschini, 1996] followed by the theoretical prediction of very high capacity of MIMO channels in rich scattering environments by Telatar [1999] and Foschini and Gans [1998] put a new perspective of exploitation of spatial dimension in wireless communications. The MIMO and massive MIMO technologies have undergone an intensive research in last decade. The 4G of wireless mobile communication has MIMO as a key technology for accessing the spatial dimension along with OFDM for frequency dimension. MIMO is considered to be a mature technology with well developed channel models, channel estimation, space-time codes, detection, precoding, hardware and commercial deployment in different communication standards. At the same time, the research is going on for future generation of mobile communication where the massive MIMO is a promising candidate. A significant work has been done on massive MIMO in last few years. However, there are a few open issues like channel modeling, channel estimation, inter-cell interference management, and system architecture where more research is needed for massive MIMO to become a matured technology for future generation of wireless mobile communication systems. In this chapter, a discussion is provided on the research activities being carried out on massive MIMO and the existing gaps where more work is needed.

## 2.1 ORIGIN AND PRELIMINARY RESEARCH

The MIMO technology projected two major gains as spatial diversity and spatial multiplexing. More specifically, in a wireless link with $N$ number of transmit antennas and $K$ number of receive antennas in a point-to-point MIMO system, the probability of outage can be written as

$$P_{outage} \propto SNR^{-NK}$$

With spatial multiplexing, the achievable rate scales as

$$R_{achievable} \propto min(N, K) \log_2(1 + SNR)$$

After these promising outcomes, the natural question is: What does stop a system to gain as much as desired by increasing the number of antennas? The major bottlenecks in doing so are the complexity of signal processing, the size constraint on MTs, and the conditioning of channel matrix. The prime step for handling these issues was to give up the point-to-point MIMO systems, which was a necessary step irrespective of losing all the research done on the space-time codes and joint processing over antennas. The Point-to-point MIMO is basically the cause of several problems in scaling the multi-antenna technology in conventional cellular systems. The small size of MTs cannot support a large number of antennas. The small computation power limited by the battery and the low cost processor in MTs is not able to support the computationally complex signal processing required for MIMO system with a large number of antennas. In the line-of-sight (LOS) propagation, the MIMO channel matrix becomes badly conditioned with rank as low as one which makes the MIMO gains unachievable. At this point, the massive MIMO plays a significant role.

The preliminary work on state-of-the-art massive MIMO systems includes [Marzetta, 2006] and [Marzetta, 2010]. The massive MIMO systems shift the large number of antennas at BS and keeps MTs with single antenna because BS has lesser constraints on size and power. The multi-user setting preserves the multiplexing gains even in the LOS scenarios provided that the angular separation of MTs is more than the Rayleigh resolution of BS antenna array. These systems have a much larger number of antennas at BS than the number of single antenna users served simultaneously in each cell. This setting creates the well conditioned hardened channel matrix based on the Marčenko-Pastur law which states that if an $N \times K$ matrix $\mathbf{H}$ has zero mean i.i.d. entries with unit variance then the empirical distribution of the eigenvalues of $\frac{1}{N}\mathbf{H}^H\mathbf{H}$ converges almost surely, as $N, K \to \infty$ with $K/N \to \psi$, to the following density function [Tulino and Verdu, 2004; Chockalingam and Rajan, 2014]:

$$f_X(x) = \left(1 - \frac{1}{\psi}\right)^+ \delta(x) + \frac{\sqrt{(x-a)^+(b-x)^+}}{2\pi\psi x}\,, \tag{2.1}$$

where $(z)^+ = max(z, 0)$, $a = (1 - \sqrt{\psi})^2$, and $b = (1 + \sqrt{\psi})^2$. For $\psi << 1$, the eigenvalues of $\frac{1}{N}\mathbf{H}^H\mathbf{H}$ are bounded significantly away from zero. With such a well conditioned channel matrix, a linear processor like zero forcing (ZF) i.e. the pseudo inverse operator $((\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H)$ can perform satisfactorily when used as a decoder or a precoder in a massive MIMO system. ZF in such a setting not only forces the intra-cell interference to zero but also averages out the additive i.i.d. random noise. The lesser computational complexity of ZF processor is suitable for a very large number of antennas at BS.

