Abstract

The thesis is motivated by what are called *choice problems* in computer science literature. Here, we define conflicts on the objects in some underlying classical problem such that it precludes some objects from being part of the solution if some others are in the solution. For example consider the GEOMETRIC COVERAGE problem. The input for this problem consists of a pair $\Sigma = (P, \mathscr{R})$, where P is a set of points in $\mathbb{R}^d$ and $\mathscr{R}$ is a set of subsets of P defined by the intersection of P with some geometric objects in $\mathbb{R}^d$. We are interested in the variations of GEOMETRIC COVERAGE problem where there are conflicts on the covering objects that restricts some objects into solution if some others are in the solution.

Similarly consider other well known problem, Frèchet distance. It is an important geometric measure that captures the distance between two curves or more generally point sets. Now let us look into a natural variant of discrete Fréchet distance problem with multiple choice (or conflicts). In the this variation, the problem consists of a set of pair of points. The objective of the problem is to select at most one point from each pair that optimizes Frèchet distance.

Our aim in this thesis is to develop a framework for modelling conflicts. We propose two natural models in which the conflict relations can be represented:
(a) Graphical model : Here we use graph called conflict graph to represent conflicts.
(b) Matroidal model : Here the conflicts are represented by representable matroid.

We consider conflicts from two perspectives, parameterized and approximation. In both the paradigms we study the conflicts when these can be modelled as matching and in other more complex graph classes.

In parameterized complexity, we first consider variation of Frèchet distance with conflicts such that conflict relation forms a matching. We show that this problem is NP-hard. Then we develop a simple randomized FPT algorithm for the problem when parametrized by the solution size, which is later derandomized using universal family of sets. We believe that our derandomization technique can be of independent interest, and can be used to solve other parameterized multiple choice problems. The randomized algorithm runs in $\mathscr{O}(2^k n \log^2 n)$ time, and the derandomized deterministic algorithm runs in $2^k k^{\mathscr{O}(\log k)} n \log^2 n$ time,. Here k is the parameter that denotes the number of elements in the solution consisting of at most one point from each pair. Finally we present a simple branching algorithm for the problem running in $\mathscr{O}(2^k n \log n)$ time. We also show that the problem does not have a polynomial sized kernel under standard complexity theoretic assumptions.

We further consider GEOMETRIC COVERAGE problem with conflicts where the conflict graph has bounded arboricity (that includes all the families of intersection graphs of low density objects in low dimensional Euclidean space). Our main result is that in these settings, if GEOMETRIC COVERAGE problem is fixed parameter tractable (FPT), then so does the conflict version. Also, if the GEOMETRIC COVERAGE problem admits a factor α-approximation, then the conflict version admits a factor α-approximation algorithm running in FPT time. As a corollary to our main result we get a plethora of approximation algorithms running in FPT time. Our other results include an FPT algorithm and a W[1]-hardness proof for the conflict version of COVERING POINTS BY INTERVALS. The FPT algorithm is for the case when the conflicts are given by a representable matroid. The W[1]-hardness result is for all the family of conflict graphs for which the INDEPENDENT SET problem is W[1]-hard.

While considering problems from approximation perspective, we provide a 3-approximation algorithm for Frèchet distance with conflicts. We further study GEOMETRIC COVERAGE problem with

conflicts where conflict relations are in form of some geometric graph. We propose a general framework where if geometric graphs representing coverage problem and conflicts satisfies some properties, then we give constant factor approximation. As application to this framework, in the first problem we consider, we are given a set of unit intervals as covering objects and the conflict graph which again is unit interval graph. We propose a 8-factor approximation algorithm for this problem and show that this problem is APX-hard. Next we consider the problem where covering objects are a set of unit intervals and conflict graph is an unit disk graph. We show that this problem admits 32-factor approximation algorithm and is APX-hard. We further show that the problem is also APX-hard when conflict graph is tree or d-arboricity graph where $d \in \mathbb{N} \geq 1$ for point interval covering. The problem is again APX hard for the case where covering objects are coins and the conflict graph is also an unit coin graph.

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