5
Power-allocation and Sum-rate in Massive MIMO Systems

In the previous chapter, the small scale fading channel matrix was explored and an estimation technique was presented for this matrix. The estimation of large scale fading plus path loss matrix (**D**) presented in our system model (2.2) is not a bottleneck due to its slow variation. However, the large variations in diagonal elements of **D**, i.e., the variations in average channel gains over space become crucial for power allocation strategies in the system. In this chapter, the distinctive nature of average channel gains is exploited to improve the sum-rate, energy consumption, and control on fairness among MTs.

Power is the most fundamental resource in wireless communication as discussed in chapter 1. The allocation of power becomes a challenge in two cases: the multi-user setting and interference limited setting. A massive MIMO system has both of these conditions as the viable system is designed to work in multi-user non-cooperative multi-cell settings where the inter-cell interference is a limiting factor influenced by power-allocation scheme of the system. Under the assumption of imperfect CSI, the inter-stream interference within a cell is also influenced by power-allocation scheme. Due to a large variations in average channel gain over MTs, the fairness and the sum-rate are also influenced by power-allocation strategy. This chapter is dedicated to analysis of sum-rate by using an average power control at MTs and to propose a power-allocation optimization scheme for improving the sum-rate, energy consumption, and the control on fairness among MTs.

As discussed in chapter 2, the system models in early literature of massive MIMO systems [Marzetta, 2010; Rusek *et al.*, 2013; Ngo *et al.*, 2013b,a,c; Kong *et al.*, 2015] consider equal transmit power at MTs which can be described by a scalar multiplier p_u as in (5.1). The use of single scalar not only limits the applicability of the system model but also raises the issue of fairness due to large variations of average channel gains across MTs. The analysis of sum-rate under average power control is not possible using existing system models in the literature resembling with (2.2) and (5.1).

$$\mathbf{y}_l = \sqrt{p_u}\mathbf{G}_{ll}\mathbf{x}_l + \sqrt{p_u}\sum_{j=1;j\neq l}^{L}\mathbf{G}_{jl}\mathbf{x}_j + \tilde{\mathbf{w}}_l \ , \tag{5.1}$$

where

- $\mathbf{G}_{jl}$ is the baseband equivalent complex channel matrix representing the channel gains from K active MTs of j^{th} cell to N antennas of BS in l^{th} cell,
- $\mathbf{y}_l$ is the received complex symbol vector across N antennas of BS in l^{th} cell,
- $\sqrt{p_u}\mathbf{x}_l$ is the transmitted complex symbol vector from K active MTs of l^{th} cell with p_u as transmit power per MT,
- $\tilde{\mathbf{w}}_l$ is a complex additive white Gaussian noise (AWGN) vector across N antennas of BS in l^{th} cell, and

- The elements of $\tilde{\mathbf{w}}_l$ are i.i.d. with zero mean and unit variance leading to the interpretation of p_u as 'transmit SNR'.

As discussed in chapter 2, a system model needs to be established which has power control factor for individual MTs to obtain an insight on how the sum-rate of the system is affected by different transmit powers from MTs. While working with inter-cell interference limited regime which is expected under the setting of unity frequency reuse factor for massive MIMO systems, the sum throughput might not be increased by increasing the power of all MTs uniformly. Under such settings, the common power scaling factor of (5.1) loses its significance. Consequently, a new system model is required which is free of such a parameter, albeit being analytically tractable for calculating sum throughput. Moreover, the inter-cell interference power is a function of the number of active MTs in the cells and the number of interfering cells. Thus, these parameters are also desired in system model so that the sum-rate analysis is more useful in overall system point of view.

First, an average power-control based system model is constructed which incorporates a simplified expression for inter-cell interference statistics. This system model is then used for theoretical analysis of MF and ZF sum-rates for a simple case of two users by extending the existing sum-rate expressions to the current system model. The theoretical analysis is followed by a numerical optimization for multi-user MF and ZF sum-rates with perfect and imperfect CSIR. The results show a significant improvement in sum-rate and power consumption over the case of equal full power assignment at MTs. The scheme explores several permutations of power-allocation to control the fairness among MTs along with keeping the sum-rate close to it's maximum value. A low complexity algorithm is proposed for numerical optimization. The performance results show that the sum-rate along with power consumption improves with increasing inter-cell interference and number of MTs in different scenarios like macro and micro cells with low and high inter-cell interference powers.

5.1 SYSTEM MODEL

Based on the Figures 1.5 and 2.1 in earlier chapters, the layout of a multi-cell massive MIMO system having MTs with power control is presented in Figure 5.1. The system has L number of cells with K active MTs in each cell at a time. The channel gain from k^{th} MT of j^{th} cell to n^{th} antenna of BS in l^{th} cell is g_{njlk} similar to the earlier definition. Since k^{th} MT in j^{th} cell is enabled with power control by introducing a factor γ_{jk}, the system model of (5.1) can be modified as follows:

$$\mathbf{y}_l = \sqrt{p_u}\mathbf{G}_{ll}\mathbf{S}_l\mathbf{x}_l + \sqrt{p_u}\sum_{j=1;j\neq l}^{L}\mathbf{G}_{jl}\mathbf{S}_j\mathbf{x}_j + \tilde{\mathbf{w}}_l \text{ or}$$

$$\mathbf{y}_l = \mathbf{G}_{ll}\mathbf{S}_l\mathbf{x}_l + \sum_{j=1;j\neq l}^{L}\mathbf{G}_{jl}\mathbf{S}_j\mathbf{x}_j + \frac{1}{\sqrt{p_u}}\tilde{\mathbf{w}}_l\ , \tag{5.2}$$

where $\mathbf{y}_l$ is $N \times 1$ received vector at BS in l^{th} cell. $\mathbf{S}_j$ is a diagonal matrix ($[\mathbf{S}_j]_{kk} = \sqrt{\gamma_{jk}}$; $0 \leq \gamma_{jk} \leq 1$) where diagonal element γ_{jk} is slowly varying power scaling factor at k^{th} MT of j^{th} cell. $\sqrt{p_u}\mathbf{S}_j\mathbf{x}_j$ is $K \times 1$ transmit symbol vector from K active MTs of j^{th} cell with p_u as 'maximum transmit power' from each MT such that the elements of $\mathbf{x}_j$ are zero mean unit variance i.i.d. random Gaussian complex symbols. $\tilde{\mathbf{w}}_l$ is an $N \times 1$ complex AWGN vector across N antennas of BS in l^{th} cell with zero mean unit variance elements such that p_u can be interpreted as 'maximum transmit SNR'.

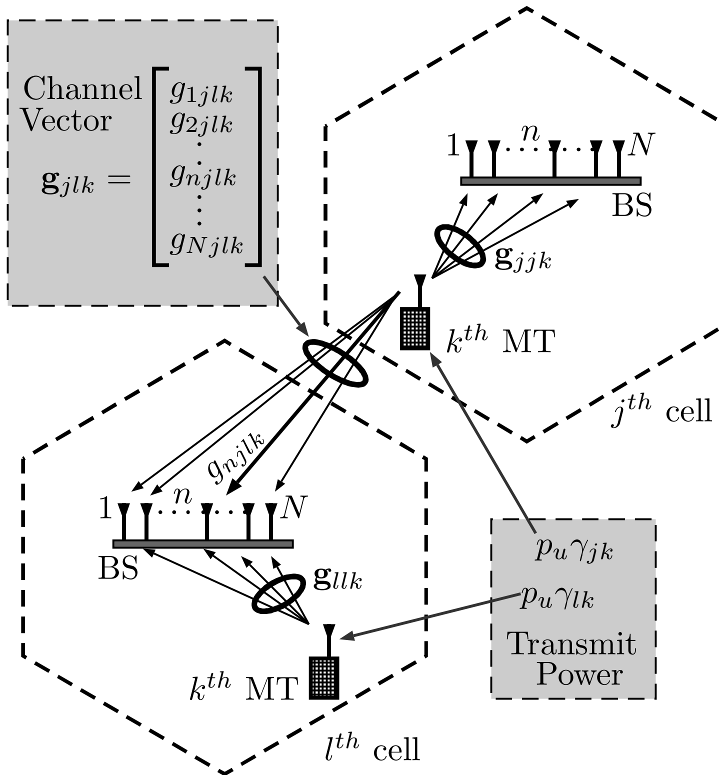

Figure 5.1. : Non-cooperative multi-cell massive MIMO system layout with average transmit power control in uplink.