Marzetta [2010] considers the multi-cell setting without inter-cell cooperation. The uplink and downlink communications work in the time division duplex (TDD) mode in a time frame smaller than the coherence time of the wireless channel. Therefore, the channel matrix remains almost constant during UL and DL transmissions. However, the inter-cell interference originated during training phase—called 'pilot contamination' [Jose *et al.*, 2009]—appears as a bottleneck in the system. Such a problem was not evident in earlier similar work on single cell setting [Marzetta, 2006]. The pilot sequences are re-used by MTs in neighboring cells. During the phase of channel estimation of its own MTs, a BS inadvertently learns the channel to MTs in other cells sharing the same pilots. When this contaminated CSI is used for decoding the user data at a BS from MTs of its own cell, the coherent combining takes place of signals coming from MTs of others cells in uplink. Similarly, in downlink, the signals with precoded user data intended for MTs of its own cell, also combine coherently at MTs of other cells. Thus, the pilot contamination leads to the inter-cell interference in data transmission in both uplink and downlink.

The detailed understanding of the massive MIMO systems appears in [Rusek *et al.*, 2013; Lu *et al.*, 2014; Larsson *et al.*, 2014; de Lamare, 2013; Ngo *et al.*, 2013a; Chockalingam and Rajan, 2014; Ngo, 2015]. The system modeling and analysis of achievable rates remain in the center of these works. They provide the system model and the achievable rates for massive MIMO systems with perfect and imperfect CSI. The discussion on propagation, antenna array, linear decoder-precoder, detection methods, and multi-cell operation is also provided in these papers. They also discuss the challenges and the issues that remain open before this technology comes to deployment.

Along with these theoretical research activities, the channel measurement campaigns and studies are also carried out in the literature [Payami and Tufvesson, 2012; Hoydis *et al.*, 2012].

The results of channel measurements have shown promising channel matrices for massive MIMO systems. However, there is a deviation in the measured channel from the channel models employed in theoretical studies, still a lot of gains are expected from the measured channel for a practical massive MIMO system. In the following sections, the particular research activities conducted for channel modeling, channel estimation, sum-rate analysis, and system design for massive MIMO are discussed. The gaps in existing research are analyzed and found out where more work is needed to bring massive MIMO systems a few steps closer to deployment.

## 2.2 MASSIVE MIMO SYSTEM MODEL

Let's consider a system having $L$ number of cells, $K$ number of MTs per cell, one BS per cell, and $N$ number of antennas per BS. Channel gain between $n^{th}$ antenna of BS in $m^{th}$ cell and $k^{th}$ MT in $l^{th}$ cell is represented by $g_{nklm}$ as shown in Figure 2.1. The basic design of a massive MIMO system is non-cooperative across cells which means BS's of different cells do not cooperate in estimating the channel and precoding/decoding of the data streams. Thus, a non-cooperative massive MIMO system estimates $g_{nklm}$ for $l = m$ only. In $m^{th}$ cell, set of channel coefficients can be represented by a channel matrix $\mathbf{G}_{mm} = [g_{nkmm}]$ where $g_{nkmm}$ is the $(n,k)^{th}$ entry of channel matrix. Linear processor matrix used in decoding/precoding — derived from $G_{mm}$ — is represented by $\mathbf{A}_m$. $K \times 1$ complex vectors $\mathbf{d}_l^r$ and $\mathbf{u}_m^r$ are received vectors at $K$ MTs in downlink and at $N$ antennas of BS in uplink respectively. Similarly, $K \times 1$ complex vectors $\mathbf{u}_l^t$ and $\mathbf{d}_m^t$ are transmitted vectors from $K$ MTs in uplink and from $N$ antennas of BS in downlink respectively. $\mathbf{w}_m^u$ is an additive white Gaussian noise (AWGN) vector and is considered to be added to signal at $N$ antennas of BS. Similarly, $\mathbf{w}_l^d$ is an AWGN vector and is considered to be added to signal at $K$ MTs.

