4

Impact of photon addition and subtraction on nonclassical and phase properties of a displaced Fock state

In this chapter, we aim to observe the nonclassical and phase properties of a PASDFS. The work done in this chapter is published in Malpani et al. [2020b].

4.1 Introduction

In Chapter 2, we have already studied nonclassical properties of PADFS and PSDFS. In Chapter 3, we have investigated phase properties of the same set state. In both chapters, we have obtained various exciting observations such as photon addition and subtraction enhance nonclassical properties of non-Gaussian DFS. Motivated by these, here we aim to study both nonclassical and phase properties for a more general quantum state. To be specific, in this chapter, we aim to study the nonclassical (both lower- and higher-order) and phase properties of a PASDFS. The reason behind selecting this particular state lies in the fact that this is a general state in the sense that in the limiting cases, this state reduces to different quantum states having known applications in continuous variable quantum cryptography (this point will be further elaborated in the next section).

As it appears from the above discussion, this investigation has two facets. Firstly, we wish to study nonclassical features of PASDFS, namely Klyshko's Klyshko [1996], Agarwal-Tara's Agarwal and Tara [1992], Vogel's Shchukin and Vogel [2005b] criteria, lower- and higher-order antibunching Pathak and Garcia [2006], squeezing Hillery [1987a]; Hong and Mandel [1985a,b], and sub-Poissonian photon statistics (HOSPS) Zou and Mandel [1990]. We subsequently study the phase properties of PASDFS by computing phase distribution function Agarwal and Singh [1996]; Beck et al. [1993], phase fluctuation parameters Carruthers and Nieto [1968]; Barnett and Pegg [1986], and phase dispersion Peřinová et al. [1998]. A detailed analysis of the obtained results will also be performed to reveal the usefulness of the obtained results.

4.2 Moments of the field operators for the quantum states of our interest

As mentioned in the previous section, this work is focused on PASDFS. A PASDFS as a Fock superposition state has already been expressed in Eq.(1.23). To study nonclassical and phase properties of this state, we have used nonclassicality witnesses introduced in Section 2.3 and phase parameters in Section 1.6, we would require analytic expression for moments of the field operators. A bit of computation yields the expression for higher-order moment of annihilation and creation operator as

$$\begin{array}{rcl}\langle\hat{a}^{\dagger t}\hat{a}^{j}\rangle & = & \langle\psi(k,q,n,\alpha)|\hat{a}^{\dagger t}\hat{a}^{j}|\psi(k,q,n,\alpha)\rangle\\ & = & \frac{N^{2}}{n!}\sum_{p,p'=0}^{n}\binom{n}{p}\binom{n}{p'}(-\alpha^{\star})^{(n-p)}(-\alpha)^{(n-p')}\\ & \times & \exp\left[-\mid\alpha\mid^{2}\right]\sum_{m=0}^{\infty}\frac{\alpha^{m}(\alpha^{\star})^{m+p-p'-j+t}(m+p+k)!(m+p+k-j+t)!}{m!(m+p-p'-j+t)!(m+p+k-q-j)!}.\end{array} \tag{4.1}$$

For different values of t and j, moments of any order can be obtained, and the same may be used to investigate the nonclassical properties of PASDFS and its limiting cases by using various moments-based criteria of nonclassicality. The same will be performed in the following section, but before proceeding, it would be apt to briefly state our motivation behind the selection of this particular state for the present study (or why do we find this state as interesting?).

