3

Quantum phase properties of photon added and subtracted displaced Fock states

In this chapter, the motive is to observe the phase properties of PADFS and PSDFS. The main findings of this chapter are published in Malpani et al. [2019b]

3.1 Introduction

In the previous chapter, the nonclassical properties of PADFS and PSDFS were studied. Here, our specific interest is to study the phase properties of PADFS and PSDFS and their limiting cases. In the recent past, the nonclassical properties of this set of engineered quantum states, many of which have been experimentally generated Lvovsky et al. [2001]; Lvovsky and Babichev [2002]; Zavatta et al. [2004, 2005, 2008], were focus of various studies (see Malpani et al. [2019a] and references therein). In Section 1.3.2, we have already expressed PADFS and PSDFS as superposition of Fock states. Further, in Section 1.6 we have described the parameters used for the study of phase properties of a quantum state. In that context we have already mentioned several applications of quantum phase distribution and quantum phase fluctuation.

To stress on the recently reported applications of quantum phase distribution and quantum phase fluctuation, we note that these have applications in quantum random number generation Xu et al. [2012]; Raffaelli et al. [2018], cryptanalysis of squeezed state based continuous variable quantum cryptography Horak [2004], generation of solitons in a Bose-Einstein condensate Denschlag et al. [2000], in phase encoding quantum cryptography Gisin et al. [2002], phase imaging of cells and tissues for biomedical application Park et al. [2018]; as well as have importance in determining the value of transition temperature for superconductors Emery and Kivelson [1995]. Keeping these applications and the general nature of engineered quantum states PADFS and PSDFS in mind, in what follows, we aim to study phase distribution, Q phase, phase fluctuation measures, phase dispersion, and quantum phase estimation using the concerned states and the states obtained in the limiting cases. As PADFS and PSDFS are already described, we may begin this study by describing limiting cases of these states as our states of interest.

We have already mentioned that our focus would be on PADFS and PSDFS. Due to the general form of PADFS and PSDFS, a large number of states can be obtained in the limiting cases.

Reduction of state	Name of the state	Reduction of state	Name of the state
$\vert\psi_+(u,n,\alpha)\rangle$	u-PADFS	$\vert\psi_-(v,n,\alpha)\rangle$	v-PSDFS
$\vert\psi_+(0,n,\alpha)\rangle$	DFS	$\vert\psi_-(0,n,\alpha)\rangle$	DFS
$\vert\psi_+(0,0,\alpha)\rangle$	Coherent state	$\vert\psi_-(0,0,\alpha)\rangle$	Coherent state
$\vert\psi_+(0,n,0)\rangle$	Fock state	$\vert\psi_-(0,n,0)\rangle$	Fock state
$\vert\psi_+(u,0,\alpha)\rangle$	u-Photon added coherent state	$\vert\psi_-(v,0,\alpha)\rangle$	v-Photon subtracted coherent state

Table 3.1: Various states that can be obtained as the limiting cases of the PADFS and PSDFS.

Some of the important limiting cases of PADFS and PSDFS in the present notation are summarized in Table 3.1. This table clearly establishes that the applicability of the results obtained in the present study is not restricted to PADFS and PSDFS; rather an investigation of the phase properties of PADFS and PSDFS would also reveal phase properties of many other quantum states of particular interest.

3.2 Quantum phase distribution and other phase properties

Quantum phase operator $\hat{\phi}$ was introduced by Dirac based on his assumption that the annihilation operator $\hat{a}$ can be factored out into a Hermitian function $f(\hat{N})$ of the number operator $\hat{N}=\hat{a}^{\dagger}\hat{a}$ and a unitary operator $\hat{U}$ Dirac [1927] as

$$\hat{a}=\hat{U}f\left(\hat{N}\right), \tag{3.1}$$

where

$$\hat{U}=e^{\imath\hat{\phi}}. \tag{3.2}$$

However, there was a problem with the Dirac formalism of phase operator as it failed to provide a meaning to the corresponding uncertainty relation. Specifically, in the Dirac formalism, the creation ($\hat{a}^{\dagger}$) and annihilation ($\hat{a}$) operators satisfy the bosonic commutation relation, $\left[\hat{a},\hat{a}^{\dagger}\right]=1$, iff $\left[\hat{N},\hat{\phi}\right]=\imath$, which leads to the number phase uncertainty relation $\Delta N\Delta\phi\geq1$. Therefore, in order to satisfy the bosonic commutation relation under Dirac formalism, the phase uncertainty should be greater than 2π for $\Delta N<\frac{1}{2\pi}$ which lacks a physical description. Subsequently, Louisell Louisell [1963] proposed some periodic phase based method, which was followed by Susskind and Glogower formalism based on Sine and Cosine operators Susskind and Glogower [1964]. An important contribution to this problem is the Barnett-Pegg formalism Barnett and Pegg [1986] which is used in this thesis. In what follows, we will also briefly introduce notions, such as quantum phase distribution, angular Q phase function, phase fluctuation parameters, phase dispersion, quantum phase estimation to study the phase properties of the quantum states of our interest.

3.3 Phase properties of PADFS and PSDFS

The description of the states of our interest given in the previous section can be used to study different phase properties and quantify phase fluctuation in the set of quantum states listed in

Table 3.1. Specifically, with the help of the quantum states defined in Eqs. (1.19)-(1.20), we have obtained the analytic expressions of phase distribution and other phase parameters defined in Section 3.2.

