Annexure B

B.1 PROOF OF LEMMA 5.1.1

Let's find the ρ for which $R|_{\gamma_2=0} = R|_{\gamma_2=1}$,

$$\log_2\left\{1+\frac{(N-1)\beta_1}{\rho}\right\} = \log_2\left\{\left(1+\frac{(N-1)\beta_1}{\beta_2+\rho}\right)\left(1+\frac{(N-1)\beta_2}{\beta_1+\rho}\right)\right\}$$

$$\Rightarrow 1+\frac{(N-1)\beta_1}{\rho} = \left(1+\frac{(N-1)\beta_1}{\beta_2+\rho}\right)\left(1+\frac{(N-1)\beta_2}{\beta_1+\rho}\right)$$

$$\Rightarrow \rho^2 + \{(N-2)\beta_1+\beta_2\}\rho - \beta_1^2 = 0. \tag{B.1}$$

The positive root of above equation ($\rho > 0$) is:

$$\rho_c = \frac{1}{2}\left[\sqrt{\{(N-2)\beta_1+\beta_2\}^2+4\beta_1^2} - (N-2)\beta_1 - \beta_2\right]. \tag{B.2}$$

The ρ_c is the critical value of ρ where sum-rate is same whether weak user is transmitting full power or not transmitting at all.

B.2 PROOF OF LEMMA 5.1.2

By setting $\frac{d}{d\gamma_2}(R) = 0$,

$$\beta_2^2\gamma_2^2 + 2\rho_c\beta_2\gamma_2 + (N-2)\rho_c\beta_1 - \beta_2^2 + \rho_c^2 = 0.$$

Solving the above equation for positive γ_2,

$$\gamma_c = \frac{1}{\beta_2}\{-\rho_c + \sqrt{\beta_1^2 - (N-2)\rho_c\beta_1}\}. \tag{B.3}$$

Since the γ_c is unique, thus sum-rate has a maxima or minima at $\gamma_2 = \gamma_c$. Numerical results show that for two users it is minima but for multi-user case, there exists a global maxima for $0 < \gamma_k < 1$; $1 \le k \le K$ for certain range of ρ.

B.3 PATH LOSS MODEL USED IN AVERAGE POWER ALLOCATION SCHEME FOR MASSIVE MIMO

Path loss is modeled as follows[Erceg *et al.*, 1999, eq:1-6]:

$$PL = A + 10B\log_{10}\left(\frac{d}{d_o}\right) + C; d \ge d_o \tag{B.4}$$

$$A \triangleq 20\log_{10}(4\pi d_o/\lambda)$$

$$B \triangleq a - bh_{BS} + \frac{c}{h_{BS}} + x_B\sigma_B; 10m \le h_{BS} \le 80m$$

$$C \triangleq x_C\sigma_C \text{ and } \sigma_C = \mu_\sigma + y_C\sigma_\sigma$$

Path loss equation can be written in terms of fixed and varying components as follows:

$$PL(d) = \{20\log_{10}(4\pi d_o/\lambda) + 10(a - bh_{BS} + \frac{c}{h_{BS}})\log_{10}(\frac{d}{d_o})\}$$

$$+\{x_B\sigma_B\log_{10}(\frac{d}{d_o}) + x_C(\mu_\sigma + y_C\sigma_\sigma)\}\ , \tag{B.5}$$

where x_B, x_C and y_C are distributed as $\mathbb{N}(0,1)$. Values of parameters are selected for flat macro cell with light tree density [Erceg *et al.*, 1999, Table(1)].

Table B.1. : Numerical values of parameters for path loss modeling (Source: [Erceg *et al.*, 1999]).

Model parameter	Scenario:A Hilly/Moderate to-Heavy tree density	Scenario:B Hilly/Light or Flat/Moderate-to Heavy Tree density	Scenario:C Flat/Light Tree density
a	4.6	4	3.6
b (in m^{-1})	0.0075	0.0065	0.0050
c (in m)	12.6	17.1	20.0
σ_B	0.57	0.75	0.59
μ_σ	10.6	9.6	8.2
σ_σ	2.3	3.0	1.6