1
INTRODUCTION

1.1 Context and identification of the problem

Each system in a natural world is born with specific characteristics, which define its behaviour under the surrounding and imposed parameters of different conditions. We live with many machines in our daily lives and have a very close relationship with them. These machines contribute our society for its proper functioning and advancement by producing outputs in forms of services or useful products. A slight error in such machines results in poor performance and it provides the products or services with deteriorating quality. Sometimes, it may be subjected to the whole system breakdown or destruction and it may harm the surrounding environment as well. One of the unwanted behaviors in such system is vibrating nature during its working life. This nature is unwanted phenomena and it is inherent to the system either due to errors at manufacturing or assembly level. As the absolute accuracy in design or manufacturing or assembly level is not possible, the presence of small errors is unavoidable. But, it can be maintained under a tolerable limit so that the output would be of acceptable quality or can avoid disaster or failure of the system from the unwanted vibration.

The rotating machine is of great demand for most the transportation applications starting from electrical power generation plants, different types of turbines, aerospace engines, special purpose or process machines in heavy industry, blowers, pumps and ship engines. Rotating machines holds huge amount of energy in a rotating form which may lead to the source of vibration. As a result, analysis the vibration characteristics and investigating the stability of the rotating system becomes essential. Vibration characteristics of the rotating components in many transportation applications can affect the overall operation of the whole set-up. For a certain range of spinning speeds, the rotating system can undergo violent behaviour which in turn causes serious harm. Therefore, understanding of the vibrational behaviour of these rotating systems is more essential. Requisite understanding and proper knowledge of vibration characteristics in the rotor-bearing system are inevitable in finding the ways to eliminate or reduce the unwanted vibrations in order to ensure safe and smooth running of the system.

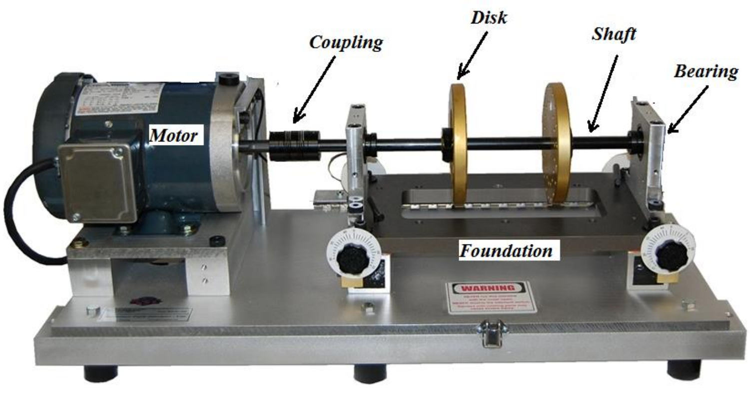
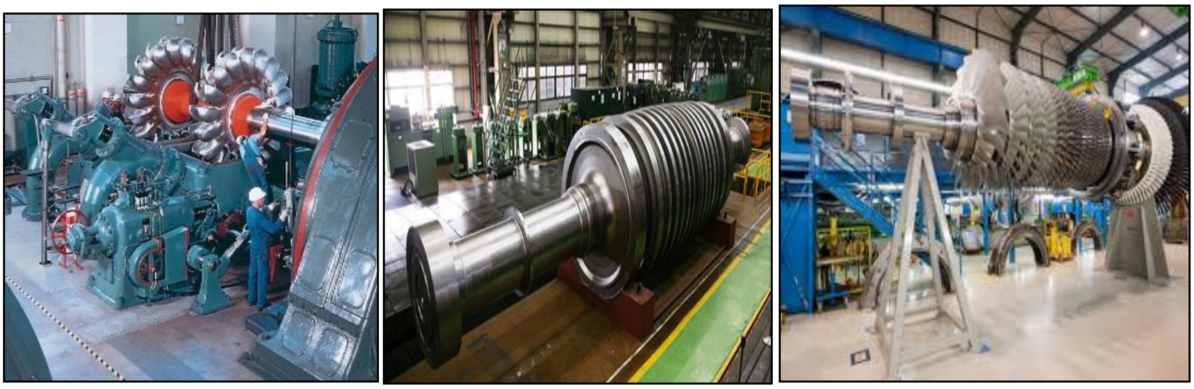


Fig.1.1: Basic model of a rotating system

A normal rotating system is composed of many components. A basic model of rotating machines is shown in following Fig. 1.1 (Ref. www. spectraquest.com). It is composed of a shaft with a disk mounted along its length. The shaft has support of the bearings at both ends. The bearings may be journal bearings/rolling contact bearings, etc. One of the ends of the shaft is coupled with a motor drive using an appropriate coupling. The whole assembly is supported by a specially constructed base. Most of the complicated rotating systems can be represented using the basic model of a rotating system. This reduces complications in understanding its behaviour by sorting out the characteristics at different levels and then summarizing effects for better insight. A rotating system has wide applications in many fields. Some applications are presented below.