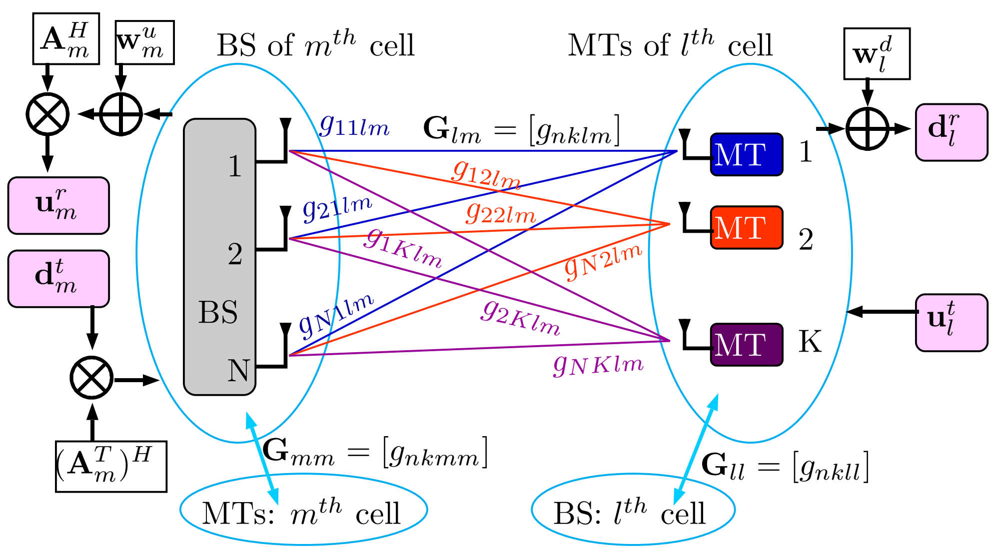


**Figure 2.1. :** Cellular massive MIMO system: base band equivalent system view.

Now system model for baseband equivalent uplink can be written by (2.2, 2.3) and for downlink by (2.4, 2.5). Channel matrix $\mathbf{G}_{lm}$ is further partitioned into two components as in (2.6). The $N \times K$ dimensional matrix $\mathbf{H}_{lm}$ represents the small-scale fading coefficients while the $K \times K$ diagonal matrix $D$ represents combined effect of path loss, large-scale fading, antenna radiation pattern, and other static constituents of the channel matrix. $p_u$ is the measure of common transmit power. If elements of $\mathbf{w}_m^u$ and $\mathbf{w}_m^d$ vectors have unit variance, the $p_u$ can be interpreted as 'transmit SNR' or 'unprocessed SNR', whereas the effective SNR is different which is calculated after involving the linear processing. Different types of decoding/precoding matrices are derived from channel matrix as shown in (2.7) using linear operations namely match filtering (MF), zero forcing (ZF) and minimum mean square error (MMSE) [Ngo *et al.*, 2013a]. The $\mathbf{I}_K$ is a $K \times K$ dimensional identity matrix.

The received signal vector at BS:

$$\mathbf{y}_m^r = \sqrt{p_u}\mathbf{G}_{mm}\mathbf{u}_m^t + (\sqrt{p_u}\sum_{l=1;l\neq m}^{L}\mathbf{G}_{lm}\mathbf{u}_l^t) + \mathbf{w}_m^u. \tag{2.2}$$

The decoded user data vector at BS:

$$\mathbf{u}_m^r = \mathbf{A}_m^H\mathbf{y}_m^r. \tag{2.3}$$

The received user vector at MTs:

$$\mathbf{d}_m^r = (\sqrt{p_u}\mathbf{G}_{mm}^T\mathbf{y}_m^t) + (\sqrt{p_u}\sum_{l=1;l\neq m}^{L}\mathbf{G}_{lm}^T\mathbf{y}_l^t) + \mathbf{w}_m^d. \tag{2.4}$$

The transmit vector (precoded user vector) at BS:

$$\mathbf{y}_m^t = (\mathbf{A}_m^T)^H\mathbf{d}_m^t. \tag{2.5}$$

$$\mathbf{G}_{lm} = \mathbf{H}_{lm}\mathbf{D}_{lm}^{1/2} \text{ or } g_{nklm} = h_{nklm}\beta_{kklm}. \tag{2.6}$$

$$\mathbf{A}_m = \begin{cases} \mathbf{G}_{mm} & MF \\ \mathbf{G}_{mm}(\mathbf{G}_{mm}^H\mathbf{G}_{mm})^{-1} & ZF \\ \mathbf{G}_{mm}(\mathbf{G}_{mm}^H\mathbf{G}_{mm} + \frac{1}{p_u}\mathbf{I}_K)^{-1} & MMSE. \end{cases} \tag{2.7}$$