Due to the difficulty in realizing single photon on demand sources, the unconditional security promised by various QKD schemes, like BB84 Bennett and Brassard [1984] and B92 Bennett et al. [1992], does not remain unconditional in the practical situations This is where continuous variable QKD (CVQKD) becomes relevant as they do not require single photon sources. Special cases of PASDFS has already been found useful in the realization of CVQKD. For example, protocols for CVQKD have been proposed using photon added coherent state ($k=1, q=0, n=0$) Pinheiro and Ramos [2013]; Wang et al. [2014], photon added then subtracted coherent states ($k=1, q=1, n=0$) Borelli et al. [2016]; Srikara et al. [2019], and coherent state ($k=0, q=0, n=0$) Grosshans and Grangier [2002]; Hirano et al. [2017]; Huang et al. [2016]; Ma et al. [2018]. Further, boson sampling with displaced single photon Fock states and single photon added coherent state Seshadreesan et al. [2015] has been reported, and an m photon added coherent state ($k=m, q=0, n=0$) has been used for quantum teleportation Pinheiro and Ramos [2013]. Apart from these schemes of CVQKD, which can be realized by using PASDFS or its limiting cases, the fact that the photon addition and/or subtraction operation from a classical or nonclassical state can be performed experimentally using the existing technology Parigi et al. [2007]; Zavatta et al. [2004] has enhanced the importance of PASDFS.

4.3 Nonclassicality witnesses and the nonclassical features of PASDFS witnessed through those criteria

The negative value of the Glauber-Sudarshan P-function characterizes nonclassicality of an arbitrary state Glauber [1963a]; Sudarshan [1963]. As P-function is not directly measurable in experiments, many witnesses of nonclassicality have been proposed, such as, negative values of

Wigner function Wigner [1932]; Kenfack and Życzkowski [2004], zeroes of Q function Husimi [1940]; Lütkenhaus and Barnett [1995], several moments-based criteria Miranowicz et al. [2010]; Naikoo et al. [2018]. An infinite set of such moments-based criteria of nonclassicality is equivalent to P-function in terms of necessary and sufficient conditions to detect nonclassicality Richter and Vogel [2002]. Here, we would discuss some of these moments-based criteria of nonclassicality and Q function (in Section 4.5) to study nonclassical properties of the state of our interest.

4.3.1 Lower- and higher-order antibunching

The relevance of photon addition, photon subtraction, Fock, and displacement parameters in the nonclassical properties of the class of PASDFSs is studied here rigorously. Specifically, using Eq. (4.1) with the criterion of antibunching (1.41) we can study the possibilities of observing lower- and higher-order antibunching in the quantum states of PASDFS class, where the class of PASDFSs refers to all the states that can be reduced from state (1.23) in the limiting cases. The outcome of such a study is illustrated in Figure 4.1. It is observed that the depth of lower- and higher-order nonclassicality witnesses can be increased by increasing the value of the displacement parameter, but large values of α deteriorate the observed nonclassicality (cf. Figure 4.1 (a)-(b)). The nonclassicality for higher-values of displacement parameter α can be induced by subtracting photons at the cost of reduction in the depth of nonclassicality witnessed for smaller α, as shown in Figure 4.1 (a). However, photon addition is always more advantageous than subtraction. Therefore, both addition and subtraction of photons illustrate these collective effects by showing nonclassicality for even higher values of α at the cost of that observed for the small values of displacement parameter. Fock parameter has completely opposite effect of photon subtraction as it shows the advantage (disadvantage) for small (large) values of displacement parameter. Figure 4.1 (b) shows benefit of studying higher-order nonclassicality as depth of corresponding witness of nonclassicality can be observed to increase with the order. The higher-order nonclassicality criterion is also able to detect nonclassicality for certain values of displacement parameter for which the corresponding lower-order criterion failed to do so.

4.3.2 Higher-order sub-Poissonian photon statistics

Variation of HOSPS nonclassicality witness for class of PASDFSs obtained by different nonclassicality inducing operations show the same effect as that of antibunching witness for all the odd orders of HOSPS, and as depicted in Figure 4.1 (c). However, this nonclassical feature disappears for even orders of HOSPS (cf. Figure 4.1 (d)). In case of the odd orders of HOSPS, though the depth of nonclassicality witness increases with the order, higher-order criterion is found to fail to detect nonclassicality for certain values of α when corresponding HOSPS criterion for smaller values of orders shown the nonclassicality.