1.1.1 Applications

a. *Electrical power generation plant*



(a) (b) (c)

Fig.1.2: Applications in electrical power generation plant

Figure 1.2a (Ref: https://en.wikipedia.org/wiki/Pelton_wheel) shows a Pelton turbine. It is used in a hydraulic power generation plant. The shaft carries two Pelton wheels and supported at both ends. The Pelton wheels are almost equivalent to a disk. This system subjected to high rotational speed. Figure 1.2b is a drive shaft used in a gas power plant. Figure 1.2b (Ref: http://www.doosan.com) is corresponding to a shaft of a rotor used in nuclear power plants and thermal power plants. Figure 1.2c (Ref: www.siemens.com) is corresponding to a rotor of a gas turbine from the Berlin manufacturing plant. Figure 1.2c is corresponding to a gas turbine having a long shaft. The shaft has many blades with different stages to draw power from the gases.

b. *Transportations*

(a) (b) (c) (d)

Fig.1.3: Applications in transportation

Figure 1.3a (Ref :https://www.ipdusa.com) is a picture of a camshaft which used in automobile industries. It is subjected to fluctuating external loading on the cams as well as rubbing. Figure 1.3 b and Fig. 1.3c (Ref: http://www.doosanheavy.com) are for crankshaft and drive shaft of ships. Their sizes are of unimaginable scale. It is subjected to a high load from the driving propeller fan as well as base motion during shipping, etc. Figure 1.3d (Ref: http://en.citizendium.org) is a rocket engine. This mechanism is subjected to a loading due to thrust, compression and expansion of the gasses at high temperature loading.

c. *Process machines in heavy industries*

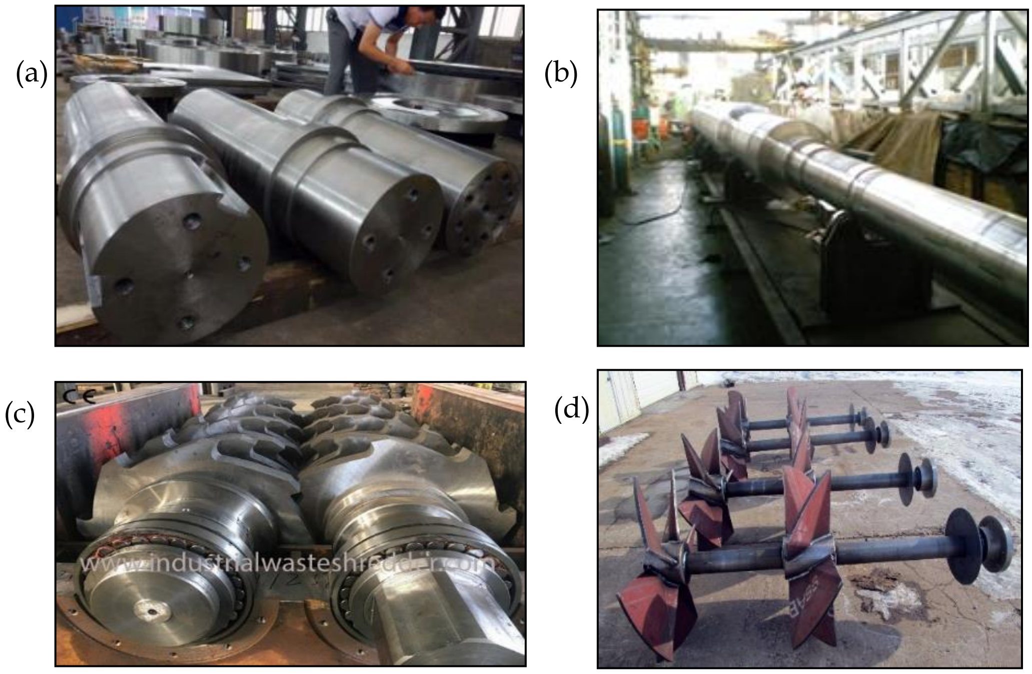


Fig.1.4: Applications in heavy industries

Figure 1.4a (Ref: http://www.citicwl.com) is corresponding to shaft from sugar industries while Fig. 1.5b (Ref: http://www.godrejprecisionengineering.com) is corresponding to the main shaft of the primary circuit sodium pump for a fast breeder reactor. Figure 1.4c (Ref: http://www.industrialwasteshredder.com) is corresponding to a waste crusher.

d. *Other applications*

A rotating system has applications in many machine tools such as lathe machine, drilling machine, boring machine, milling machine, grinding machine as well as in medical equipments e.g. Small electric motors. They are hugely employed in various other industrial applications such as powertrains, steam turbines, internal combustion engines, compressor, propulsion systems, household appliances, and disk drives.

