7
Conclusion

7.1 Summary

The interface of quantum mechanics and graph theory is an emerging field of research. It utilizes the tools of mathematics, physics and computer science. We began this thesis with a very brief survey at the interface of the two fields: graph theory, and quantum mechanics. Interesting topics in this field include quantum graphs, tensor network, and graph C* algebras, among others. In this thesis, we have exploited the concept of graph Laplacian quantum states.

Given a Laplacian matrix of a graph there is a corresponding density matrix. We considered this quantum state as a graph Lapacian quantum state. In general it is a mixed state. This work is bidirectional. On one hand, we have investigated properties of a class of mixed quantum states in terms of graphs, such as, unitary operations, quantum entanglement and quantum discord. On the other hand, we have employed tools and techniques of quantum information theory in graph theoretical problems.

In the chapter 3, we have discussed a number of local unitary operations on graphs. Quantum gates are unitary operations. They act as local unitary operators when we apply them in a multi qubit system. An n qubit quantum state is represented by a graph consisting of 2^n vertices. Action of the quantum gate is equivalent to a set of changes in the edge set of the graph. Here, we have discussed procedures of X, Y, Z, H and $CNOT$ gate operations in terms of graphs. Also, we have developed graph theoretic procedures for Bell state generations.

We have discusses quantum entanglement, in chapter 4. Partial transpose is a tool of linear algebra used in entanglement detection. Using the idea of partial transpose we have introduced the idea of partially symmetric graphs. In higher dimensional Hilbert spaces, $\mathcal{H}^{(m)} \otimes \mathcal{H}^{(n)}$, partial symmetric graphs generated separable graph Laplacian quantum states. Also, we have explored the idea that graph isomorphism may generate mixed entangled states from mixed separable states.

Chapter 5 is dedicated to quantum discord. A quantum state has zero discord if the blocks of its density matrix form a family of commuting normal matrices. We have employed this idea in the graph theoretic context. We have constructed the condition on graphs for commutativity and normality. These conditions are in terms of neighbourhood of vertices. We classified quantum states with zero and non-zero discord using the above graph theoretic conditions. Then we generated a measure of quantum discord using graph theoretic parameters.

The idea of graph theoretic partial transpose and graph theoretic counterpart of commuting normal matrices was used in chapter 6 to generate a family of non-isomorphic cospectral graphs. We have shown that graph theoretical partial transpose is a useful tool for generating such classes of graphs. We have also attempted to solve a system of Lyapunov equations in this chapter.