4.3.3 Lower- and higher-order squeezing

Out of all the nonclassicality inducing operations used in PASDFS only photon subtraction is squeezing inducing operation as shown in Figure 4.2, which is consistent with some of our recent observations Malpani et al. [2019a]. With photon addition higher-order squeezing can be induced for large values of modulus of displacement parameter at the cost of squeezing observed for small $|\alpha|$ as long as the number of photon subtracted is more than the value of Fock parameter. As far as higher-order squeezing is concerned, the observed nonclassicality disappears for large values of real displacement parameter with increase in the depth of the nonclassicality witness. Squeezing being a phase dependent nonclassical feature depends on the phase θ of the displacement parameter

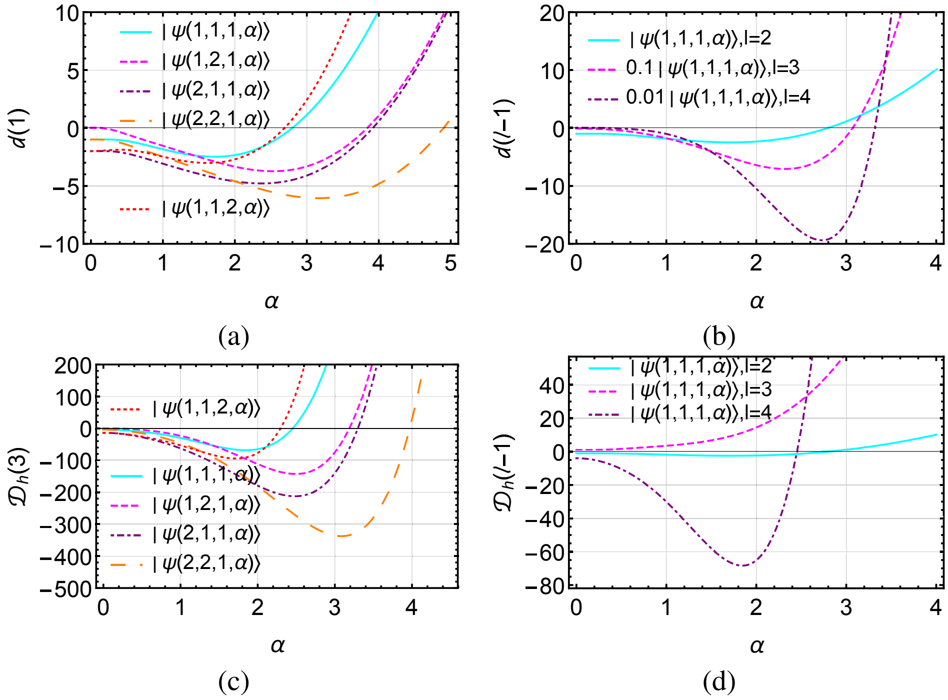


Figure 4.1: For PASDFS the lower-and higher-order antibunching is given as a function of displacement parameter α. (a) Lower-order antibunching for different values of parameters of the state. (b) Higher-order antibunching for particular state. HOSPS for PASDFS for different values of (c) state parameters and (d) order of nonclassicality.

$\alpha = |\alpha|\exp[\imath\theta]$ (shown in Figure 4.2 (c) for lower-order squeezing).