In the decoding process in uplink, MF is an operation of multiplying the conjugate transpose of channel matrix with baseband equivalent received signal vector from $N$ antennas of BS. Mathematically, this operation is the projection of received signal onto the direction of transmitted vector which maximizes the SNR of desired signal streams. ZF is an operation of multiplying the pseudo inverse of channel matrix with baseband equivalent received signal vector from $N$ antennas of BS. Mathematically, this operation is the projection of received signal in the direction which is orthogonal to the subspace of interference signal. This operation forces the inter-stream interference to zero. MMSE is similar to ZF operation except for that the output of MMSE operation is bounded which prevents the effective SNR from falling down during ill-conditioning of the channel matrix. This operation minimizes the mean square error of desired signal streams. The process at BS in downlink is equivalent to the process in uplink except for that the transpose of uplink channel matrix is used for the operation.

## 2.3 CHANNEL MODELS

The channel modeling in theoretical research for massive MIMO is based on the propagation of the wireless signal in rich scattered environment [Tse and Viswanath, 2005; Goldsmith, 2005]. The elements of channel matrix are assumed circularly symmetric complex Gaussian distributed i.i.d. random variables ($\sim \mathbb{CN}(0,\sigma^2)$) with zero mean for establishing the expressions for maximum achievable rates in early works. The channel with such a distribution results into classical Rayleigh faded amplitudes of the channel coefficients, therefore, also called i.i.d. Rayleigh faded channel. Recently, a significant research has been carried out on channel models for massive MIMO systems [Zheng *et al.*, 2014]. The channel models for massive MIMO can be broadly divided in two categories: correlation-based stochastic channel models (CBSCMs) and geometry-based stochastic channel models (GBSCMs). The early channel model with i.i.d. Rayleigh faded channel is a variant of CBSCMs but without correlation. Other models in CBSCM category are Kronecker-based stochastic channel model (KBSCM), the Weichselberger model [Weichselberger *et al.*, 2006], and the virtual channel representation (VCR). GBSCMs can be further divided into two sub-categories: 2-D based and 3-D based models.

Figure 2.2 provides a brief classification of existing channel models for massive MIMO systems. CBSCMs are widely used for evaluation of achievable rates and performance of massive MIMO systems because of their lower implementation complexity and mathematical tractability. However, they have compromised accuracy due to oversimplification. On the other hand, GBSCMs are more accurate and consistent with measured channel data but they are not suitable for system evaluation due to their higher complexity and large number of input parameters.

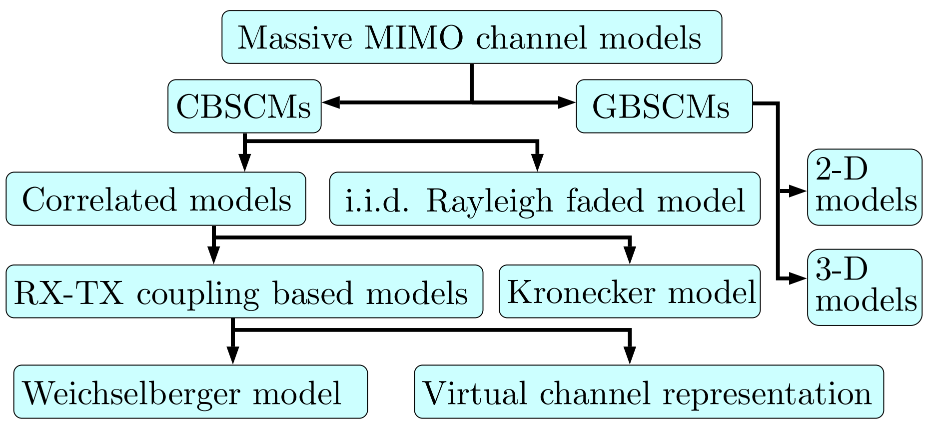


**Figure 2.2. :** Classification of channel models for massive MIMO systems.

### 2.3.1 Correlation-Based Stochastic Channel Models

In [Marzetta, 2010; Mohammed and Larsson, 2013; Rusek *et al.*, 2013], the classical CBSCM without correlation, i.e., i.i.d. Rayleigh faded channel model is utilized. The opportunity with such models is that the central limit theorem and results of large random matrix theory can easily be applied for the analysis of the system. However, this model ignores the correlation across the antennas of BS and across the users. Therefore, they are suitable for BS's with largely separated antennas and a rich scattered environment in surroundings.