3.3.1 Phase distribution function

From the definition of the phase distribution (1.54), it can be observed that for a Fock state, $P_\theta = \frac{1}{2\pi}$, implying it has a uniform distribution of phase. Interestingly, the states of our interest, PADFS and PSDFS, are obtained by displacing the Fock state followed by photon addition/subtraction. Therefore, we will study here what is the effect of application of displacement operator on a uniformly phase distributed (Fock) state and how subsequent photon addition/subtraction further alters the phase distribution. Using phase distribution function, the information regarding uncertainty in phase and phase fluctuation can also be obtained. To begin with, we compute the analytic expressions of P_θ for the PADFS and PSDFS, using Eq. (1.54) as

$$\begin{aligned} P_\theta(u,n) &= \frac{1}{2\pi}\frac{|N_+|^2}{n!}\sum_{p,p'=0}^{n}\binom{n}{p}\binom{n}{p'}\exp\left[-\mid\alpha\mid^2\right]|\alpha|^{2n-p-p'} \\ &\times \sum_{m,m'=0}^{\infty}\frac{(-|\alpha|)^{m+m'}\sqrt{(m+p+u)!(m'+p'+u)!}}{m!m'!}\exp[\iota(\theta-\theta_2)(m'+p'-m-p)], \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} P_\theta(v,n) &= \frac{1}{2\pi}\frac{|N_-|^2}{n!}\sum_{p,p'=0}^{n}\binom{n}{p}\binom{n}{p'}\exp\left[-\mid\alpha\mid^2\right]|\alpha|^{2n-p-p'} \\ &\times \sum_{m,m'=0}^{\infty}\frac{(-|\alpha|)^{m+m'}(m+p)!(m'+p')!}{m!m'!\sqrt{(m+p-v)!(m'+p'-v)!}}\exp[\iota(\theta-\theta_2)(m'+p'-m-p)], \end{aligned} \tag{3.4}$$

respectively. Here, θ_2 is the phase associated with the displacement parameter α ($\alpha = |\alpha|e^{\iota\theta_2}$). Since the obtained expressions in Eqs. (3.3) and (3.4) are complex in nature, we depict numerical (graphical) analysis of the obtained results in Figs. 3.1 and 3.2 for PADFS and PSDFS, respectively. Specifically, in Figure 3.1 (a), we have shown the variation of phase distribution with phase parameter θ for different number of photon added in the displaced single photon Fock state ($D(\alpha)|1\rangle$) for $\theta_2 = 0$. A uniform phase distribution for Fock state (with a constant value of $\frac{1}{2\pi}$) is found to transform to one that decreases for higher values of phase and possess a dip in the phase distribution for $\theta = 0$, which can be thought of as an approach to the Fock state. In fact, in case of classical states, P_θ has a peak at zero phase difference $\theta - \theta_2$, and therefore, this contrasting behavior can be viewed as signature of quantumness of DFS. However, with the increase in the number of photons added to the DFS, the phase distribution of the PADFS is observed to become narrower. In fact, a similar behavior with increase in the mean photon number of coherent state was observed previously Agarwal et al. [1992]. It is imperative to state that P_θ in case of higher number of photon added to DFS has similar but narrower distribution than that of coherent state. In contrast, with increase in the Fock parameter, the phase distribution is observed to become broader (cf. Figure 3.1 (b)). Thus, the increase in the number of photons added and the increase in Fock parameter have opposite effects on the phase distribution. The same is also illustrated through the polar plots in Figure 3.1 (c)-(d), which not only reestablish the same fact, but also illustrate the dependence of P_θ on the phase of the displacement parameter. Specifically, the obtained phase distribution remains symmetric along the value of phase