4.3.4 Klyshko's Criterion

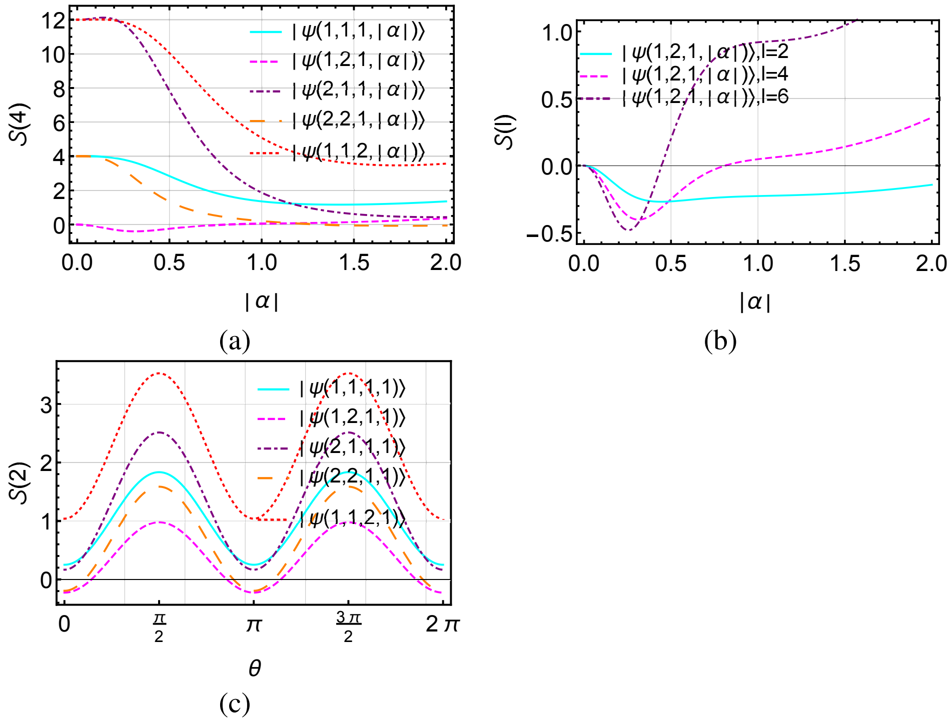
For PASDFS p_z can be obtained from Eq. (4.1). Nonclassicality reflected through Klyshko's criterion can be controlled by all the state engineering operations used here as shown in Figure 4.3. The depth of this nonclassicality witness increases at higher values of photon numbers z due to increase in photon addition and/or Fock parameter. In contrast, depth of witness increases at smaller photon numbers z due to photon subtraction. The Klyshko's nonclassicality witness is positive for some photon numbers only if $k+n>q$. Additionally, with increase in displacement parameter the depth of nonclassicality witness decreases, and the weight of the distribution of witness shift to higher values of z.

4.3.5 Agarwal-Tara's criterion

This nonclassicality witness is able to detect nonclassicality in all the quantum states in the class of PASDFSs (cf. Figure 4.4 (a)). Note that for $|\psi(1,2,1,\alpha)\rangle$ with small α, A_3 parameter is close to zero, which is due to very high probability for zero photon states.

4.3.6 Vogel's criterion

The negative value of the determinant dv of matrix v in Eq. (1.47) is signature of nonclassicality. Fock parameter has adverse effect on the nonclassicality in PASDFS detected by this criterion. This averse effect can be compensated by photon subtraction and can be further controlled by photon



(a) (b) (c)

Figure 4.2: Dependence of the Hong-Mandel-type higher-order squeezing witness on displacement parameter for (a) different state parameters and (b) order of squeezing. (c) Lower-order squeezing as a function of phase parameter of the state, i.e., phase θ of displacement parameter $\alpha = 1.\exp[\imath\theta]$.

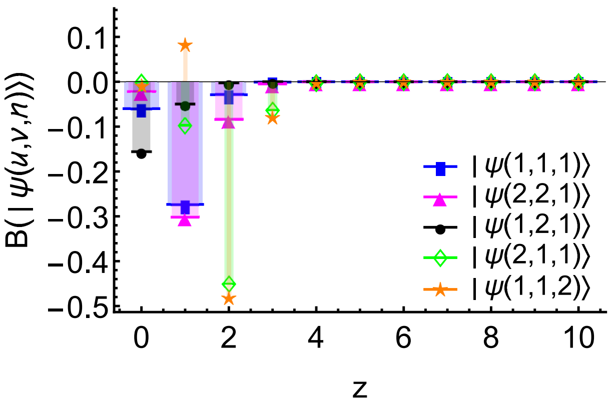
addition (as shown in Figure 4.4 (b)). Notice that the nonclassical behavior illustrated by Agarwal-Tara's (Vogel's) criterion is related to higher-order antibunching (squeezing) criterion. However, nonclassicality witness of Vogel's criterion is a phase independent property unlike squeezing.

