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Conclusions And Scope For Future Work

This concluding chapter aims to briefly summarize the results obtained in this thesis work, and it also aims to provide some insights into the scope of future works. To begin with, we may note that this thesis is a theoretical work focused on nonclassical and phase properties of some of the engineered quantum states of radiation field. Here, lower- and higher-order nonclassical properties of PADFS, PSDFS, PASDFS, ECS, VFECS, PAECS, BS, VFBS, PABS, KS, VFKS and PAKS have been witnessed through lower- and higher-order antibunching, higher-order sub-Poissonian photon statistics, higher-order squeezing, Klyshko's criterion, Vogel's criterion, Agarwal-Tara's criterion, Qfunction, Mandel Q_M parameter, etc. Further, phase properties of these states have been investigated with the help of phase distribution function, phase dispersion, phase fluctuation, phase uncertainty parameter and angular Q function. These investigations have revealed that the state engineering processes may help us to introduce and manipulate the nature and amount of nonclassicality present in a quantum state. Keeping this in mind, at the end of the thesis two quantum state engineering processes, which can be used to generate holes at vacuum in photon number distribution, have been compared. This systematic and rigorous study of the nonclassical and phase properties of the above mentioned engineered quantum states have led to many new findings, some of them are already mentioned in the end of the individual chapters. In what follows, we list the major findings of the present thesis.

6.1 Conclusion

The main observations of the present thesis may be summarized as follows:

- 1. It is observed that photon addition and subtraction are not only non-gaussianity and nonclassicality inducing operations but they can also boost the nonclassicality present in the DFS.
- 2. The results indicate that the amount of nonclassicality in PADFS and PSDFS can be controlled by the Fock state parameter, displacement parameter, the number of photon addition and/or subtraction.
- 3. Higher-order squeezing witness and Q function are observed to be dependent on the phase of the displacement parameter. However, only higher-order squeezing criterion was found to be able to detect nonclassicality, and thus established that this phase parameter can also be used to control the amount of nonclassicality.

- It is observed that the depth of nonclassicality witnesses increases with order of nonclassicality.
- 5. The phase distribution and angular Q functions are found to be symmetric along the value of the phase of the displacement parameter.
- 6. Photon addition/subtraction and Fock parameters are found to induce opposite effects on phase distribution. Between photon addition and subtraction operations, subtracting a photon modifies the phase properties more than photon addition. Interestingly, phase properties are associated with average photon number of the state as well. Photon subtraction increases the average photon number as photon addition does. However, photon addition creates a hole at vacuum unlike photon subtraction.
- 7. The three phase fluctuation parameters given by Carruthers and Nieto reveal phase properties of PADFS and PSDFS, although one of them, *U* parameter indicates antibunching in both PADFS and PSDFS.
- 8. Phase dispersion quantifying phase fluctuation remains unity for Fock state reflecting uniform distribution, which can be observed to decrease with increasing displacement parameter. This may be attributed to the number-phase complimentarity as the higher values of variance with increasing displacement parameter lead to smaller phase fluctuation.
- 9. The present investigation has revealed the advantage of the PADFS and PSDFS in quantum phase estimation and has obtained the set of optimized parameters in the PADFS/PSDFS.
- 10. The nonclassicality and non-Gaussianity of PASDFS viewed with the help of a quasidistribution function, namely Q function is shown in the present thesis.
- 11. The present study also provides a flavor of the significance of the hole burning processes in inducing particular nonclassical features in the family of engineered and parent quantum states. The hole burning operations are observed to be potentially relevant as the quantum states studied in this work are observed to be highly nonclassical when quantification is done through a measure of nonclassicality.

6.2 Scope for future work

The works reported in the present thesis give us a general idea for the investigation of phase properties and nonclassical features present in a family of engineered quantum states. This work can be further extended in various ways. Some of the possible extension of the present work are listed below with a focus on the possibilities that may be realized in the near future.

- The work may be continued to find out the non-Gaussianity of the studied states. Subsequently, the nonclassicality and non-Gaussianity observed in these states can be used to realize various applications in quantum information processing tasks. Therefore, specific applications of the non-classical properties of the aforesaid states which (the applications) are otherwise impossible to achieve using other types of states (classical/nonclassical).
- 2. A major part of the results presented here can be experimentally verified using the available

technology. Along this line, it would be interesting to perform resource comparison (e.g., total number of beam splitters, photo detectors, nonlinear gadgets, etc.) in generation of the aforesaid nonclassical states using quantum state engineering methods.

- 3. The work can be extended to quantify the amount of nonclassicality present in quantum states using different nonclassicality measures.
- 4. The methods adopted here and the results obtained here can be helpful in further theoretical studies on nonclassical and phase properties of other engineered quantum states (both finite as well as infinite dimensional). There could be many such states using several other quantum state engineering tools, for instance, squeezing, photon catalysis, etc.
- Attempts can be made to observe the effect of noise in these states. Specifically, further study of the robustness of observed nonclassical properties of PADFS, PSDFS, PASDFS, BS, VFBS, PABS, KS, VFKS, PAKS under photon loss as well as inefficiency of photo-detectors.

We expect that theoretical work done in this thesis will be performed experimentally and that will lead to some important applications. We also hope this work will be very useful in quantum optics. With these hopes this thesis is concluded.