$\mathbf{G}_{jl}$ is the overall channel matrix from K active MTs of j^{th} cell to N BS antennas of l^{th} cell. This matrix represents the small-scale fading, geometric attenuation, and large-scale shadow fading. As discussed in chapter 2, $\mathbf{G}_{jl}$ can be represented as follows:

$$\mathbf{G}_{jl} = \mathbf{H}_{jl}\mathbf{D}_{jl}^{1/2}\ , \tag{5.3}$$

where $\mathbf{H}_{jl}$—the CSI at receiver (CSIR) in uplink—is the channel matrix of small-scale fading coefficients having i.i.d. zero mean unit variance Gaussian random distribution. $\mathbf{D}_{jl}$ is a diagonal matrix where diagonal elements ($[\mathbf{D}_{jl}]_{kk} = \beta_{jlk}$) vary slowly with space and time specifying the path loss plus large scale fading from k^{th} MT in j^{th} cell to BS of l^{th} cell. β_{jlk} is assumed to be constant over a block of communication and calculated using the existing path loss model of [Erceg *et al.*, 1999, eq:1-6] as follows:

$$\beta_{jlk} = 10^{-\frac{PL(d_{jlk})}{10}}\ , \tag{5.4}$$

where d_{jlk} is the distance of k^{th} MT in j^{th} cell from BS of l^{th} cell and $PL(d_{jlk})$ is corresponding overall path loss. The details of the path loss model are given in Annexure B.3. The inter-cell interference signal at any of BS antennas is a sum of a large number ($(L-1)K$) of random signals. Therefore, the inter-cell interference signals at different antennas of BS can be modeled as i.i.d. Gaussian distributed random variables with an interference power ρ_l according to the central limit theorem. The system model of (5.2) then can be written as:

$$\mathbf{y}_l \approx \mathbf{G}_{ll}\mathbf{S}_l\mathbf{x}_l + \sqrt{\rho_l}\mathbf{w}_l + \frac{1}{\sqrt{p_u}}\tilde{\mathbf{w}}_l\ , \tag{5.5}$$

where

$$\rho_l = \mathbb{E}\left[\Big|\sum_{j=1;j\neq l}^{L}\sum_{k=1}^{K} h_{njlk}\sqrt{(\beta_{jlk})(\gamma_{jk})}x_{jk}\Big|^2\right]$$
$$\Rightarrow \rho_l \stackrel{(a)}{=} \sum_{j=1;j\neq l}^{L}\sum_{k=1}^{K}\mathbb{E}\left[\left|h_{njlk}\right|^2\right]\mathbb{E}\left[(\beta_{jlk})\right]\mathbb{E}\left[(\gamma_{jk})\right]\mathbb{E}\left[\left|x_{jk}\right|^2\right]$$
$$\Rightarrow \rho_l \stackrel{(b)}{=} \sum_{j=1;j\neq l}^{L}\sum_{k=1}^{K}\mathbb{E}\left[\beta_{jlk}\right]\mathbb{E}\left[\gamma_{jk}\right]$$
$$\Rightarrow \rho_l \stackrel{(c)}{=} (L-1)K\mathbb{E}\left[\beta_{jlk}\right]\mathbb{E}\left[\gamma_{jk}\right]\ , \tag{5.6}$$

where n is the BS antenna index and $\mathbb{E}$ is mathematical expectation which corresponds to the process of sample realization over time. Step (a) follows from the fact that h_{njlk}, β_{jlk}, γ_{jk}, and x_{jk} all are independent for $j \neq l$ and x_{jk} is zero mean. Step (b) follows from the fact that h_{njlk} and x_{jk} are zero mean and unit variance random variables. Step (c) follows from the non-cooperative settings such that the user scheduling is random and independent across cells. In such settings, β_{jlk} and γ_{jk}—looking from l^{th} cell, i.e., $j \neq l$—change their values randomly (due to random locations) at random time which makes them time varying random variables. Therefore, β_{jlk} and γ_{jk} make two sets of $(L-1)K$ independent and identically distributed random variables. The results on the rates in massive MIMO systems without average power-control exist in the literature [Ngo *et al.*, 2013a]. However, a brief explanation of the procedure is given in the context of system model with average power-control and inter-cell interference for the convenience. As discussed in chapter 2, a linear processor is defined as follows:

$$\tilde{\mathbf{A}}_l = \begin{cases}\tilde{\mathbf{G}}_{ll} & \text{for MF and}\\ \tilde{\mathbf{G}}_{ll}(\tilde{\mathbf{G}}_{ll}^H\tilde{\mathbf{G}}_{ll})^{-1} & \text{for ZF.}\end{cases}$$

The processed vector from (5.5) can be written as follows:

$$\mathbf{r}_l = \tilde{\mathbf{A}}_l^H\{\tilde{\mathbf{G}}_{ll}\mathbf{x} + \sqrt{\rho_l}\mathbf{w}_l + \tfrac{1}{\sqrt{p_u}}\tilde{\mathbf{w}}_l\}.$$

Let $\tilde{\mathbf{a}}_{lk}$ and $\tilde{\mathbf{g}}_{lk}$ be k^{th} column vector of $\tilde{\mathbf{A}}_l$ and $\tilde{\mathbf{G}}_{ll}$ respectively. The received symbol corresponding to k^{th} MT then can be written as:

$$r_{lk} = \tilde{\mathbf{a}}_{lk}^H\tilde{\mathbf{g}}_{lk}x_{lk} + \sum_{i=1,i\neq k}^{K}\tilde{\mathbf{a}}_{lk}^H\tilde{\mathbf{g}}_{il}x_i + \sqrt{\rho_l}\tilde{\mathbf{a}}_{lk}^H\mathbf{w}_l + \tfrac{1}{\sqrt{p_u}}\tilde{\mathbf{a}}_{lk}^H\tilde{\mathbf{w}}_l.$$

For a given realization of channel above equation can be written as follows:

$$r_{lk} = \sqrt{p_{lk}^s}x_{lk} + \sqrt{p_{lk}^a}z_{1lk} + \sqrt{p_{lk}^b}z_{2lk} + \sqrt{p_{lk}^c}z_{3lk}\ , \tag{5.7}$$

where

$$p_{lk}^s \triangleq |\tilde{\mathbf{a}}_{lk}^H\tilde{\mathbf{g}}_{lk}|^2,\ p_{lk}^a \triangleq \sum_{i=1,i\neq k}^{K}|\tilde{\mathbf{a}}_{lk}^H\tilde{\mathbf{g}}_{il}|^2,$$
$$p_{lk}^b \triangleq \rho_l\|\tilde{\mathbf{a}}_{lk}\|^2, \text{ and } p_{lk}^c \triangleq \frac{1}{p_u}\|\tilde{\mathbf{a}}_{lk}\|^2.$$

The random variables z_{1lk}, z_{2lk}, and z_{3lk} have zero mean, unit variance, and Gaussian distribution. By modeling the interference and noise parts in (5.7) as AWGN and considering the span of channel codes over independent realizations of the channel, the ergodic rate can be given by:

$$R_{P,lk} = \mathbb{E}\Big\{log_2\Big(1+\frac{p^s_{lk}}{p^a_{lk}+p^b_{lk}+p^c_{lk}}\Big)\Big\}. \tag{5.8}$$

The extension of the existing results on rates in massive system for our system model is straight-forward by replacing β_{llk} with $\beta_{llk}\gamma_{lk}$, β_{lli} with $\beta_{lli}\gamma_{li}$, and adding the inter-cell interference power term ρ_l to the noise power. The analysis, problem formulation, and solution remain around the average transmit power-control factors γ_{lk}.