4.4 Phase properties of PASDFS

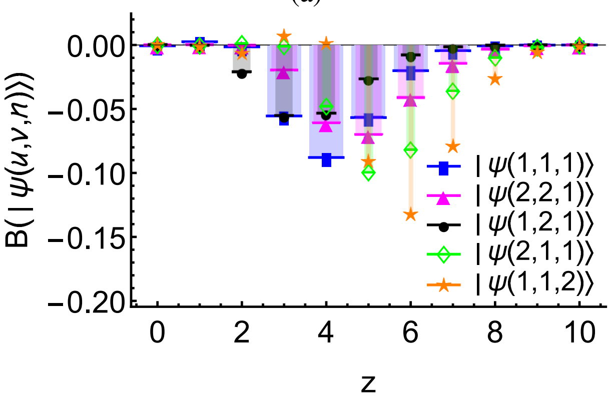
The nonclassicality inducing operations are also expected to impact the phase properties of a quantum state Banerjee and Srikanth [2007]. Recently, we have reported an extensive study on the role that such quantum state engineering tools can play in application oriented studies on quantum phase Malpani et al. [2019b]. Specifically, relevance in quantum phase estimation, phase fluctuation, and phase distribution were discussed which can play an important role in quantum metrology Giovannetti et al. [2011]. Here, we briefly discuss some of the phase properties of the class of PASDFSs.

4.4.1 Phase distribution function

The analytical expression for phase distribution function for PASDFS can be computed as



(a)



(b)

Figure 4.3: Illustration of Klyshko's parameter $B(z)$ with respect to the photon number z for different values of state parameters with (a) $\alpha = 0.5$ and (b) $\alpha = 1$.

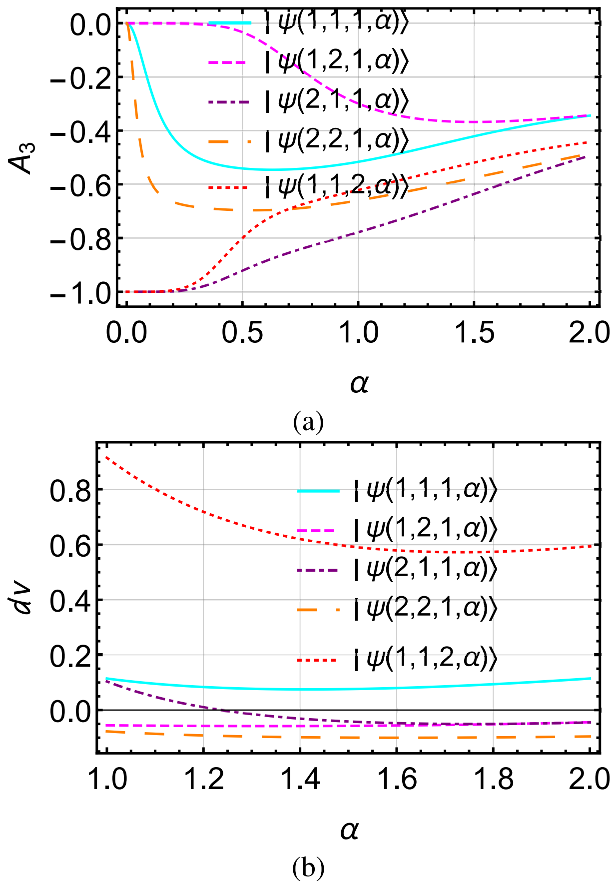
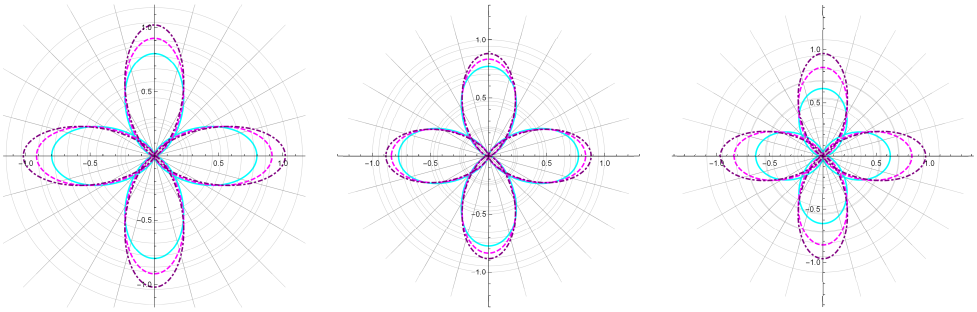