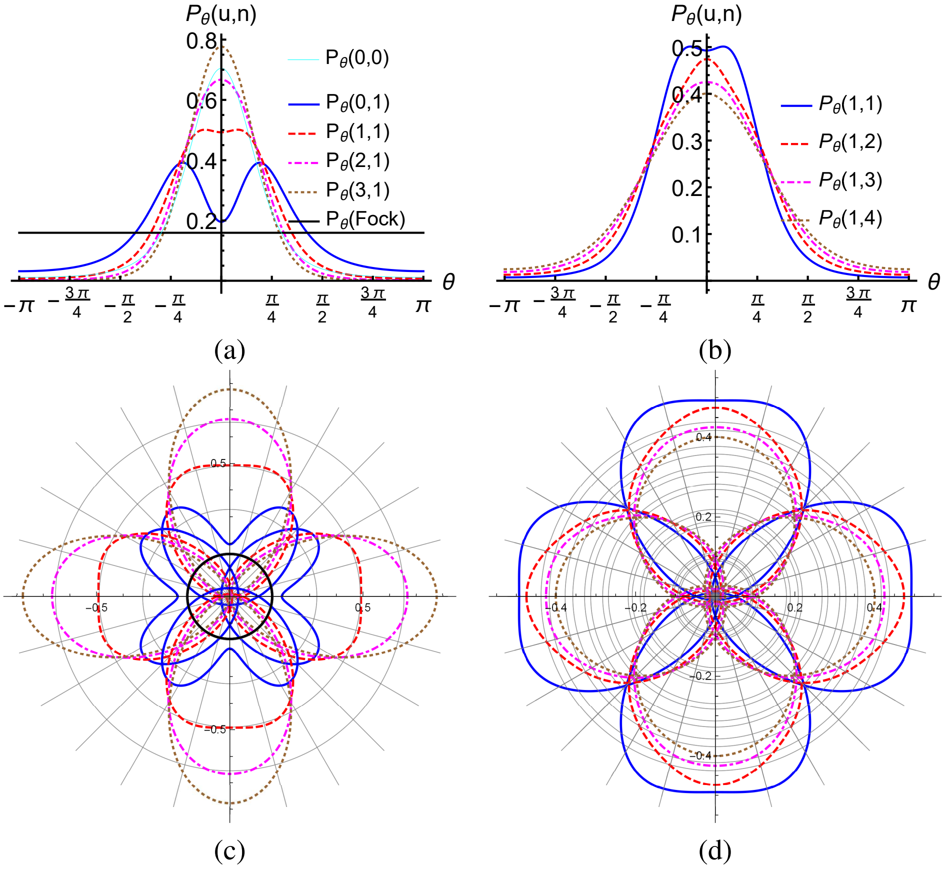

Figure 3.1: Variation of phase distribution function with phase parameter for PADFS with displacement parameter $|\alpha| = 1$ for different values of photon addition ((a) and (c)) and Fock parameters ((b) and (d)). The phase distribution is shown using both two-dimensional ((a) and (b) with $\theta_2 = 0$) and polar ((c) and (d)) plots. In (c) and (d), $\theta_2 = \frac{n\pi}{2}$ with integer $n \in [0, 3]$, and the legends are same as in (a) and (b), respectively.

θ_2 (i.e., P_θ is observed to have a mirror symmetry along $\theta = \theta_2$) of the displacement parameter. The phase distribution of Fock state is shown by a black circle in the polar plot.

Instead of photon addition, if we subtract photons from the DFS, a similar effect on the phase distribution to that of photon addition is observed. Further, a comparison between photon addition and subtraction on the phase distribution establishes that a single photon subtraction has a prominent impact on phase distribution when compared to that of single photon addition, i.e., the distribution can be observed to be narrower than that of coherent state in most of the cases for $u = v$. For instance, single photon added (subtracted) DFS is broader (narrower) than corresponding coherent state. Similarly, with the increase in the value of Fock parameter, we can observe more changes on PSDFS than what was observed in PADFS, i.e., the phase distribution broadens more with Fock parameter for PSDFS. Note that P_θ has a peak at $\theta = \theta_2$ only for photon addition $u > n$, while in case of photon subtraction it can be observed for $v \geq n$. With the increase in the amplitude of displacement parameter ($|\alpha|$) initially the phase distribution becomes narrower, which is further supported by both addition and subtraction of photons, but it becomes broader again for very high $|\alpha|$ (figure is not shown here).

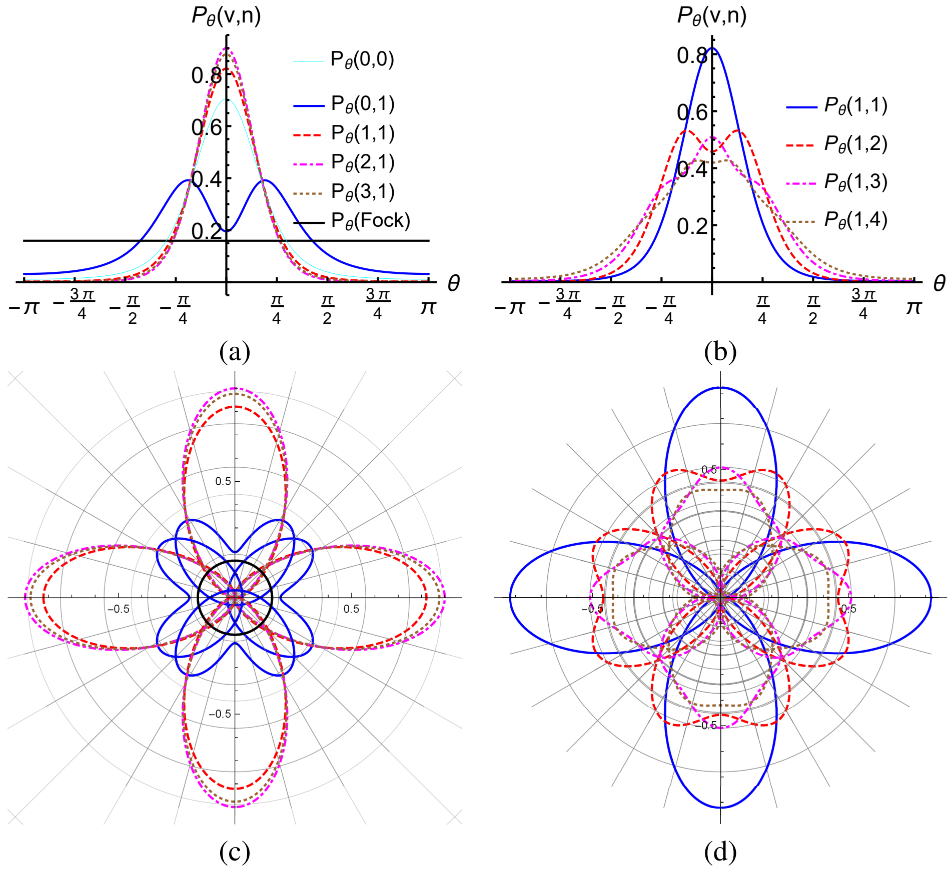

Figure 3.2: Variation of phase distribution function with phase parameter for PSDFS with displacement parameter $|\alpha| = 1$ for different values of photon subtraction ((a) and (c)) and Fock parameters ((b) and (d)). The phase distribution is shown using both two-dimensional ((a) and (b) with $\theta_2 = 0$) and polar ((c) and (d)) plots. In (c) and (d), $\theta_2 = \frac{n\pi}{2}$ with integer $n \in [0, 3]$, and the legends are same as in (a) and (b), respectively.