5.1.1 Sum-Rate with Perfect CSIR

In the case of perfect CSIR, it is considered that the channel matrix is hypothetically known at BS all the time with full accuracy. By using (5.8) and the results of [Ngo *et al.*, 2013a], lower bounds on MF and ZF rates for k^{th} MT with perfect CSIR can be written as follows:

$$\tilde{R}^{MF}_{l,k} = \log_2\left\{1+\frac{(N-1)\beta_{llk}\gamma_{lk}}{\sum_{i=1,i\neq k}^{K}\beta_{lli}\gamma_{li}+\rho_l+\frac{1}{p_u}}\right\} \text{ and} \tag{5.9}$$

$$\tilde{R}^{ZF}_{l,k} = \log_2\left\{1+\frac{(N-K)\beta_{llk}\gamma_{lk}}{\rho_l+\frac{1}{p_u}}\right\}. \tag{5.10}$$

First, we put a theoretical analysis on sum-rate with MF processing based simple case of two users (MT_1 and MT_2) and high SNR ($p_u \to \infty$) which will be followed by a numerical optimization for multi-user settings. With simplified notations ($\beta_{llk} \to \beta_k$, $\gamma_{lk} \to \gamma_k$, and $\rho_l \to \rho$), the sum-rate in this case in a given cell is:

$$R_{MF} = \log_2\left\{1+\frac{(N-1)\beta_1\gamma_1}{\beta_2\gamma_2+\rho}\right\}+\log_2\left\{1+\frac{(N-1)\beta_2\gamma_2}{\beta_1\gamma_1+\rho}\right\}.$$

Let $\beta_1 > \beta_2$, i.e., MT_1 has a better channel gain, $\gamma_1 = 1$, i.e., MT_1 transmits its full power, and $0 \leq \gamma_2 \leq 1$, i.e., the power of MT_2 is varied. The sum-rate then becomes:

$$R_{MF} = \log_2\left\{1+\frac{(N-1)\beta_1}{\beta_2\gamma_2+\rho}\right\}+\log_2\left\{1+\frac{(N-1)\beta_2\gamma_2}{\beta_1+\rho}\right\}. \tag{5.11}$$

Lemma 5.1.1. *For two users' uplink MF sum-rate with perfect CSIR, there exists a unique* $\rho = \rho_c$ *where the sum-rate is same whether the weak user is transmitting full power or not transmitting at all.*

Proof: See Annexure B.1.

In the case of 2-users MF rate, ρ_c defines a critical value of inter-cell interference power as shown in Figure 5.2. The range of ρ is selected on the basis of (5.6). Next, it is analyzed if there exists a maxima/minima on the sum-rate for some $0 < \gamma_2 < 1$ with $\rho = \rho_c$ by solving $\frac{d}{d\gamma_2}(R_{MF}) = 0$ to produce the following lemma.

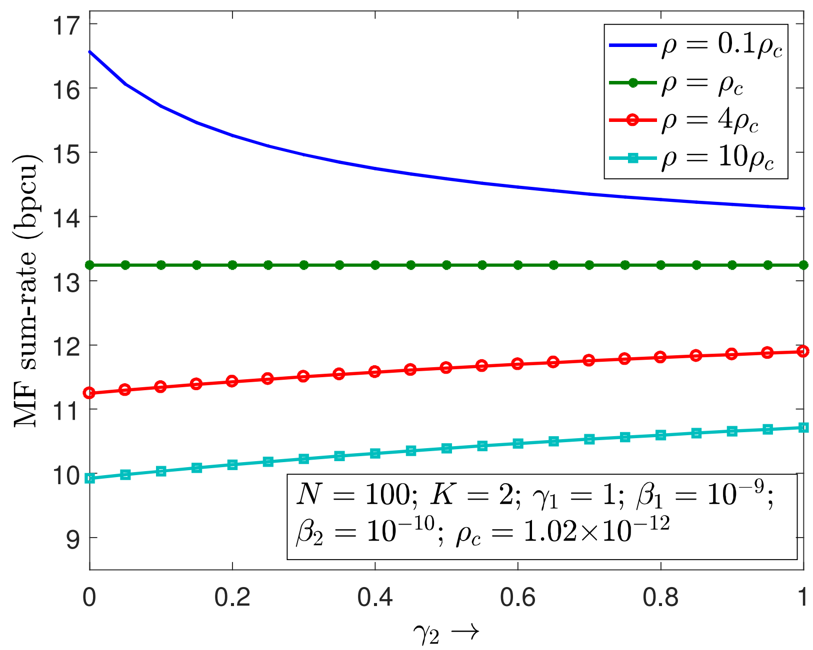

Figure 5.2. : Sum-rate versus transmit power factor (γ_2) for 2 users with MF processor and perfect CSIR.

Lemma 5.1.2. *For two users' uplink with perfect CSIR, MF sum-rate is not flat with $0 \leq \gamma_2 \leq 1$ for $\rho = \rho_c$.*

Proof: See Annexure B.2.

The results corresponding to (5.11) show that $\gamma_2 = \gamma_c$ (solution of $\frac{d}{d\gamma_2}(R_{MF}) = 0$) for $\rho = \rho_c$ corresponds to a minima in case of 2 users. However, the value of sum-rate at this minima is approximately same as the sum-rate at $\gamma_2 = 0$ and $\gamma_2 = 1$ as shown in Figure 5.2 (plot with $\rho = \rho_c$). The behavior of sum-rate with respect to γ_2 is determined by the value of ρ.

Theorem 5.1.3. *The power-allocation optimization for multi-user MF sum-rate with perfect CSIR under the system model given by (5.5) is possible simultaneously for both sum-rate maximization and reduction of inter-cell interference by selecting appropriate $0 < \gamma_2 < 1$ for certain range of inter-cell interference.*

Proof: Proof of Lemma 5.1.2 in Annexure B.2 leads to a unique solution for $\rho = \rho_c$ where $R_{MF}|_{\gamma_2=0}$ is equal to $R_{MF}|_{\gamma_2=1}$ and the sum-rate in (5.11) is monotonically decreasing function of ρ which means that there exist two regions of ρ where one corresponds to $R_{MF}|_{\gamma_2=0} < R_{MF}|_{\gamma_2=1}$ (the region with $\rho > \rho_c$ in Figure 5.2) and other corresponds to $R_{MF}|_{\gamma_2=0} > R_{MF}|_{\gamma_2=1}$ (the region with $\rho < \rho_c$ in Figure 5.2). Since the sum-rate for one region ($\rho < \rho_c$) is $R_{MF}|_{\gamma_2=0} > R_{MF}|_{\gamma_2=1}$, thus there exists a range of γ_2 as a subset of $[0, 1]$ for which the sum-rate is greater than $R_{MF}|_{\gamma_2=1}$. Therefore, the total transmit power in the cell reduces, consequently, the average inter-cell interference reduces considering the similar operation in all cells.

In the case of ZF processing with perfect CSIR, there is no intra-cell interference. Therefore, a few properties of inter-cell interference are exploited in order to draw a useful analysis on the sum-rate. As the cells are symmetric and homogeneous, the values of total average transmit power from different MTs in different cells are equal, i.e., $\mathbb{E}[\gamma_{lk}] = \mathbb{E}[\gamma_{jk}]$. Therefore, the inter-cell interference power ρ_l from (5.6) can be approximated as follows:

$$\rho_l \approx (L-1)\mathbb{E}[\beta_{jlk}] \sum_{k=1}^{K} \gamma_{lk}. \tag{5.12}$$

With the simplified notations ($\beta_{llk} \to \beta_k$, $\gamma_{lk} \to \gamma_k$, and $\rho_l \to \rho$), by using (5.10) and (5.12), the two users sum-rate in the case of ZF processor with perfect CSIR for high SNR setting ($p_u \to \infty$) can be written as follows:

$$R_{ZF} = \log_2\left\{1 + \frac{A_{ZF}\beta_1\gamma_1}{\gamma_1+\gamma_2}\right\} + \log_2\left\{1 + \frac{A_{ZF}\beta_2\gamma_2}{\gamma_1+\gamma_2}\right\},$$

where

$$A_{ZF} = \frac{(N-2)}{(L-1)\mathbb{E}[\beta_{jlk}]} \text{ with } j \neq l. \tag{5.13}$$

However, equation (5.12) is a good approximation for a large K because the term ρ_l comes from the statistics of the channel. For two users' case, it can be considered a valid approximation for analysis with the assumption of symmetry across the cells in terms of channel gains and transmit power factors.