Figure 4.4: Nonclassicality reflected through the negative values of (a) Agarwal-Tara's and (b) Vogel's criteria as a function of α or different state parameters.



(a) (b) (c)

Figure 4.5: Polar plot of phase distribution function for PASDFS $|\psi(k,q,n,\alpha)\rangle$ with respect to variation in displacement parameter for (a) $n=1$, $k=2$ and $q=1$, 2, and 3 represented by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively; (b) $n=2$, $q=2$ and $k=1$, 2, and 3 illustrated by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively; and (c) $n=1$ with $k=q=1$, 2, and 3 shown by the smooth (cyan), dashed (magenta), and dot-dashed (purple) lines, respectively.

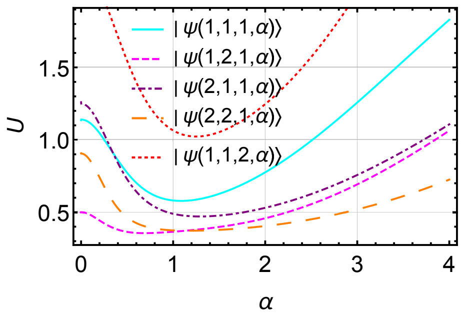


Figure 4.6: Variation of phase fluctuation parameter with displacement parameter for various state parameters in PASDFS.

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}\frac{N^2}{n!}\sum_{p,p'=0}^{n}\binom{n}{p}\binom{n}{p'}(-\alpha^{\star})^{(n-p)}(-\alpha)^{(n-p')}\exp\left[-\mid\alpha\mid^2\right] \\ &\times \sum_{m,m'=0}^{\infty}\frac{\alpha^m(\alpha^{\star})^{m'}(m+p+k)!(m'+p'+k)!}{m!m'!\sqrt{(m+p+k-q)!(m'+p'+k-q)!}}\exp[\imath\theta(m'+p'-m-p)]. \end{aligned} \tag{4.2}$$

Photon subtraction can be observed to be a more effective tool to alter phase properties of PASDFS than photon addition, as shown in Figure 4.5. Interestingly, photon addition shows similar behavior, though less prominent, as photon subtraction, Fock parameter has opposite effect.

4.4.2 Phase Fluctuation

Here, we focus only on the first phase fluctuation parameter U, which is related to antibunching if U is below its value for coherent state (i.e., 0.5), remaining consistent with Barnett-Pegg formalism Gupta and Pathak [2007]; Pathak and Mandal [2000]. One can observe that the phase fluctuation parameter is able to detect nonclassicality (specifically antibunching) only in three cases where the role of the photon subtraction is relevant (cf. Figure 4.6). The observation can be seen analogous to that observed for Vogel's nonclassicality criterion.

4.5 Quasidistribution function: Q function

Here, We will establish non-Gaussianity inducing behavior of photon addition and Fock parameter (cf. Figure 4.7), which are so far illustrated as nonclassicality inducing and phase altering operations. Clearly, with photon addition tendency of quasidistribution away from Gaussian behavior is visible, while with photon subtraction squeezing along particular phase angle chosen by displacement parameter can be observed. This squeezing can be noticed to be more appreciable for higher values of displacement parameter (cf. Figure 4.7 (c)-(d)). From Figure 4.7 (e)-(f), it can be observed that Fock parameter and photon addition have a similar effect in the phase space. As zeros of Q function are signature of nonclassicality, PASDFS shows nonclassicality in Figure 4.7 (b), (e), and (f). This establishes that use of more than one state engineering tool may be helpful in generation of nonclassical states. It would be interesting to verify whether one more tools (say squeezing or photon catalysis) may further enhance the nonclassical properties.

