Annexure A

A.1 PROOF OF THE THEOREM 4.2.1

Let,

$$\tilde{\mathbf{Y}} = \lim_{\substack{N\to\infty \\ \text{constant } K}} \sqrt{1+\alpha^2}\{(\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}})^H(\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}})\}^{-1}(\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}})^H\mathbf{Y}.$$

Columns of $\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}}$ are asymptotically orthogonal, consequently

$\lim_{N\to\infty,\ \text{constant } K}\{(\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}})^H(\frac{\mathbf{H}+\Delta\mathbf{H}}{\sqrt{1+\alpha^2}})\}/N \to \mathbf{I}$, therefore,

$$\tilde{\mathbf{Y}} = \lim_{\substack{N\to\infty \\ \text{constant } K}} \frac{(\mathbf{H}+\Delta\mathbf{H})^H\mathbf{Y}}{N} \text{ or}$$

$$\tilde{\mathbf{Y}} = \lim_{\substack{N\to\infty \\ \text{constant } K}} \frac{(\mathbf{H}+\Delta\mathbf{H})^H(\mathbf{HX})}{N} + \frac{(\mathbf{H}+\Delta\mathbf{H})^H\mathbf{W}}{N\sqrt{\rho_u}} \text{ or}$$

$$\tilde{\mathbf{Y}} = \lim_{\substack{N\to\infty \\ \text{constant } K}} \frac{\mathbf{H}^H\mathbf{HX}}{N} + \frac{\Delta\mathbf{H}^H\mathbf{HX}}{N} + \frac{(\mathbf{H}+\Delta\mathbf{H})^H\mathbf{W}}{N\sqrt{\rho_u}}.$$

By representing matrix $\mathbf{H}$ as $[h_{ij}]$ with $1 \le i \le N$ & $1 \le j \le K$, $\Delta\mathbf{H}$ as $[\delta h_{ij}]$ with $1 \le i \le N$ & $1 \le j \le K$ and $\mathbf{W}$ as $[w_{ij}]$ with $1 \le i \le N$ & $1 \le j \le T$,

$$\tilde{\mathbf{Y}} = \mathbf{X} + \lim_{\substack{N\to\infty \\ \text{constant } K}} [\frac{1}{N}\sum_{n=1}^{N}\delta h_{ni}^* h_{nj}]\mathbf{X} + \lim_{\substack{N\to\infty \\ \text{constant } K}} [\frac{1}{N\sqrt{\rho_u}}\sum_{n=1}^{N}(h_{ni}^* + \delta h_{ni}^*)w_{nj}].$$

Since, δh_{ni}^* and h_{nj} as well as $(h_{ni}^* + \delta h_{ni}^*)$ and w_{nj} are independent, therefore,

$$\tilde{\mathbf{Y}} = \mathbf{X} + [0]\mathbf{X} + [0] \text{ or}$$

$$\tilde{\mathbf{Y}} = \mathbf{X}.$$

A.2 COMPLEXITY CALCULATION FOR PROPOSED CHANNEL ESTIMATION METHOD AND EVD BASED METHOD

Block indexes (t^b and f^b) are omitted to keep the notation simple.

A.2.1 Complexity Calculation for equation 4.29

Table A.1 shows the complexity calculation for equation 4.29.

Table A.1. : Number of multiplications for pseudo inverse operation using channel matrix.

Operation	No. of multiplications
$\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$	K^2N
$(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}$	K^2N+K^3
$(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^H$	$K^2N+K^3+K^2N$
$(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^H Y$	$K^2N+K^3+K^2N+NKT$
Complexity per symbol:	$(2KN+K^2+NT)/T$

A.2.2 Complexity Calculation for equation 4.30

Table A.2 shows the complexity calculation for equation 4.30.

Table A.2. : Number of multiplications for pseudo inverse operation using data matrix.

Operation	No. of multiplications
$\tilde{\mathbf{X}}_d\tilde{\mathbf{X}}_d^H$	K^2T
$(\tilde{\mathbf{X}}_d\tilde{\mathbf{X}}_d^H)^{-1}$	K^2T+K^3
$\tilde{\mathbf{X}}_d^H(\tilde{\mathbf{X}}_d\tilde{\mathbf{X}}_d^H)^{-1}$	$K^2T+K^3+K^2T$
$\mathbf{Y}\tilde{\mathbf{X}}_d^H(\tilde{\mathbf{X}}_d\tilde{\mathbf{X}}_d^H)^{-1}$	$K^2T+K^3+K^2T+NKT$
Complexity per symbol:	$2K+K^2/T+N$

A.2.3 Complexity Calculation for EVD-based Channel estimation

Definitions of variables in Table A.3 follows from [Ngo and Larsson, 2012]. Using Table A.3, the total number of multiplication associated with EVD-based channel estimation are:

$$N^2T+N^2K+NK^2+NKT+K^2T+8T^3+8NT^2\nu+4NTL+4T^2.$$

With $\nu=1$ and $L=1$, complexity per symbol is:

$$\frac{N^2}{K}+\frac{N^2}{T}+\frac{NK}{T}+N+K+\frac{8T^2}{K}+\frac{8NT}{K}\nu+\frac{4N}{K}+\frac{4T}{K}.$$

Table A.3. : Number of multiplications for EVD-based channel estimation algorithm. [Ngo and Larsson, 2012]

Operation	No. of multiplications
$\mathbf{R}_y=\frac{1}{T}\sum_{t=1}^{T}\mathbf{y}\mathbf{y}^H$	N^2T
$\mathbf{R}_y\mathbf{H}_{ll}$	N^2K
$\mathbf{U}_{ll}\mathbf{\Xi}$	NK^2
$\mathbf{A}_n=\sqrt{P_t}\mathbf{U}_{ll}\mathbf{D}^{-1/2}\mathbf{X}_n$	$NKT+K^2T$
$\hat{\xi}=(\sum_{n=1}^{\nu}\bar{\mathbf{A}}_n^T\bar{\mathbf{A}}_n)^{-1}\sum_{n=1}^{L}\bar{\mathbf{A}}_n^T\mathbf{y}_n$	$8T^3+8T^2N\nu+4NTL+4T^2$