Let $\beta_1 \geq \beta_2$, i.e., MT_2 does not have better channel than MT_1, $\gamma_1 = 1$, i.e., MT_1 transmits its full power and $0 \leq \gamma_2 \leq 1$, i.e., the power of MT_2 is varied. The sum-rate then becomes:

$$R_{ZF} = \log_2\left\{1 + \frac{A_{ZF}\beta_1}{1+\gamma_2}\right\} + \log_2\left\{1 + \frac{A_{ZF}\beta_2\gamma_2}{1+\gamma_2}\right\}. \tag{5.14}$$

Theorem 5.1.4. *For two users' uplink with perfect CSIR, the ZF sum-rate for some $0 < \gamma_2 < 1$ is higher than the sum-rate for $\gamma_2 = 1$ for certain range of the parameters (i.e. β_1, β_2, and A_{ZF}) in (5.14). Therefore, the maximization of sum-rate is possible for $0 < \gamma_2 < 1$ along with a reduction in inter-cell interference.*

Proof: By solving $\frac{d}{d\gamma_2}(R_{ZF}) = 0$ for γ_2, critical value γ_c is obtained as follows:

$$\gamma_c = \frac{\beta_2 + \beta_1(A_{ZF}\beta_2 - 1)}{\beta_1 + \beta_2(A_{ZF}\beta_1 - 1)}. \tag{5.15}$$

The results show that γ_c (solution of $\frac{d}{d\gamma_2}(R_{ZF}) = 0$) corresponds to a maxima on the sum-rate. There is a range of parameters A_{ZF}, β_1, and β_2 for which this maxima lies within 0 and 1, i.e., $\gamma_c \in [0,1]$. These ranges can be found as follows:

$$\begin{aligned}
&0 \leq \frac{\beta_2 + \beta_1(A_{ZF}\beta_2 - 1)}{\beta_1 + \beta_2(A_{ZF}\beta_1 - 1)} \leq 1 \\
&\Rightarrow 0 \overset{(a)}{\leq} \beta_2 + \beta_1(A_{ZF}\beta_2 - 1) \overset{(a)}{\leq} \beta_1 + \beta_2(A_{ZF}\beta_1 - 1) \\
&\Rightarrow 0 \overset{(b)}{\leq} \frac{\beta_2}{\beta_1} + A_{ZF} - \frac{\beta_1}{\beta_2} \overset{(b)}{\leq} \frac{\beta_1}{\beta_2} + A_{ZF} - \frac{\beta_2}{\beta_1},
\end{aligned}$$

where (a) follows from the fact that denominator $\left(\beta_1 + \beta_2(A_{ZF}\beta_1 - 1)\right)$ is always positive. The operation (b) follows from the fact that $\beta_1 > 0$ and $\beta_2 > 0$. The right-hand side inequality is always true because it leads to $\beta_2 \leq \beta_1$ which is already true. The left-hand side inequality leads to the following.

$$\left(\frac{\beta_1}{\beta_2}\right)^2 - \frac{\beta_1}{\beta_2}A_{ZF} - 1 \leq 0. \tag{5.16}$$

The roots of the quadratic expression in (5.16) are given by the following:

$$\left.\frac{\beta_1}{\beta_2}\right|_{roots} = \frac{A_{ZF} \pm \sqrt{A_{ZF}^2 + 4}}{2}.$$

The quadratic expression in (5.16) obtains negative values between these roots. Therefore, the maximization of sum-rate is possible for the following range of parameters.

$$\frac{A_{ZF} - \sqrt{A_{ZF}^2 + 4}}{2} \leq \frac{\beta_1}{\beta_2} \leq \frac{A_{ZF} + \sqrt{A_{ZF}^2 + 4}}{2} \tag{5.17}$$

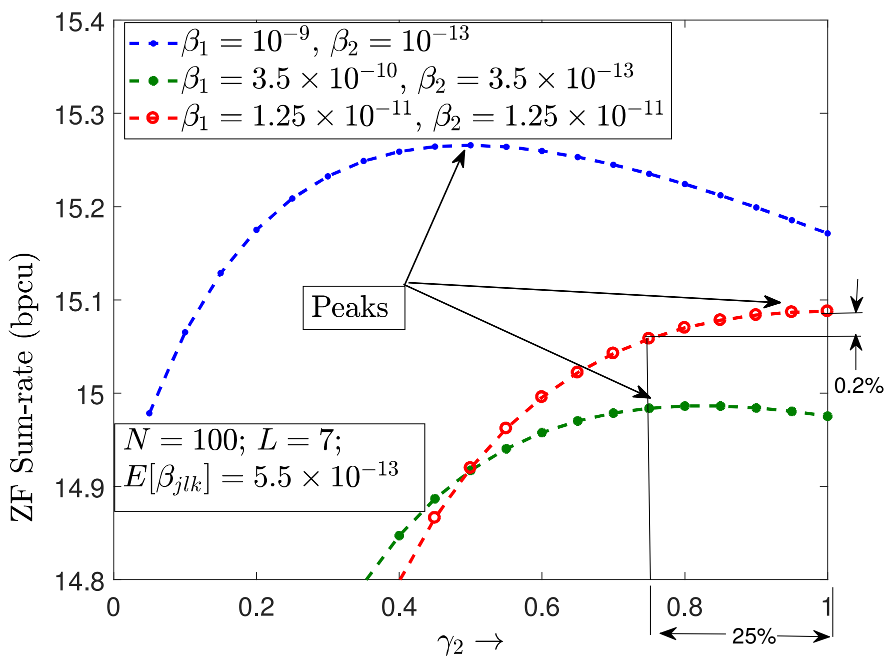

Figure 5.3. : Sum-rate versus transmit power factor (γ_2) for 2 users with ZF processor and perfect CSIR.

The value of A_{ZF} depends on $\mathbb{E}[\beta_{jlk}]$ which is calculated using an existing path loss model [Erceg *et al.*, 1999]. The numerical value of A_{ZF} is very high in practice. The lower limit in (5.17) is negative which is always smaller than β_1/β_2 and the upper limit is very high. Therefore, the maximization is possible for almost all values of parameters. With $N = 100$, $L = 7$, and different values of β_1 & β_2, the sum-rate versus γ_2 is plotted in Figure 5.3. There are cases where sum-rate maximizes when second user's transmit power is smaller than maximum transmit power (for $\gamma_2 = 1$). Moreover, for the cases with monotonically increasing sum-rate, there is a possibility of a significant reduction in the transmit power by sacrificing a small fraction of sum-rate as shown in the figure.

5.1.2 Impact of the Assumptions of System Modeling

The modeling of inter-cell interference power ρ_l in (5.6) follows the assumption of the large number of interfering MTs. This assumption allows the modeling of interference-distribution as Gaussian and the use of the sample mean instead of expectation. In this way, a tractable analysis is done for the sum-rate of two users case. Typically, an interference limited multi-cell massive MIMO system has several MTs per cell. Therefore, this assumption is easy to maintain in a typical system. Further, the applicability of the results is not limited to 2 users as shown in the performance results obtained by the simulation.

5.1.3 Sum-Rate with Imperfect CSIR

Sum-rate for perfect CSIR is useful for theoretical analysis as performed in the previous subsection. However, a practical system has to bear with imperfect CSIR. This imperfect CSIR affects the performance as shown in various plots for pilot-based and blind channel estimation methods in previous chapter. In order to use well established sum-rate expressions from the literature, a pilot based channel training is considered where a time period of length τ is dedicated for uplink pilots in every T symbols (an RB). The estimate of the channel is obtained by MMSE estimation. For such a setting, the sum-rate results of [Ngo *et al.*, 2013a, Sec III] are easily extendible for average power-control based current system model similar to the case of perfect CSIR. Lower bounds on MF and ZF rates for k^{th} MT with imperfect CSIR are given by (5.18) and (5.19).

$$\begin{aligned}\tilde{R}^{MF}_{IP,k} =& \frac{T-\tau}{T}\log_2\Bigg\{1+\tau(N-1)\beta^2_{llk}\gamma^2_{lk}\Big/ \\ & \Big\{(\tau\beta_{llk}\gamma_{lk}+\rho_l+\frac{1}{p_u})\sum_{i=1,i\neq k}^{K}\beta_{lli}\gamma_{li}+(\tau+1)\beta_{llk}\gamma_{llk}(\rho_l+\frac{1}{p_u})+(\rho_l+\frac{1}{p_u})^2\Big\}\Bigg\}. \end{aligned} \tag{5.18}$$

$$\begin{aligned}\tilde{R}^{ZF}_{IP,k} =& \frac{T-\tau}{T}\log_2\Bigg\{1+\tau(N-K)\beta^2_{llk}\gamma^2_{lk}\Big/ \\ & \Big\{(\rho_l+\frac{1}{p_u})\Big\{(\tau\beta_{llk}\gamma_{lk}+\rho_l+\frac{1}{p_u})\sum_{i=1}^{K}\frac{\beta_{lli}\gamma_{li}}{\tau\beta_{lli}\gamma_{li}+\rho_l+\frac{1}{p_u}}+\tau\beta_{llk}\gamma_{lk}+\rho_l+\frac{1}{p_u}\Big\}\Big\}\Bigg\}. \end{aligned} \tag{5.19}$$

The lower bounds in (5.18) and (5.19) follow the convexity of $log_2(1+1/x)$, Jensen's inequality, and channel hardening in massive MIMO systems [Ngo *et al.*, 2013a]. The deviation of these bounds vary from around 12% to 3% for MF and 5% to 1% for ZF as the number of base station antennas grows from 100 to 250 (cf. [Ngo *et al.*, 2013a, Fig. 1]). The training phase is considered to be non-synchronous across cells so that the average inter-cell interference is equal in the training phase and the data phase. Consequently, the transmit power in training phase and data phase is considered to be equal. The theoretical analysis is not tractable for sum-rate in the case of imperfect CSIR. Therefore, the numerical optimization is preferred directly in this case.