4.6 Conclusions

In this chapter, we have investigated the nonclassical behavior of PASDFS using different witnesses of lower- and higher-order nonclassicality. The significance of this choice of state underlies the fact that a class of engineered quantum states can be achieved as the reduced case of PASDFS $|\psi(k,q,n,\alpha)\rangle$, like photon added DFS $(q=0)$, photon subtracted DFS $(k=0)$, DFS $(q=k=0)$, photon added coherent state $(n=q=0)$, photon subtracted coherent state $(n=k=0)$, coherent state $(n=k=q=0)$, and Fock state $(n=k=q=\alpha=0)$. Some of the reduced states have been experimentally realized and in some cases optical schemes for generation have been proposed, so this family of states is apt for various challenging tasks to establish quantum dominance. The state under consideration requires various non-Gaussianity inducing quantum engineering operations and thus our focus here was to analyze the relevance of each operation independently in the nonclasscial features (listed in Table 4.1) observed in PASDFS. To study the nonclassical properties of PASDFS, a set of moments-based criteria for Klyshko's, Agrwal-Tara's, and Vogel's criteria, as well as lower- and higher-order antibunching, HOSPS, and squeezing. Further, phase properties for the same state are also studied using phase distribution function and phase fluctuation. Finally, non-Gaussianity and nonclassicality of PASDFS is also studied using Q function.

The present study reveals that with an increase in the order of nonclassicality the depth of nonclassicality witnesses increase. Additionally, higher-order nonclassicality criteria were able to detect nonclassicality in the cases when corresponding lower-order criteria failed to do so. Different nonclassical features are observed for smaller values of displacement parameter, which can be sustained for higher values by increasing the number of subtracted photon. Photon addition generally improves nonclassicality, and this advantage can be further enhanced for the higher (smaller) values of displacement parameter using photon subtraction (Fock parameter). The HOSPS nonclassical feature is only observed for the odd orders. As far as squeezing is concerned, only photon subtraction could induce this nonclassicality. Large number of photon addition can be used to observe squeezing