3.3.2 Angular Q function of PADFS and PSDFS

The relevance of the Q function as witness of nonclassicality Thapliyal et al. [2015] and in state tomography Thapliyal et al. [2016] is well studied. On top of that, non-Gaussianity of the PADFS and PSDFS using Q function was recently reported by us Malpani et al. [2019a]. We further discuss a phase distribution based on Q function using Eq. (1.57). In this particular case, we have obtained the angular Q function from the Q functions of the PADFS and PSDFS reported as Eqs. (15)-(16) in Malpani et al. [2019a]. Specifically, we have shown the effect of photon addition on the DFS ($D(\alpha)|1\rangle$) for a specific value of the displacement parameter in Figure 3.3 (a) for angular Q function. One can clearly see that the polar plots show an increase in the peak (located at $\theta_1 = \theta_2$) of the distribution with photon addition. Further, one can compare the behavior of Q_{θ_1} with P_θ in Figure 3.1 and observe that they behave quite differently (as reported in Agarwal et al. [1992] for the coherent states), other than increase in the peak of the distribution. Specifically, P_θ has a peak at $\theta = \theta_2$ only for $u > n$, while Q_{θ_1} is always peaked at the phase of the displacement parameter which also becomes a line of symmetry. Interestingly, the effect of increase in the Fock parameter of PADFS on $Q_{\theta_1}$is similar but less prominent in comparison to photon addition. This is in quite contrast of that observed for P_θ (in Figs. 3.1 and 3.3 (b)). In case of PSDFS, both photon subtraction and Fock parameter have completely different effects on Q_{θ_1} (cf. Figure 3.3 (c)-(d)) which is also in contrast to that on corresponding P_θ (shown in Figure 3.2). Specifically, with increase in photon subtraction the angular Q function becomes narrower peaked at $\theta = \theta_2$, but for larger number of photon subtraction the peak value decreases quickly. However, with increasing Fock parameter (cf. Figure 3.3 (d)), Q_{θ_1} behaves much like photon addition on DFS (shown in Figure 3.3 (a)). The observed behavior shows the relevance of studying both these phase distributions due to their independent characteristics.

3.3.3 Quantum phase fluctuation of PADFS and PSDFS

Note that Carruthers and Nieto Carruthers and Nieto [1968] had introduced these parameters in terms of Susskind and Glogower operators Susskind and Glogower [1964]; here we use them in Barnett-Pegg formalism to remain consistent with Gupta and Pathak [2007], where U parameter is shown relevant as a witness of nonclassicality Gupta and Pathak [2007]. Specifically, U is 0.5 for coherent state, and reduction of U parameter below the value for coherent state can be interpreted as the presence of nonclassical behavior Gupta and Pathak [2007]. In what follows, we will study quantum phase fluctuations for PADFS and PSDFS by computing analytic expressions of U, S and Q parameters in Barnett-Pegg formalism, with a specific focus on the possibility of witnessing nonclassical properties of these states via the reduction of U parameter below the coherent state limit. Carruthers and Nieto Carruthers and Nieto [1968] introduced three parameters to study quantum phase fluctuation (1.60)-(1.62). It was only in the recent past that Gupta and Pathak provided a physical meaning to one of these parameters by establishing its relation with antibunching and sub-Poissonian photon statistics Gupta and Pathak [2007]. Thus, the quantum phase fluctuation studied here using three parameters will also be used to witness the nonclassical nature of the quantum states under consideration. Here, the effect of photon addition/subtraction and displacement parameters on these fluctuation parameters is also studied (shown in Fig. 3.4). Specifically, Figure 3.4 (a)-(c) show variation of the three parameters of quantum phase fluctuation for different values of the number of photons added in the displaced Fock state ($D(\alpha)|1\rangle$) with displacement parameter $|\alpha|$. It may be clearly observed that two of the quantum phase fluctuation parameters, namely $U(u,n)$ and $Q(u,n)$ decrease with the value of displacement parameter, while $S(u,n)$ increases with $|\alpha|$. Interestingly, the photon addition and increase in the displacement parameter exhibit the same effect on all three quantum phase fluctuation parameters for PADFS, while for higher values of displacement parameter