5.2 POWER-ALLOCATION OPTIMIZATION FOR K-USERS

The transmit power factor affects both the numerator and denominator parts in sum-rate expression. Therefore, it is sensible to formulate an optimization problem on the sum-rate for all four cases (MF & ZF with perfect CSIR and MF & ZF with imperfect CSIR) as follows:

$$R_{opt} = \max_{\gamma_{lk},\gamma_{li}} \left\{ \sum_{l=1}^{L} \sum_{k=1}^{K} R_{lk} \right\} \tag{5.20}$$

with $0 \le \gamma_{lk} \le 1 \; \forall \; 1 \le l \le L$ and $1 \le k \le K$,

where R_{lk} is the uplink rate of k^{th} MT in l^{th} cell.

The optimization problem in (5.20) deals with the optimization of average power-allocation based on the statistics of the channel leading to the infrequent execution of optimization algorithm where the impact of the complexity is not severe. A combinatorial optimization with discrete levels of power scaling factors (γ) is considered here. Under the non-cooperative settings, similar nature of the channel, and same optimization process across the cells, the optimization over individual cells is approximately equivalent to the joint optimization over all cells. Considering the inter-cell interference limited (or high SNR, i.e., $p_u \to \infty$) regime, the optimization problem (5.20) can be re-defined for MF and ZF with perfect CSIR as follows:

$$R_{l,P,opt}^{MF} = \max_{\gamma_{lk},\gamma_{li}} \left\{ \sum_{k=1}^{K} \log_2 \left\{ 1 + \frac{(N-1)\beta_{llk}\gamma_{lk}}{\sum_{i=1,i\neq k}^{K} \beta_{lli}\gamma_{li} + \rho_l} \right\} \right\} \tag{5.21}$$

$0 \le \gamma_{lk} \le 1 \; \forall \; 1 \le k \le K$ and

$$R_{l,P,opt}^{ZF} = \max_{\gamma_{lk},\gamma_{li}} \left\{ \sum_{k=1}^{K} \log_2 \left\{ 1 + \frac{(N-K)\beta_{llk}\gamma_{lk}}{\rho_l} \right\} \right\} \tag{5.22}$$

$0 \le \gamma_{lk} \le 1 \; \forall \; 1 \le k \le K$.

The value of ρ_l follows from (5.12). Similar to the case of perfect CSIR, the optimization problem can be formulated for the case of imperfect CSIR using (5.18) and (5.19).

$$R_{l,IP,opt}^{MF} = \max_{\gamma_{lk},\gamma_{li}} \left\{ \sum_{k=1}^{K} \tilde{R}_{IP,k}^{MF} \right\} \tag{5.23}$$

$0 \le \gamma_{lk} \le 1 \; \forall \; 1 \le k \le K$ and

$$R_{l,IP,opt}^{ZF} = \max_{\gamma_{lk},\gamma_{li}} \left\{ \sum_{k=1}^{K} \tilde{R}_{IP,k}^{ZF} \right\} \tag{5.24}$$

$0 \le \gamma_{lk} \le 1 \; \forall \; 1 \le k \le K$.

To proceed with discrete level combinatorial optimization, γ-space is divided into $N_{s\gamma}$ partitions such that γ_{lk} can get a value from a set of discrete values between 0 and 1 as following example.

$$\gamma_{lk} \in \{0.2,\ 0.4,\ 0.6,\ 0.8,\ 1.0\}. \tag{5.25}$$

An array AR_1 (length $= (N_{s\gamma})^K$) of diagonal matrices $\mathbf{S}_l^2$ with diagonal entries coming from the set (5.25) is constructed by using all possible permutations of γ_{lk} with repetition allowed as follows:

$$\mathbf{S}_l^2 = \begin{bmatrix} \gamma_{l1} & 0 & \dots & 0 \\ 0 & \gamma_{l2} & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & \gamma_{lK} \end{bmatrix} \text{ and}$$

$$AR_1 = \left[\mathbf{S}_l^2(1), \mathbf{S}_l^2(2), ..., \mathbf{S}_l^2((N_{s\gamma})^K)\right]. \tag{5.26}$$

An other array AR_2 of length $(N_{s\gamma})^K$ is created with entries $tr\{\mathbf{S}_l^2\}$ corresponding to each $\mathbf{S}_l^2$ of AR_1 as follows:

$$AR_2 = \left[tr\{\mathbf{S}_l^2(1)\}, tr\{\mathbf{S}_l^2(2)\}, ..., tr\{\mathbf{S}_l^2((N_{s\gamma})^K)\}\right], \tag{5.27}$$

where $tr\{\mathbf{S}_l^2\} = \sum_{k=1}^{K} \gamma_{lk}$.

Corresponding to each value of $\mathbf{S}_l^2$, MF and ZF sum-rates with perfect CSIR are calculated with help of (5.9), (5.10), and (5.12) with $1/p_u = 0$. Similarly, MF and ZF sum-rates with imperfect CSIR are calculated with help of (5.18), (5.19), and (5.12) with $1/p_u = 0$. The calculated values of sum-rate arrays UR_{MF} for MF and UR_{ZF} for ZF are sorted in ascending order to obtain the arrays SR_{MF} and SR_{ZF} respectively.

$$SR_{MF} = \textit{Sort Ascending } UR_{MF} \text{ and}$$
$$SR_{ZF} = \textit{Sort Ascending } UR_{ZF}.$$

The array AR_1 is aligned with sorted sum-rate array SR_{MF} and SR_{ZF} to produces arrays of γ-matrices GM_{MF} and GM_{ZF} respectively as follows:

$$GM_{MF} = AR_1 \textit{ Index aligned w.r.t. } SR_{MF} \text{ and}$$
$$GM_{ZF} = AR_1 \textit{ Index aligned w.r.t. } SR_{ZF}.$$

The array AR_2 is aligned with sorted sum-rate array SR_{MF} and SR_{ZF} to produces arrays of sum-power factors SP_{MF} and SP_{ZF} respectively as follows:

$$SP_{MF} = AR_2 \textit{ Index aligned w.r.t. } SR_{MF} \text{ and}$$
$$SP_{ZF} = AR_2 \textit{ Index aligned w.r.t. } SR_{ZF}.$$

The results on numerical analysis of this power-allocation optimization scheme are given in Section 5.4.1.