One step ahead to the i.i.d. Rayleigh faded channel model in accuracy and in complexity is the KBSCM which has been used in [Couillet *et al.*, 2011; Noh *et al.*, 2014] with correlation across BS antennas and across MTs. The two correlation matrices are multiplied from left and right with the i.i.d. Rayleigh faded channel matrix to give the Kronecker channel matrix as output. However, this model keeps the transmitter's and receiver's correlation matrices separable. Therefore, the joint processing of transmitter-receiver correlation is not possible in this model.

Weichselberger channel model [Wen *et al.*, 2011] relaxes the separability restriction of Kronecker channel model. This model introduces a coupling matrix which is multiplied using Hadamard product with the i.i.d. Rayleigh faded matrix in Kronecker channel model and the correlation matrices are replaced by their SVD unitary matrices [Chockalingam and Rajan, 2014, Ch:11]. The coupling matrix helps to model the joint processing of the correlation of transmitter and receiver and can be obtained by channel measurement.

The VCR model uses the discrete Fourier transform (DFT) matrices to simulate the massive MIMO channel. This model is based on the angular transformation of the signal over uniform linear array (ULA). The accuracy of this model increases with increasing elements in ULA [Ozcelik *et al.*, 2005]. However, this model is only applicable to ULA with single polarization [Sayeed, 2002].

### 2.3.2 Geometry-Based Stochastic Channel Models

GBSCMs are propagation based models. A narrow band fading channel for massive MIMO is modeled [Chen and Lau, 2014] using an extended one-ring channel model (relatively simplified GBSCM) for MIMO from [Zhang *et al.*, 2007]. In this model, almost no scatterers are considered in surrounding of BS and an infinite number of uniform randomly distributed scatterers is considered on a ring around MTs. Apart from this, cluster-based approach is frequently employed for GBSCMs. The measured channel data are utilized in [Gao *et al.*, 2013] for cluster-based channel modeling. The number of the clusters, the visibility region (VR) of the clusters, and the visibility gain of the clusters is used for modeling the channel in this work. Another variant of cluster-based modeling with multi-link simulation is presented in [Poutanen *et al.*, 2012] by introducing the concept of common clusters.

A novel non-stationary three dimensional (3-D) wideband twin-cluster model for massive MIMO is proposed in [Wu *et al.*, 2014]. The non-stationary properties of the channel over array-axis and over the time-axis are simulated by introducing appearance and disappearance of clusters over array and time axes. The appearance and disappearance of clusters are simulated by death-birth process. In addition, the near field effect using spherical wavefront is also incorporated in this model. The spherical wavefront becomes more significant for a large antenna array and for clusters closer to BS. As being a 3-D model, the impact of elevation angles of the clusters are taken into account for correlation properties.

Using similar death-birth process framework, Wu *et al.* [2015] propose a wideband two dimensional (2-D) GBSCM for massive MIMO systems. In this model, the clusters are located on several ellipses which are confocal but have different lengths of major axes. With such a model, the angles of arrival (AoAs) and angles of departure (AoDs) are dependent which provides a joint modeling of transmitter and receiver correlations. Recently, a new GBSCM is proposed under project "Mobile and wireless communications Enablers for the Twenty-twenty Information Society" (METIS) [Raschkowski *et al.*, 2015]. The METIS channel model is developed based on WINNER II channel model [Kyösti *et al.*, 2007] but with consideration of spherical wavefront, AoAs, and AoDs as in [Wu *et al.*, 2014, 2015].

### 2.3.3 Need for a New Channel Model

Existing classes of channel models i.e. CBSCMs and GBSCMs have their own limitations. GBSCMs are mainly developed for simulation of a realistic channel with inclusion of realistic propagation parameters like geometry of clusters, evolution of clusters along time and antenna array, AoAs, AoDs, and multipath components (MPCs). But, they are not suitable for evaluation and understanding of communication schemes and system designs due to their higher complexity.

On the other hand, the CBSCMs are important channel models for such purposes. Correlation across BS antennas, across MTs, and along time remains the central idea in these models. CBSCMs assume channel coefficients as random variables sampled along space, time and frequency.