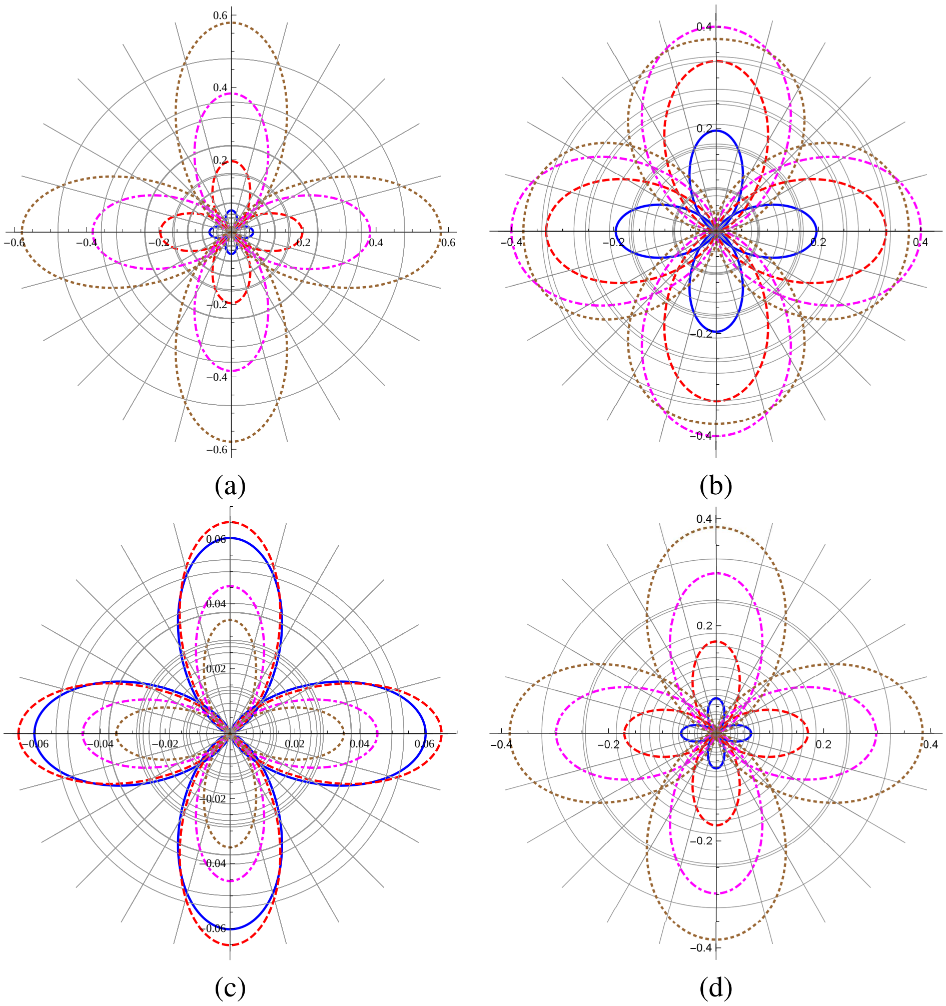

Figure 3.3: The polar plots for angular Q function for PADFS (in (a) and (b)) and PSDFS (in (c) and (d)) for displacement parameter $|\alpha|=1$ and $\theta_2 = \frac{n\pi}{2}$ with integer $n \in [0,3]$ for different values of photon addition/subtraction and Fock parameters. In (a) and (c), for $n = 1$, the smooth (blue), dashed (red), dot-dashed (magenta), and dotted (brown) lines correspond to photon addition/subtraction 0, 1, 2, and 3, respectively. In (b) and (d), for the single photon added/subtracted displaced Fock state, the smooth (blue), dashed (red), dot-dashed (magenta), and dotted (brown) lines correspond to Fock parameter 1, 2, 3, and 4, respectively.

$S(u,n)$ show completely opposite effect of photon addition. In contrast, $U(v,n)$ for v subtracted photons from $D(\alpha)|1\rangle$ is found to increase (decrease) with photon subtraction while decrease (increase) with the displacement parameter for small (large) value of $|\alpha|$ (cf. Figure 3.4 (d)). On the other hand, parameter $S(v,n)$ is also observed to increase (decrease) with $|\alpha|$ (v) as shown in Figure 3.4 (e). The third parameter $Q(v,n)$ shows slightly complex behavior for PSDFS with both $|\alpha|$ and v (cf. Figure 3.4 (f)) as it behaves analogous to PADFS for each subtracted photon for both small and large values of the displacement parameter (when it increases with $|\alpha|$), but for intermediate values the behavior is found to be completely opposite.

As mentioned previously, $U(i,n)\ \forall i \in \{u,v\}$ has a physical significance as a witness of antibunching for values of this parameter less than $\frac{1}{2}$, Figure 3.4 (a) and (d) can be used to perform similar studies for PADFS and PSDFS, respectively. In case of PADFS, we can observe this relevant parameter to become less than $\frac{1}{2}$, and thus to illustrate the presence of antibunching, only at higher values of the displacement parameter and photon added to the displaced Fock state. In contrast, PSDFS shows the presence of this nonclassical feature in all cases.Thus, occurrence of antibunching in PADFS and PSDFS is established here through this phase fluctuation parameter. Interestingly, a similar dependence of antibunching in PADFS and PSDFS Eq. (1.41) has been recently reported by us Malpani et al. [2019a] using a different criterion. Further, one can observe from the expression of U in Eq. (1.60) that it is expected to be independent of the phase of the displacement parameter, which can also be understood from the use of this parameter as a witness for an intensity moments based nonclassical feature. In contrast, S and Q in Eqs. (1.61)-(1.62) show dependence on the phase of displacement parameter. Here, we have not discussed the effect of Fock parameter in detail, but in case of photon addition, u and n have same (opposite) effects on S (U and Q) parameter(s). Fock parameter has always shown opposite effect of photon subtraction on all three phase fluctuation parameters, and thus nonclassicality revealed by U can be enhanced with Fock parameter. The relevance of Fock parameter can also be visualized by observing the fact that the single photon subtracted coherent state has $U = 0.5$ (which is consistent with the value zero of the antibunching witness reported in Thapliyal et al. [2017b]). Thus, in this case, the origin of the induced antibunching can be attributed to the non-zero value of Fock parameter.