5.3 LOW COMPLEXITY ALGORITHM FOR POWER-ALLOCATION OPTIMIZATION

However, the proposed power-allocation optimization is based on the statistics of channel where the need for an optimization operation is infrequent. Still, a low complexity algorithm in contrast to exhaustive search based optimization will be useful in the point-of-view of a large number of MTs and the operational cost. The complexity of combinatorial optimization process presented in previous section is exponential in the number of users. In this section, a low complexity optimization scheme is presented as outlined in Algorithm 1. The core of the algorithm is same as the process described in the previous section but the iterative architecture of the algorithm reduces the computational complexity by several orders. The details of steps are given as follows:

Algorithm 1: Power-allocation optimization algorithm

Data: $k = \{1, 2, ..., K\}$;
$\gamma_k \in \left\{\gamma_{st}^k, (\gamma_{st}^k + \frac{\gamma_{mx}^k}{N_{stp}-1}), (\gamma_{st}^k + \frac{2\gamma_{mx}^k}{N_{stp}-1}), ..., (\gamma_{st}^k + \gamma_{mx}^k)\right\}$
$\mathbf{S}_l^2[i] :=$ i^{th} permutation of power scaling matrix
$I_{MF/ZF} :=$ Index array of sorted MF/ZF sum-rate array
$\gamma_{sum}[i] := \sum_{k=1}^{K}(\gamma_k)[i]$ (i^{th} permutation)
Result: $\gamma_1, \gamma_2, ..., \gamma_K$
initialization: γ_{st}^k; γ_{mx}^k; N_{stp}; C; γ_k;
while $C < C_{max}$ **do**
  Construct permutations:
  $AR_1 \leftarrow \mathbf{S}_l^2[1], \mathbf{S}_l^2[2], ..., \mathbf{S}_l^2[(N_{stp})^K]$;
  Calculate:
  $AR_2 \leftarrow \gamma_{sum}[1], \gamma_{sum}[2], ..., \gamma_{sum}[(N_{stp})^K]$;
  Calculate $UR_{MF/ZF}$;
  $[SR_{MF/ZF}, I_{MF/ZF}] \leftarrow$ **Sort**$(UR_{MF/ZF})$;
  $GM_{MF/ZF} \leftarrow AR_1$ indexed by $I_{MF/ZF}$;
  $SP_{MF/ZF} \leftarrow AR_2$ indexed by $I_{MF/ZF}$;
  Desired set $\leftarrow$ **Select***$[GM_{MF/ZF}$, $SP_{MF/ZF}$, and $SR_{MF/ZF}]$;
  $C \leftarrow C + 1$;
  Adjust (γ_{st}^k and γ_{mx}^k);
end
* Selection Criteria: Close to maximum rate, lowest γ sum, and desired fairness.

---

1. Use the following values of γ_{lk} for the algorithm:

$$\gamma_{lk} \in \left\{\gamma_{st}^{lk}, (\gamma_{st}^{lk} + \frac{\gamma_{mx}^{lk}}{N_{stp}-1}), (\gamma_{st}^{lk} + \frac{2\gamma_{mx}^{lk}}{N_{stp}-1}), ..., (\gamma_{st}^{lk} + \gamma_{mx}^{lk})\right\},$$

   where γ_{st}^{lk} is the starting value of γ_{lk}, $(\gamma_{st}^{lk} + \gamma_{mx}^{lk})$ is the last value of γ_{lk}, and N_{stp} defines the number of partitions in this range of γ-space. N_{stp} depends on the capability of signal processor and vary from 2 to 5. There are $K(N_{stp})^K$ calculations of the rate expression. The values of γ_{lk} are selected in such a way so that they divide the γ-space in equal partitions.

2. Create a permutation of power scaling matrix $\mathbf{S}_l^2$ by selecting values of γ_{lk} from the set given in step (1) for all $1 \leq k \leq K$. Calculate the rate expression for all K users and add up to get the sum-rate. Also calculate the sum of power scaling factors ($\sum_{k=1}^{K} \gamma_{lk}$).

3. Repeat the calculation of step (2) for different $\mathbf{S}_l^2$ matrix permutations to obtain the array of $\mathbf{S}_l^2$ matrices (AR_1), the array of sum-power scaling factors (AR_2), and the sum-rate array. Sort the sum-rate array in ascending order (to get SR_{MF} or SR_{ZF}) and align the original indexes of

the unsorted array with the sorted array to get an index array.

4. Align AR_1 and AR_2 with sorted array of sum-rate (SR_{MF} or SR_{ZF}) using the index array mentioned in step (3) to obtain the aligned array of $\mathbf{S}_l^2$ (GM_{MF} or GM_{ZF}) and the aligned sum-power factor array (SP_{MF} or SP_{ZF}) respectively.

5. Select one set of $\mathbf{S}_l^2$ matrix permutation to optimize the power-allocation for a better sum-rate (i.e. close to maximum sum-rate) and an improved energy consumption (i.e. smaller $\sum_{k=1}^{K}\gamma_k$) for controlled fairness (i.e. relative values of γ_{lk} for $1 \leq k \leq K$). There are several permutations of γ_k for improving the energy consumption by sacrificing a small fraction of sum-rate.

6. Re-run the above process after adjusting the γ_{st}^k and γ_{mx}^k close to selected permutation of γ_k to achieve a fine optimization. Run C_{max} number of iterations.

The selection of a permutation in step (5) of the algorithm depends on the amount of inter-cell interference, the desired fairness, the cost of the transmit power, and the cost of spectral efficiency.

5.4 RESULTS

This section presents the results corresponding to numerical analysis of presented scheme, the performance results of proposed low complexity algorithm for the scheme, and the complexity analysis of proposed low complexity algorithm.

5.4.1 Numerical Analysis of Sum-Rate for the K-Users Case

For numerical analysis of sum-rate for the K-users case, the path loss is simulated for a flat macro cell with light tree density [Erceg *et al.*, 1999, Table(1)] with BS height $25m$ and uniform random user locations in 7 cells with $1km$ of radius measured from center to vertex. Using these simulated path loss values, the obtained average of large scale fading coefficients ($\mathbb{E}\left[\beta_{jlk}\right]$) for interfering links (i.e. $j \neq l$) is approximately 5×10^{-13}. The average channel gains for desired links (i.e. β_{jlk} for $j = l$) vary approximately from 10^{-13} to 10^{-10}. Other settings include the number of BS antennas: $N = 100$, the number of MTs per cell: $K = 6$, and the number of cells in the system: $L = 7$. For the combinatorial optimization, a set of the values of γ_{lk} is constructed with 5 values between 0 and 1 as given in (5.25). The values of the large scale fading coefficients for 6 MTs are chosen as follows:

$$\beta_{llk} \in \left\{\{0.5, 1, 5, 10, 50, 100\} \times 10^{-12}\right\}. \tag{5.28}$$

The last few values of SR_{MF} and SP_{MF} with perfect CSIR and the last few values of SR_{ZF} and SP_{ZF} with perfect CSIR are plotted in Figure 5.4. Similarly, the last few values of SR_{MF} and SP_{MF} with imperfect CSIR and the last few values of SR_{ZF} and SP_{ZF} with imperfect CSIR are plotted in Figure 5.5. The summary of results from Figures 5.4 and 5.5 is given as follows:

- The maximum sum-rate is obtained at significantly lower sum-power in all cases compared to the case of full sum-power $\left(\sum_{k=1}^{K}\gamma_{lk} = 6\right)$ which is obtained by using full power at each MT, i.e., by putting $\gamma_{lk} = 1\ \forall\ 1 \leq k \leq K$.

- There are several permutations of power scaling factors close to maximum sum-rate which can be used to reduce average transmit power by sacrificing a small fraction of sum-rate.

- Due to the large number of permutations of power scaling factors close to maximum sum-rate, the fairness among MTs can be controlled by using an appropriate scheduler.