The variations in the channel over time are caused by Doppler spread in the signal. In a typical environment, the mobility of MTs is a major contributor to the Doppler spread. Further, the surroundings of MTs are typically rich scattered and the signal is the sum of a large number of MPCs with different Doppler shifts. Therefore, it is reasonable to consider the variations being random along time. The spectrum of these variations is bounded by the Doppler spectrum. Therefore, there is a correlation between consecutive channel realizations along time axis when the time scale is taken at the order of the coherence time of the channel. These underlying phenomena support the CBSCMs for simulation of channel along time.

The variations in the channel over frequency are caused by delay spread. Similar to the temporal variations, the variations over frequency can be modeled as random variables because the resultant signal has several MPCs of different path lengths. Similar to the temporal correlation, there is a correlation between consecutive channel realizations along the frequency axis when the frequency scale is of the order of the coherence bandwidth of the channel. Thus, the underlying phenomena support the CBSCMs also for the simulation of channel along frequency.

The variations in the channel over BS array axis are caused by variations in the phase of incoming MPCs from antenna to antenna. The change in the phase of a given MPC for a given time and frequency, depends primarily on AoA of that MPC. When several MPCs come from different directions, the change in phase of an individual MPC is different which leads to random variations in the channel amplitude and phase. However, if angular spread of the signal is small, then the changes in the phase of all MPCs from one antenna to next antenna are similar which does not change the interference pattern of MPCs significantly along a spatial dimension. Consequently, there is only change in phase of the overall channel over the array. In such cases, the variation in the channel over array obtains a shape of complex sinusoidal function as the angular spread becomes narrower. Modeling of such a variation by a random variable is not suitable in the point of view of accuracy. Line-of-sight (LOS) propagation is one of such cases where correlation across antenna is high. But, the inner product of the measured channel in [Payami and Tufvesson, 2012] for a large antenna array in multi-user settings shows a very small cross user correlation for LOS scenario. Therefore, there is need for analytically tractable channel model which can accurately model the spatial variations along with temporal and spectral variations of the channel.

## 2.4 CHANNEL ESTIMATION

As discussed in the chapter 1, channel matrix is the prime requirement for processing the signal in massive MIMO systems. Thus, there exists an extensive research on the channel estimation for massive MIMO systems. The channel estimation can be broadly classified into two categories: pilot-based estimation and blind estimation. In pilot-based estimation, a known signal is transmitted by MTs which is orthogonal across MTs. BS uses this signal to estimate the channel. In blind estimation, the uplink data is the basis for channel estimation. BS uses the uplink data along with some properties of channel—sometimes with partial training signals—to estimate the channel.

### 2.4.1 Pilot-Based Methods and Associated Issues

The pilot-based estimation has been considered for channel estimation from the preliminary work on massive MIMO systems [Marzetta, 2006, 2010]. Further, an enhanced design of pilot signals is presented in [So *et al.*, 2015] where training-based received SNR maximization approach is used. A low complexity channel estimation with pilot-based method is proposed in [Shariati *et al.*, 2014] where Polynomial ExpAnsion CHannel (PEACH) estimators are introduced for arbitrary channel and interference statistics. However, the pilot-based estimation have two major drawbacks. First, they consume a number of symbols in each RB which is at least equal to the number of MTs served simultaneously. In the scenarios where the size of an RB is small due to low coherence time, the pilot signals significantly overload the system. Consequently, the spectral efficiency of the system reduces drastically.

Another drawback of pilot-based estimation is that during the channel estimation stage, albeit being orthogonal inside the cell, the pilot sequences are re-used in neighboring cells which creates an interference and contaminates the estimated channel. During the channel estimation, the advantages of a massive MIMO system are not available because the array gain is absent in this stage. The contaminated channel estimate is not mere the deviated version of the desired estimate. Rather, this estimated channel when used for decoding or precoding the user data, introduces inter-cell interference in both uplink and downlink. In uplink, the usage of such an estimate constructively combines the inter-cell interference coming from the MTs of neighboring cells, that share the same pilots. Similarly during downlink, the usage of such an estimate makes a BS transmit the beam-formed signal to MTs of neighboring cells, that share the same pilots. Thus, an inter-cell interference is caused in the system.