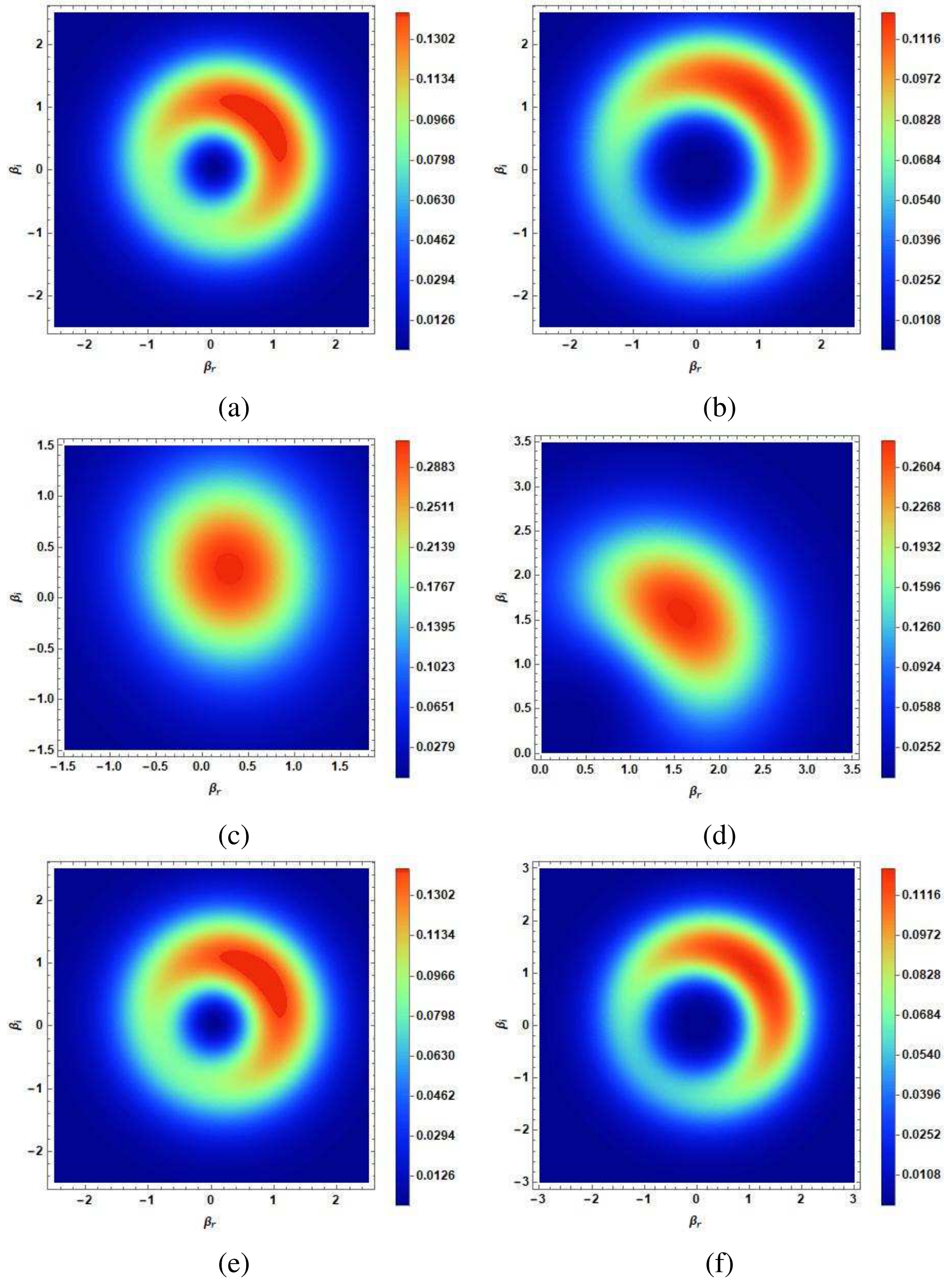


Figure 4.7: Q function for PASDFS $|\psi(k,q,n,\alpha)\rangle$ with (a) $k = q = n = 1$, (b) $k = 2, q = n = 1$, and (c) $q = 2, k = n = 1$ with $\alpha = \frac{1}{5\sqrt{2}} \exp(\imath\pi/4)$. (d) Similarly, Q function of PASDFS with $q = 2, k = n = 1$ and $\alpha = \sqrt{2}\exp(\imath\pi/4)$. Q function for $|\psi(k,q,n,\alpha)\rangle$ with (e) $k = q = 1, n = 2$ and (f) $q = 1, k = n = 2$ for $\alpha = \frac{1}{5\sqrt{2}} \exp(\imath\pi/4)$.

S. No.	Nonclassical Properties	Observed in PASDFS
1	Lower-order and higher-order Antibunching	yes
2	Higher-order sub Poissionian photon statistics	yes
3	Lower-order and higher-order squeezing	yes
4	Klyshko's criterion	yes
5	Agarwal-Tara's criterion	yes
6	Vogel's criterion	yes
7	Phase distribution function	-
8	Phase fluctuation	yes
9	Q function	yes

Table 4.1: Summary of the nonclassical properties of PASDFS.

at higher values of displacement parameter at the cost of that present for smaller α. Photon subtraction alters the phase properties more than photon addition, while Fock parameter has an opposite effect of the photon addition/subtraction. The nonclassicality revealed through phase fluctuation parameter shows similar behavior as Vogel's criterion. Finally, we have shown the nonclassicality and non-Gaussianity of PASDFS with the help of a quasidistribution function, namely Q function.