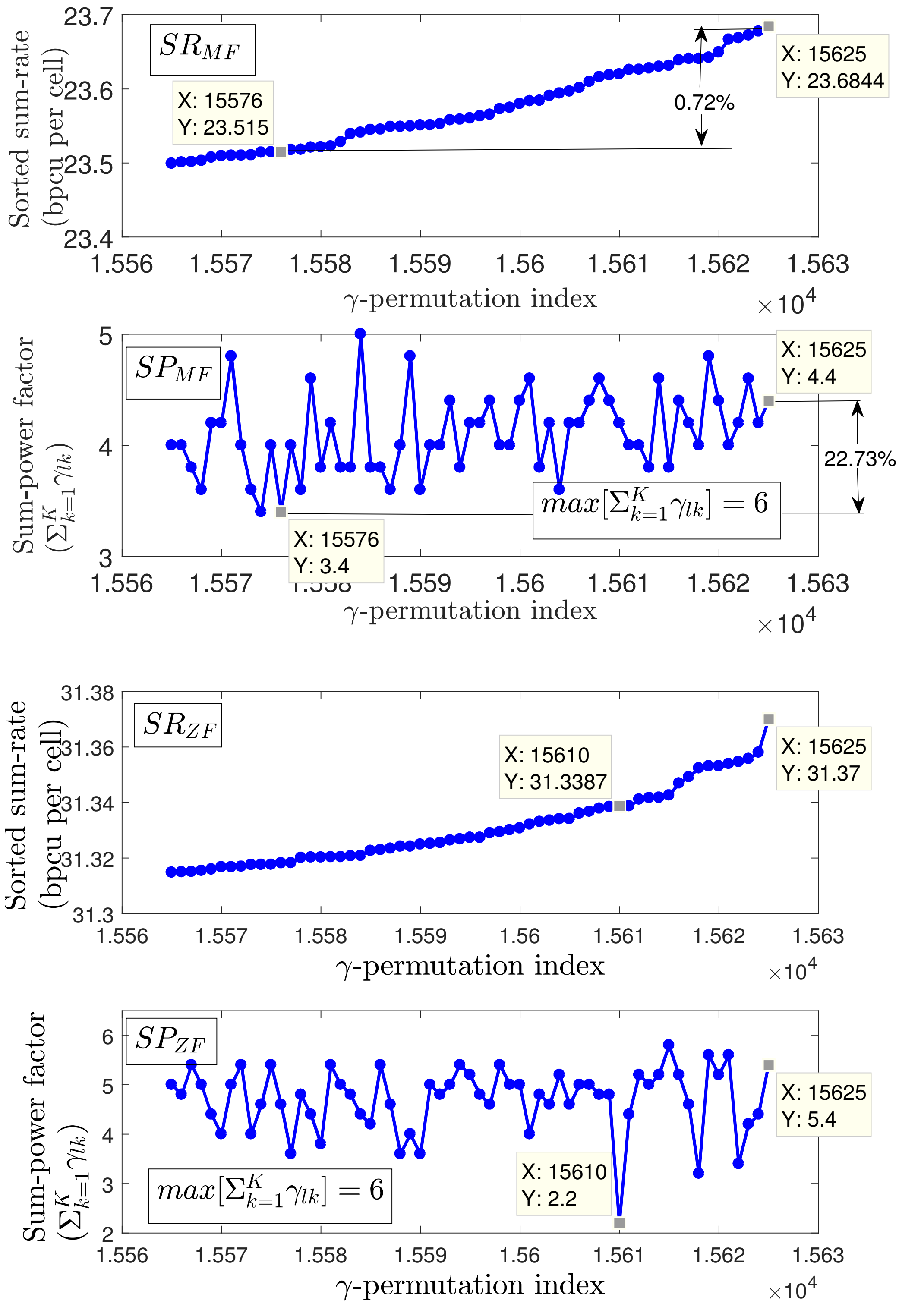

Figure 5.4. : Sorted sum-rate versus index of permutation vector of $\gamma_k; 1 \leq k \leq K$ for MF and ZF processors with perfect CSIR; SR:Sum-rate; SP: Sum-power.

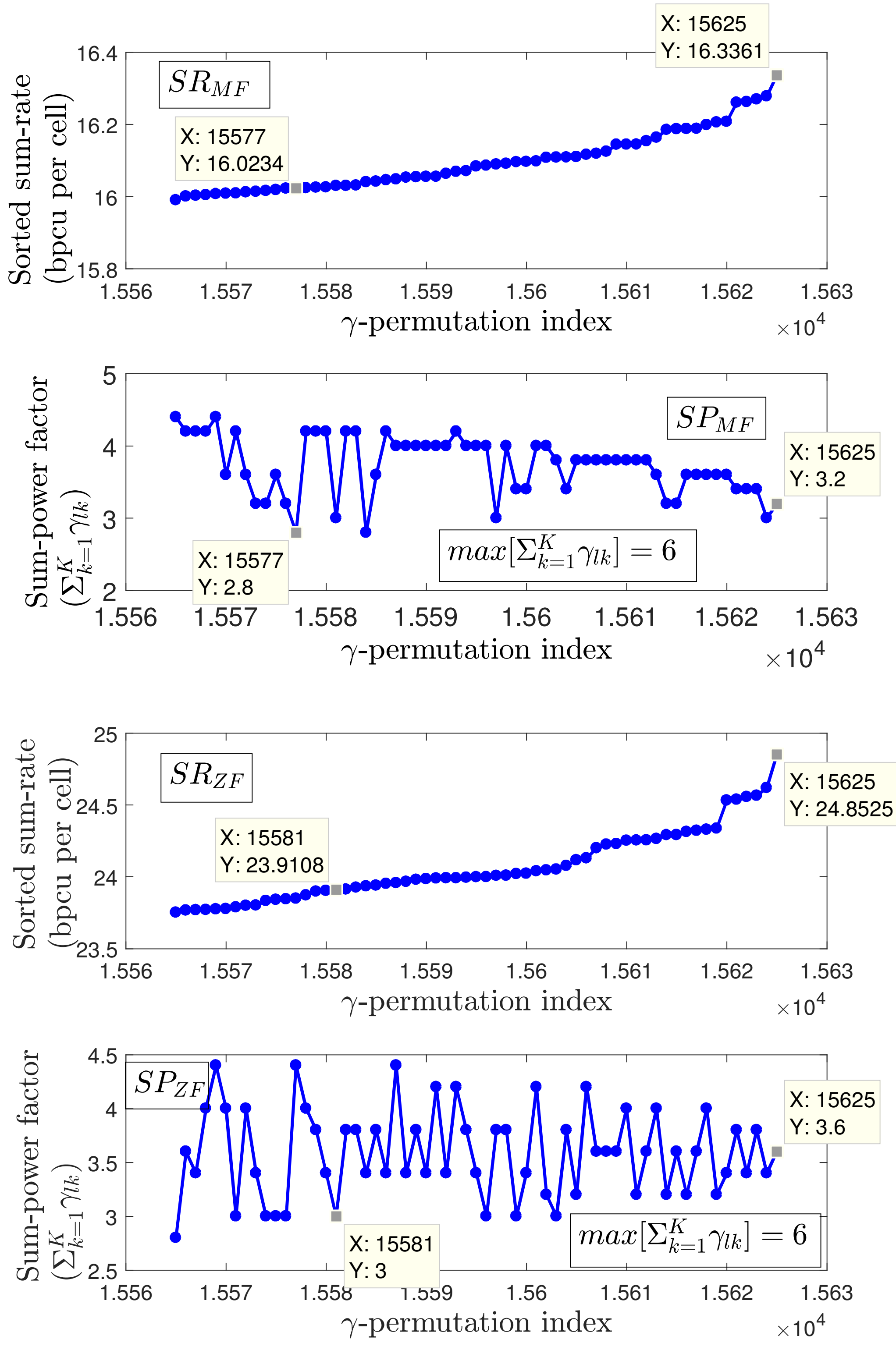

Figure 5.5. : sum-rate versus index of permutation vector of $\gamma_k; 1 \leq k \leq K$ for MF and ZF processors with imperfect CSIR; γ_k is transmit power scaling factor for k^{th} MT; ρ_s is a measure of inter-cell interference power; SR: Sum-rate; SP: Sum-power.

5.4.2 Performance Results of Algorithm

The results on the sum-rate improvement and transmit power reduction by the proposed scheme are quantified in Tables 5.1 and 5.2. In these results, the criterion for the selection of permutation in Algorithm 1 is kept to maximize the sum-rate. For averaging the results, uniform random user locations are simulated. The channel gains β_{llk} (for cell under consideration) and β_{jlk} (for neighboring cells) are simulated using the path loss model referenced in Annexure B.3. The obtained values of per-user average inter-cell interference power ($\mathbb{E}[\beta_{jlk}]$) are 5×10^{-13} and 1×10^{-10} for macro and micro cells respectively. The iteration count for the algorithm is kept at 4. The range of γ-space in first iteration is 0 to 1 which becomes half after each iteration in 2-partitions setting and becomes one third in 3-partitions setting, and so on.

Table 5.1. : Comparison of sum-rate (bpcu per cell) for different number of the partition of γ-space. (MAC: Macro cell; MIC: Micro cell; HI, LI: High, Low Inter-cell interference power; 2P, 3P, 5P: two, three, and five partitions of γ-space, $**$: Not calculated due to high simulation complexity).

