Abstract

The modern theory of dynamical systems originated in the late nineteenth century with the work of Poincaré, where he investigated the dynamics of the three body problem in celestial mechanics. Later, Birkhoff used dynamical systems to address several classical problems in ergodic theory. Smale also introduced the topological tools and methods in the mathematical theory of dynamical systems. Since then, the theory has gained attention and has been used to approximate various natural and physical systems in several branches of science and engineering. Although good approximations have been obtained using the theory of autonomous systems, better estimates can be attained using non-autonomous dynamical systems.

In this work, we investigate the dynamical behaviour of a general non-autonomous dynamical system generated by a finite family $\mathbb{F} = \{f_1, f_2, \cdots f_k\}$. We derive conditions under which the dynamics of the system $(X, \mathbb{F})$ can be characterized in terms of the dynamics of the autonomous system $(X, f_k \circ \cdots \circ f_2 \circ f_1)$. We derive relation between properties like equicontinuity, minimality, proximality and various forms of mixing for two systems. In the process, we analogously extend many of the results known for the autonomous case. We also derive relation between various forms of sensitivities for the two systems. We prove that, while topological transitivity is not equivalent for two systems, weakly mixing is equivalent for two systems when the family $\mathbb{F}$ is commutative. We also derive necessary and sufficient conditions for the non-autonomous system to exhibit stronger forms of mixing.

Further, we study the case when the non-autonomous system is generated by a uniformly convergent sequence of continuous surjective self maps on X. We derive condition under which the dynamics of the non-autonomous system can be characterized in terms of limiting system (autonomous system). We relate properties like the existence of periodic points, transitivity, weak mixing, topological mixing, various forms of sensitivities and denseness of proximal cells (pairs) for two systems. We prove that if the limit map is an isometry and (f_n) converges to f at a sufficiently fast rate, many of the dynamical notions for the non-autonomous system can be characterized using the limiting (autonomous) system.

We also study the dynamics of a general non-autonomous dynamical system. In particular, we relate the dynamical behaviour of the non-autonomous system with the dynamics of its generating functions. We prove that dynamics of the generating functions need not carry forward to the non-autonomous system generated (and vice-versa). Finally, we investigate the dynamics of various possible rearrangements of a general non-autonomous dynamical system. In the process, we derive conditions under which properties like various forms of mixing, sensitivity, Li-Yorke sensitivity and proximality are preserved under finite rearrangements. We also give an example to show that dynamics of the system need not be preserved under infinite rearrangement.

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