3.3.4 Phase Dispersion

Here, it is worth stressing that both Carruthers-Nieto parameters and phase dispersion D correspond to phase fluctuation, Our primary focus is to study phase fluctuation and further to check the correlation between these measures of phase fluctuation. Thus, it would be interesting to study phase fluctuation from the two perspectives. We compute a measure of quantum phase fluctuation based on quantum phase distribution, the phase dispersion (1.56), for both PADFS and PSDFS to perform a comparative study between them. Specifically, the maximum value of dispersion is 1 which corresponds to the uniform phase distribution, i.e., $P_\theta = \frac{1}{2\pi}$. Both PADFS and PSDFS show a uniform distribution for the displacement parameter $\alpha = 0$ (cf. Figure 3.5). It is a justified result as both the states reduce to the Fock state in this case. However, with the increase in the value of displacement parameter quantum phase dispersion is found to decrease. This may be attributed to the number-phase complimentarity Banerjee and Srikanth [2010]; Srikanth and Banerjee [2009, 2010], which leads to smaller phase fluctuation with increasing variance in the number operator at higher values of displacement parameter. Thus, with an increase in the average photon number by increasing the displacement parameter, phase dispersion decreases for both PADFS and PSDFS. Addition of photons in DFS leads to decrease in the value of phase dispersion, while subtraction of photons has more complex effect on phase dispersion (cf. Figure 3.5 (a) and (c)). Specifically, for

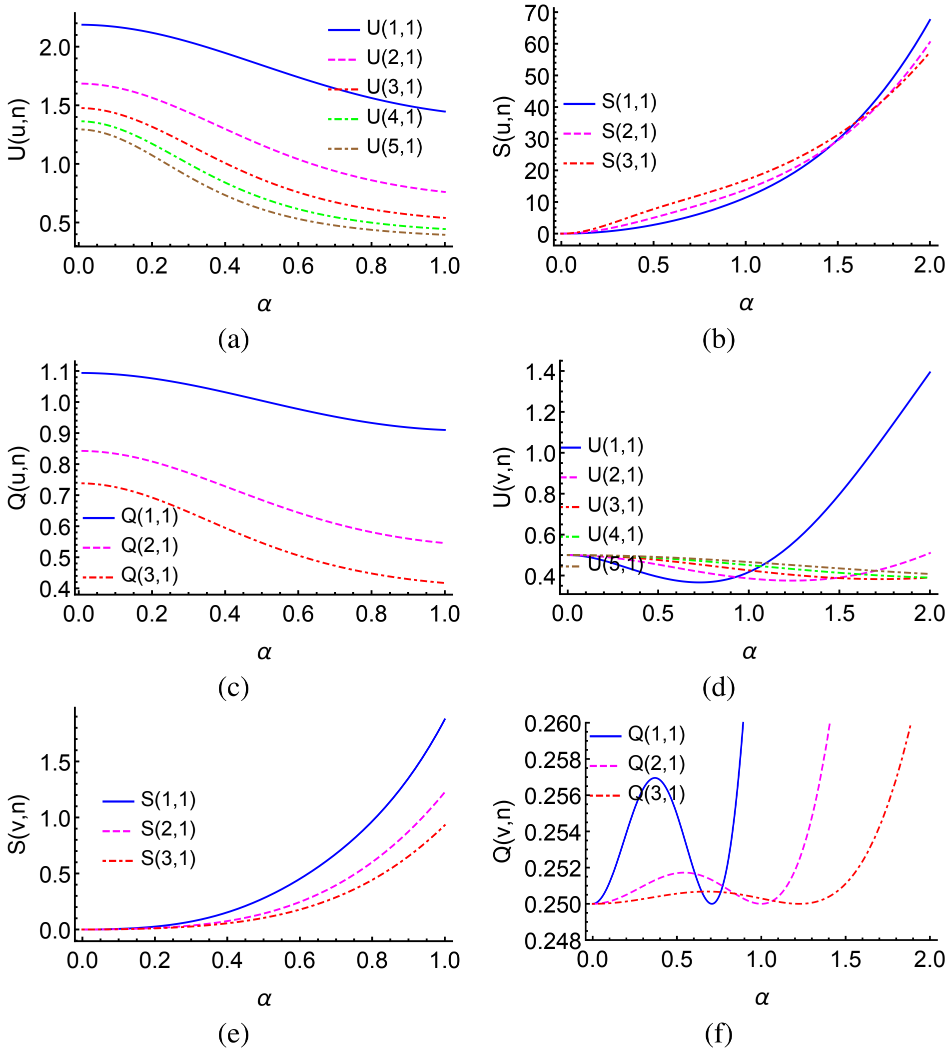

Figure 3.4: Variation of three phase fluctuation parameters introduced by Carruthers and Nieto with the displacement parameter with $\theta_2 = 0$. The values of photon addition (u), subtraction (v), and Fock parameter $n = 1$ are given in the legends. Parameter $U\,(i,n)\;\forall i \in \{u,v\}$ also illustrates antibunching in the states for values less than $\frac{1}{2}$.