Setting	K=5,HI	K=10,HI	K=5,LI	K=10,LI
MF,MAC,2P	14.99	21.25	17.23	24.82
MF,MAC,3P	15.02	21.33	17.26	24.89
MF,MAC,5P	15.03	**	17.26	**
ZF,MAC,2P	18.88	28.11	23.43	35.84
ZF,MAC,3P	18.91	28.23	23.44	35.91
ZF,MAC,5P	18.92	**	23.45	**
MF,MIC,2P	9.25	11.52	11.56	14.50
MF,MIC,3P	9.30	11.69	11.59	14.62
MF,MIC,5P	9.32	**	11.60	**
ZF,MIC,2P	9.90	12.28	12.83	16.14
ZF,MIC,3P	9.96	12.50	12.86	16.31
ZF,MIC,5P	9.98	**	12.87	**

The summary of the simulation results of proposed low complexity algorithm is given as follows:

- The algorithm provides a significant improvement in sum-rate and power consumption over the full power assignment scheme.
- The sum-rate as well as the percentage improvement in sum-rate over the case of full power assignment to each MT increases with the number of MTs.
- The percentage power reduction over full power assignment scheme increases with number of MTs.
- The maximized sum-rate by two partitions of γ-space is a lower bound on the global maximum sum-rate. However, the improvement in sum-rate by increasing the number of partitions of γ-space is small. The higher number of partitions of γ-space brings the scheme closer to the straightforward combinatorial optimization.

Along with these results, following can be concluded regarding the proposed scheme in comparison to existing results in the literature.

- The proposed algorithm is based on combinatorial optimization such that the selection

Table 5.2. : Summary of % increment in sum-rate and % decrement in total transmit power over the case of full power assignment under imperfect CSIR and two partitions of γ-space. (RI: Rate improvement; PD: Power decrement; MAC: Macro cell; MIC: Micro cell; HI, LI: High, Low inter-cell interference power; Inter-cell interference descending order: MIC HI, MIC LI, MAC HI, MAC LI).

Setting	K=5	K=10	K=15	K=20
RI-MF,MIC,HI	8.51	32.85	51.49	73.37
RI-MF,MIC,LI	1.87	14.20	25.56	38.20
RI-MF,MAC,HI	6.77	10.08	14.78	14.70
RI-MF,MAC,LI	7.44	10.47	12.94	12.54
RI-ZF,MIC,HI	9.68	39.90	64.85	95.35
RI-ZF,MIC,LI	2.10	18.96	35.27	54.53
RI-ZF,MAC,HI	4.48	10.11	19.57	22.50
RI-ZF,MAC,LI	1.57	4.68	10.37	12.93
PD-MF,MIC,HI	40.17	52.90	57.51	63.63
PD-MF,MIC,LI	42.61	43.45	50.01	55.25
PD-MF,MAC,HI	45.77	44.41	50.29	53.06
PD-MF,MAC,LI	43.51	42.35	46.38	47.63
PD-ZF,MIC,HI	38.39	54.75	59.96	65.38
PD-ZF,MIC,LI	39.19	44.98	51.40	57.88
PD-ZF,MAC,HI	47.65	31.82	43.68	45.06
PD-ZF,MAC,LI	55.68	24.91	32.07	37.38

step (function with superscript * in Algorithm 1) can be modified to improve any one or the combination of sum-rate, sum-power and individual user-specific rates. Consequently, the scheme can be modified to match its performance with existing other power-allocation schemes that maximize one of the parameters like minimum achievable rate, EE, and sum-rate [Zhang *et al.*, 2016; Zarei *et al.*, 2017; Dai and Dong, 2016].

- The performance of proposed scheme is better than that of an existing scheme with similar approach [Dai and Dong, 2016]. The last column of Table 5.2 (i.e. K=20) can be compared with the results (Table 5.3) of the existing power-allocation scheme [Dai and Dong, 2016]. However, the existing scheme allocates half of the full power at MTs for equal power-allocation scheme in contrast to proposed scheme where improvement is shown over full power assignment. Moreover, the current work considers inter-cell interference limited multi-cell setting in contrast to [Dai and Dong, 2016].

- The results corresponding to MF in micro-cell (proposed scheme) are also compared with the results of existing SIF based power-allocation scheme [Zhang *et al.*, 2016, Algorithm 1] reproduced in Table 5.4. Due to the fairness as a priority, the aforementioned existing scheme shows negative improvements in sum-rate. Similarly, due to the sum-rate as a priority in the current setup of proposed scheme, the power saving is smaller than that of SIF based power-allocation scheme.

Table 5.3. : Summary of % increment in sum-rate of an existing scheme [Dai and Dong, 2016, Fig. 6] over equal power-allocation scheme (Notation and parameters follow from [Dai and Dong, 2016]). P_p : Pilot power; K= 20; Equal Power per user = $P_0/2$ (P_0: full power per user).

Setting	P_p=-8dB	P_p=-4dB	P_p=0dB	P_p=4dB
MF	24.1	23.7	22.2	20.4
ZF	50.0	48.6	39.0	22.0
Setting	P_p=8dB	P_p=12dB	P_p=16dB	
MF	14.6	11.8	10.0	
ZF	19.6	19.2	18.9	

Table 5.4. : Summary of % improvement of an existing scheme [Zarei *et al.*, 2017] over full power assignment scheme for MF processor in micro cell; HI/LI: High/Low inter-cell interference; RI: rate increment; PD: power decrement.

Setting	$K=5$	$K=10$	$K=15$	$K=20$
RI-MF,HI	-48.25	-41.98	-45.13	-50.18
PD-MF,HI	95.76	93.96	94.16	94.61
Setting	$K=5$	$K=10$	$K=15$	$K=20$
RI-MF,LI	-33.93	-36.22	-46.48	-52.68
PD-MF,LI	90.36	90.27	90.04	93.40

5.4.3 Complexity Analysis of Algorithm

If γ-space (i.e. 0 to 1) is divided into $N_{s\gamma}$ partitions in the straightforward combinatorial optimization process, the number of the operations of sum-rate calculation will be $K(N_{s\gamma})^K$. Whereas the proposed algorithm takes the advantage of the fact that the most of the combinations are far from the maximum achievable rate by optimization. Therefore, the search space can be reduced iteratively. In each iteration, γ-space is divided into N_{stp} ($< N_{s\gamma}$) partitions. Based on the selected sum-rate permutation, the new boundaries for γ-space are defined for next iteration.

The resolution of exhaustive search over γ-space is $1/N_{s\gamma}$. If the search space in each iteration is reduced to $\varepsilon < 1$ times the search space in previous iteration and algorithm runs for C_{max} number of iterations, the resolution of iterative search over γ-space then will be:

$$\frac{\varepsilon^{(C_{max}-1)}}{N_{stp}}. \tag{5.29}$$

For equal performance of the exhaustive search and the iterative search schemes, the resolutions of both schemes should be equal, i.e.,

$$\frac{\varepsilon^{(C_{max}-1)}}{N_{stp}} = \frac{1}{N_{s\gamma}}. \tag{5.30}$$

The required number of iterations, calculated from (5.30), is:

$$C_{max} = \left\lceil \log_{\varepsilon}\left\{\frac{N_{stp}}{N_{s\gamma}}\right\} \right\rceil + 1, \tag{5.31}$$

where $\lceil . \rceil$ is integer ceiling function. The complexity for proposed algorithm is $C_{max}K(N_{stp})^K$. By taking a few typical values like $K = 15$, $N_{s\gamma} = 5$, $N_{stp} = 2$, and $\varepsilon = 0.5$, the calculated C_{max} is 3 which reduces the complexity by the following factor:

$$\frac{K(N_{s\gamma})^K}{C_{max}K(N_{stp})^K} = 3.1 \times 10^5. \tag{5.32}$$

By using the above low complexity algorithm with slowly varying channel statistics (inherent), the proposed scheme imposes an insignificant computation burden on the system.

In this chapter, the average channel gains are exploited for their distinct values over space caused by the large scale fading and path loss. A multi-cell system model with power control at MTs was constructed and simplified to obtain an insightful inter-cell interference expression. An analysis on the sum-rate was presented to find out the possibilities for exploitation of average channel gains in average power control for improving the sum-rate and energy consumption. The improvement in the sum-rate varied from 1% to 100% and in the energy consumption from 40% to 60% for different scenarios. The scheme also enabled the control of fairness among MTs by switching among several permutations of transmit power close to maximum sum-rate. Finally, a low complexity optimization algorithm for the scheme was proposed having 3 to 6 orders lower complexity compared to exhaustive search algorithm for typical values of the parameters.

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