Abstract

My doctoral research is at the interface of quantum information and graph theory. A Laplacian of a graph is a positive semidefinite, Hermitian matrix. A quantum state is represented by a density matrix that is a positive semidefinite Hermitian matrix with unit trace. Therefore, corresponding to every graph G there is a quantum state which is represented by the density matrix,

$$\rho(G) = \frac{1}{\text{trace}(K(G))} K(G),$$

where $K(G)$ is the combinatorial or signless Laplacian matrix of G. Here, we call these quantum states as graph Laplacian quantum states.

Analogous to classical computation, in quantum computation, we utilise quantum gates, for instance, the Pauli X, Y, Z, Hadamard, and CNOT gates. A graph theoretic counterpart of these operations is modelled by the graph-theoretic technique, called graph switching. We have developed a combinatorial procedure for each of these quantum gates. As a result, it is possible to generate a graph theoretical construction of Bell states, widely used in quantum information theory.

Detection of entanglement in a quantum state is one of the fundamental problems in quantum mechanics. Partial transpose is a crucial tool in entanglement detection. We have constructed a graph theoretical counterpart of this tool and used it to detect entanglement in higher dimensional quantum states. A number of graph isomorphisms can be utilised in generating entanglement from mixed separable states of arbitrary dimensions.

Apart from quantum entanglement, another well-known class of quantum correlations is quantum discord, which is also used as a resource, in quantum information processing. We have demonstrated graph theoretical criterion for non-zero discord in quantum states. On the basis of these conditions, we have provided a graph theoretic measure of quantum discord applicable for graph Laplacian quantum states. We have generalized some of these results for weighted digraphs.

My doctoral work is bidirectional. It attempts a connection between graph theory and quantum mechanics. Graph theoretic partial transpose is a valuable contribution of this effort. Constructing non-isomorphic cospectral graphs is one of the oldest challenging problems, in spectral graph theory. Using partial transpose we may construct a number of varieties of these graphs.

The broad theme of my work is to develop and exploit the connections between quantum mechanics and graph theory in such a way that both these, hitherto distinct fields behave in a symbiotic manner.