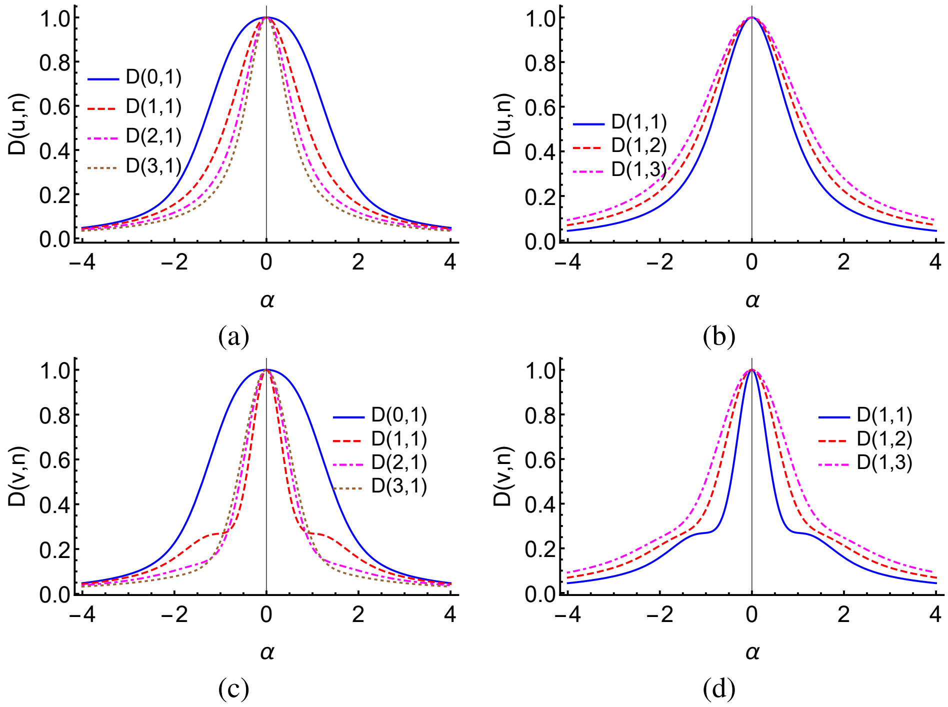

Figure 3.5: Variation of phase dispersion for PADFS (in (a) and (b)) and PSDFS (in (c) and (d)) with displacement parameter for an arbitrary θ_2. Dependence on different values of photon added/subtracted and the initial Fock state $|1\rangle$ (in (a) and (c)), while on different values of Fock parameter for single photon added/subtracted state (in (b) and (d)).

the smaller values of the displacement parameter ($|\alpha| < 1$), the phase dispersion parameter behaves differently for $v \leq n$ and $v > n$. This can be attributed to the sub-Poissonian photon statistics for $v \leq n$ with $|\alpha| < 1$ as well as the small value of average photon number (Figure 3.4 (d)). However, at the higher values of the displacement parameter D for the PSDFS behaves in a manner analogous to the PADFS. Interestingly, increase in the Fock parameter shows similar effect on PADFS and PSDFS in Figure 3.5 (b) and (d), respectively.

3.3.5 Phase sensing uncertainity for PADFS and PSDFS

We finally discuss quantum phase estimation using Eq. (1.69), assuming the two mode input state in the Mach-Zehnder interferometer as $|\psi_i(j,n,\alpha)\rangle \otimes |0\rangle$. The expressions for the variance of the difference in the photon numbers in the two output modes of the Mach-Zehnder interferometer for input PADFS and PSDFS and the rest of the parameters required to study phase sensing are reported in Appendix.

The obtained expressions allow us to study the optimum choice of state parameters for quantum phase estimation using PADFS and PSDFS. The variation of these parameters is shown in Figure 3.6. Specifically, we have shown that PSDFS is preferable over coherent state for phase estimation (cf. Figure 3.6 (b)). However, with the increase in the photon subtraction this phase uncertainty parameter is found to increase although remaining less than corresponding coherent state value. In contrast, with photon addition, advantage in phase estimation can be attained as the reduction of the phase uncertainty parameter allows one to perform more precise measurement. This advantage can be enhanced further by choosing large values of photon addition and Fock parameter (cf. Figure 3.6