The issue of pilot contamination is addressed in [Yin *et al.*, 2013; Bogale and Le, 2014; Neumann *et al.*, 2014]. Yin *et al.* [2013] have considered the coordinated approach for channel estimation for suppression of pilot contamination. Bogale and Le [2014] consider the optimization algorithm for creating an optimized set of pilots by converting the problem of channel estimation into the weighted sum mean square error (WSMSE) minimization problem. Neumann *et al.* [2014] utilize the uplink data for suppression of pilot contamination which is based on maximum a-posteriori criterion. Although, these methods perform well over the previously available simple pilot-based methods. But the efforts put for pilot decontamination result into a higher computational complexity. Further, these methods address only a part of the problem. The issue of spectral efficiency remains unaddressed in these methods. Therefore, the need for blind estimation methods becomes significant.

### 2.4.2 Blind Methods and Associated Issues

Joint processing of pilots and uplink data has been proposed as an efficient approach for MIMO systems in early literature [Jindal *et al.*, 2009]. Blind estimation methods for massive MIMO systems available in the literature, in general, depend on the properties of large random matrices and asymptotic settings [Muller *et al.*, 2014; Ngo and Larsson, 2012; Alshamary and Xu, 2016]. Muller *et al.* [2014] have proposed a sub-space based blind pilot decontamination method aimed at removing the pilot contamination. However, the pilot usage is same as in conventional MIMO systems. Ngo and Larsson [2012] propose an eigenvalue decomposition (EVD) based method with a small amount of pilot training. The iterative least square projection (ILSP) from [Talwar *et al.*, 1996] is further embedded with EVD based method to enhance the performance. The optimal joint decoding of data and channel without pilots is investigated in [Alshamary and Xu, 2016] with non-exponential complexity in coherence time based on generalized likelihood ratio test (GLRT).

There are a few issues with the existing blind estimation methods for massive MIMO systems. The aforementioned methods rely on asymptotic settings either in the number of symbols in an RB ($T$) or in the number of the base station antennas ($N$) or in both. The method of [Ngo and Larsson, 2012] has an inherent bottleneck in judging the ordering of channel vectors in estimated channel matrix. This method uses a distinct mapping between the eigenvalues of channel-covariance matrix and MTs in the cell. This mapping is obtained by ordering of average channel gains ($\beta$) for MTs. However, ideally such a mapping is feasible for infinitely large $N$ and $T$. When this solution is applied with finite $N$ and $T$, then the point-to-point distinctive mapping is not possible, rather a range based distinct assignment is possible. For practical settings, this mapping can support only a few number of MTs ($K$) in a cell which specifically have average channel gains that are at least separated by this allowed range. For a larger $K$, there is a high probability that separation between average channel gains of two or more MTs will be less than the required minimum separation for distinct mapping. Several random locations in a cell have similar average channel gains due to the significant contribution of shadow fading in the statistics of the channel. Due to these reasons, there is an insignificant probability that the increasing number of MTs will have required distinct intervals of average channel gains. Moreover, the required stationarity of the channel over time has to be addressed before using statistics for such a mapping. The complexity of EVD-ILSP method is another issue which imposes a need for more work on blind channel estimation.

The method of [Muller *et al.*, 2014] is blind in a sense that it aims at removing the pilot contamination without CSI. In this method, the received signal matrix is projected on a subspace of the unitary matrix obtained from the singular value decomposition (SVD) of the received signal matrix. This projection converts the $N \times K$ dimensional massive MIMO system into a $K \times K$ dimensional conventional MIMO system which needs pilot-based channel estimation. The major claim of the scheme is the array gain without complete CSI. However, the scheme relies on asymptotic settings. For a realistic ratio of $N$ and $K$ (for example $N/K = 10$), the array gain for a desired link is slightly higher than the gains for interfering links. Since, this process converts the $N \times K$ massive MIMO system into a $K \times K$ conventional MIMO system, therefore, no further massive MIMO gains are possible.

The scheme proposed in [Alshamary and Xu, 2016] has lower complexity compared to the exhaustive search methods used for comparison in their work. However, the computational complexity is still very high which limits the scheme usage to a small number of symbols and MTs. The higher dependency on asymptotic settings is again reflected in this work during performance and complexity evaluation. In general, the dependency of blind estimation methods for massive MIMO systems on higher value of $N/K$ and $T/K$ has been found in the literature. For a practical system, $T/K$ may not be very high due to limited coherence time and $N/K$ also need to be small for cost effectiveness. Therefore, there is a need for a new blind channel estimation method which can reduce these dependencies by some means with a manageable complexity and lesser limitations on system parameters.

## 2.5 SUM-RATE AND AVERAGE POWER-ALLOCATION

Higher rate with limited resources is one of the driving forces behind the extensive research on massive MIMO systems. As multiple MTs are served using the same time-frequency resources, the sum-rate of the system is an attractive parameter which the researchers want to analyze and improve. When switching from point-to-point MIMO to massive MIMO, the asymptotic settings and user specific statistics of channel come into picture while analyzing the rates. The fundamental work on non-cooperative cellular massive MIMO systems along with preliminary results on system architecture, capacity, and communication strategy is proposed by Thomas L. Marzetta [2010]. The detailed understanding of such systems in the context of system modeling and capacity appears in [Rusek *et al.*, 2013; Lu *et al.*, 2014] where several practical aspects of wave propagation and signal processing along with perfect CSI are considered. However, the assumption of availability of perfect CSI is theoretical and useful for demonstration of the potentials of massive MIMO systems.

For a practical setting, a different sum-rate analysis is required because as CSI varies with time and frequency, it has to be estimated frequently at a cost of a few resources. Further, this estimated CSI is imperfect. The work on sum-rate for practically affordable linear signal processing techniques is presented by Ngo *et al.* [2013a] where the performance of MF, ZF, and MMSE along with the spectral efficiency of the massive MIMO systems are discussed. A brief discussion on multi-cell settings is also provided. The analysis of uplink sum-rate of a multi-cell multi-user SIMO system with ZF processing is provided in [Ngo *et al.*, 2013c]. The analysis on uplink sum-rate for a multi-cell multi-user massive MIMO systems with finite dimensional channel appears in [Ngo *et al.*, 2013b] with consideration of pilot contamination and inter-cell interference. The work on sum-rate is further extended with channel aging and correlated channels in [Kong *et al.*, 2015] and [Liu *et al.*, 2016a] respectively.

The system models in aforementioned literature consider equal average transmit power at MTs described by a single scalar multiplier (cf. $p_u$ in (2.2) and (2.4)) for representing the average transmit power per MT [Marzetta, 2010; Rusek *et al.*, 2013; Ngo *et al.*, 2013b,a,c; Kong *et al.*, 2015]. These existing system models implicitly (sometimes explicitly) leave the average power-control for future work. Since the different MTs have largely different channel gains [Erceg *et al.*, 1999], thus the average power-control may play an interesting role in maximizing the sum-rate. In case of imperfect CSI at receiver in uplink, the change in rate and interference are highly varying with user locations without power control. The fairness among MTs can also be improved along with energy efficiency (EE) of the system by average power-control.

The average channel gains have been used as a part of the system models in existing literature of massive MIMO systems. However, the exploitation of average channel statistics has received an attention recently. A power-allocation scheme (termed Max-Min power-allocation) based on the maximization of minimum achievable rate is provided in [Zarei *et al.*, 2017]. The power-allocation schemes for cell-free and small cell massive MIMO systems are explored in [Nayebi *et al.*, 2017] where full power assignment scheme is shown to be better than Max-Min power-allocation scheme for cellular massive MIMO systems. A power-allocation scheme based on standard interference function (SIF) is provided in [Zhang *et al.*, 2016] which maximizes the EE. Another power-allocation scheme by optimizing the EE over number of BS antennas and active users is provided in [Björnson *et al.*, 2015]. The maximization of sum-rate along with power-allocation scheme is explored in [Nguyen *et al.*, 2015] where an improvement in sum-rate is shown by pilot assignment algorithm.

The sum-rate maximization by power-allocation is discussed in [Dai and Dong, 2016] where a multi-pair massive MIMO two-way amplify-and-forward relay with MF/ZF is considered with single-cell setting. Maximization of sum-rate and EE for a full duplex single-cell massive MIMO system under the effect of self interference and co-channel interference is discussed in [Li *et al.*, 2017]. Considering the existing literature, there is a possibility for further research to bring out the simultaneous implementation of sum-rate, EE and fairness optimization by average power-allocation in inter-cell interference limited multi-cell massive MIMO systems. Meanwhile, the sum-rate maximization could be kept central to the power-allocation scheme which lacks in existing literature.

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