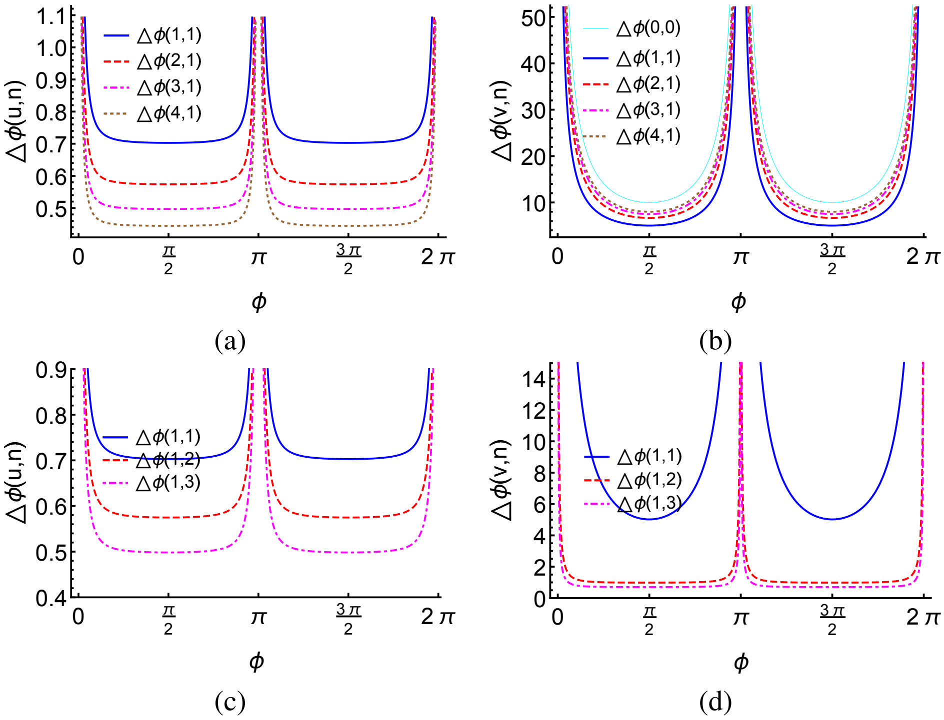

Figure 3.6: Phase sensing uncertainty for (a) PADFS and (b) PSDFS as a function of phase to be estimated ϕ for different number of photon addition/subtraction with $n = 1$. The dependence for (c) PADFS and (d) PSDFS is also shown for different values of Fock parameters with $u = 1$ and $v = 1$, respectively. In all cases, we have chosen $\alpha = 0.1$.

(a) and (c)). In a similar sense, appropriate choice of Fock parameter would also be advantageous in phase estimation with PSDFS as it decreases the phase uncertainty parameter, but still PADFS remains preferable over PSDFS. This can further be controlled by an increase in $|\alpha|$ which decreases (increases) phase uncertainty parameter for PADFS (PSDFS).

3.4 Conclusions

A set of engineered quantum states can be obtained as the limiting cases from the PADFS and PSDFS, e.g., DFS, coherent state, photon added/subtracted coherent state, and Fock state. Specifically, PADFS/PSDFS are obtained by application of unitary (displacement) and non-unitary (addition and subtraction of photons) operations on Fock state. In view of the fact that the Fock states have uniform phase distribution, the set of unitary and non-unitary quantum state engineering operations are expected to affect the phase properties of the generated state. Therefore, here we have calculated quantum phase distribution, which further helped in quantifying phase fluctuation as phase dispersion. We have also computed the phase distribution as the angular Q function. We have further studied phase fluctuation using three Carruthers and Nieto parameters, and have used one of them to reveal the existence of antibunching in the quantum states of our interest.

Both the phase distribution and angular Q functions are found to be symmetric along the value of the phase of the displacement parameter. The phase distribution is observed to become narrow and peak(s) to increase with the amplitude of the displacement parameter ($|\alpha|$), which further becomes broader for higher values of $|\alpha|$. Further, photon addition/subtraction and Fock parameters are observed to have opposite effects on phase distribution, i.e., distribution function becomes narrower (broader) with photon addition/subtraction (Fock parameter). Among photon addition and subtraction operations, subtracting a photon alters the phase properties more than that of photon addition. Specifically, at the small values of the displacement parameter ($|\alpha| < 1$), the phase properties of PSDFS for $v \leq n$ and $v > n$ behave differently. This can be attributed to the fact that for $v \leq n$ with $|\alpha| < 1$, the average photon number becomes very small. Further, the peak of the phase distribution remains at the phase of displacement parameter only when the number of photons added/subtracted is more than that of the Fock parameter. However, in case the number of photons subtracted (added) is same as the Fock parameter, the peak of the phase distribution is observed (not observed) at the phase of displacement parameter. The angular Q function can be observed to show similar dependence on various parameters, but the peak of the distribution remains located at the value of phase of the displacement parameter. The three phase fluctuation parameters introduced by Carruthers and Nieto Carruthers and Nieto [1968] show phase properties of PADFS and PSDFS, while one of them, U parameter also reveals antibunching in both PADFS and PSDFS. In this case, the role of Fock parameter as antibunching inducing operation in PSDFS is also discussed. Phase dispersion quantifying phase fluctuation remains unity for Fock state reflecting uniform distribution, which can be observed to decrease with increasing displacement parameter. This may be attributed to the number-phase complimentarity as the higher values of variance with increasing displacement parameter lead to smaller phase fluctuation. Fock parameter and photon addition/subtraction show opposite effects on the phase dispersion as it increases (decreases) with n (u/v).

Finally, we have also discussed the advantage of the PADFS and PSDFS in quantum phase estimation and obtained the set of optimized parameters in the PADFS/PSDFS. Both photon addition and Fock parameter decrease the uncertainty in phase estimation, while photon subtraction, though performs better than coherent state is not as advantageous as u or n. In Ou [1997], it was established that signal-to-noise ratio is significant only when the phase shift to measure is of the same order as multiplicative inverse of the average photon number. Therefore, in case of PADFS this limitation of quantum measurement is expected to play an important role. Thus, we have shown here that state engineering tools can be used efficiently to control the phase properties of the designed quantum states for suitable applications. The study performed in this chapter can be extended for other such operations, like squeezing, photon addition followed by subtraction or vice versa.