1.1.2 Sources of nonlinearity and uncertainties

At critical conditions, the vibration behaviour of a rotating system is such that it cannot be predicted using the linear theory. The unpredictable behaviour is mainly due to inherent nonlinear properties of the system which are getting induced due to a nonlinear interaction of the dynamic behaviour with the system characteristic. Some types of nonlinearities are mentioned below.

a. Inertial nonlinearity: This effect can be observed when nonlinear terms in an equation of motion contain velocities and/or accelerations. The kinetic energy of the system is the source of the inertial nonlinearity. Examples are centripetal and Coriolis acceleration terms in a rotating coordinate system.

b. Geometric nonlinearities: This effect can be observed when any mechanical component is subjected to large deflections/deformations. Potential energy is a source of this nonlinearity. Nonlinear relation exists between deflection and strain relationships. This is found during mid-plane stretching of the shaft/beam, nonlinear curvature and inextensibility in an axial direction.

c. Nonlinearity due to Damping: A linear model of damping is an ideal form. Damping introduces a nonlinear phenomenon due to nonlinear interaction between two different contacting surfaces. Such as internal friction (i.e., hysteretic damping) due to sliding between neighboring layers of vibrating component. Nonlinear fluid interaction (i.e., viscous damping) in journal bearing leads to oil whirl/oil whip phenomenon, Coulomb friction and aerodynamic drag.

d. Nonlinear properties of boundary conditions: Presence of nonlinear terms in boundary conditions such as nonlinear spring/damper effect at ends of the support. Clearance in bearings. Loose end support or rigid contacts.

e. Material or Physical nonlinearity: The nonlinear relationship between strain-stress causes this type of nonlinearity.

f. Nonlinear fluid-vibrating surface interaction: Nonlinear interaction between fluid and a vibrating surface can induce nonlinear behavior such as oil whirl and oil whip phenomenon in journal bearings, air and vibrating surface interaction in aircraft propellers, fluid flow on rotating blades in turbines and flutter instability.

In most of the high speed applications such as turbo-pumps, jet engine etc, the rotor rotates at a significantly high speed, especially at super-critical speed. At such high speed, a lightweight rotating system can be useful to reduce the critical speed. The lightweight system is prone to lower stiffness and consequently a lower critical speed of the system. At a high speed, the rotating system often undergoes the large deformation under the effect of an external disturbance. Major source of the nonlinearity in the rotating system is due to the large deformation.

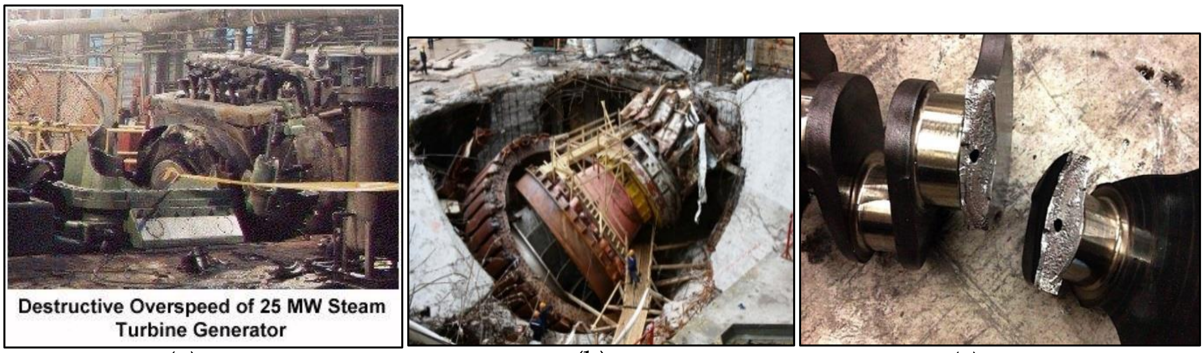
1.2 Significance of nonlinear analysis

Most machines have a rotating system that transforms one form of energy such as linear to rotational energy or translate it from one location to another. These rotating bodies have a huge amount of energy in a rotational form. The presence of an error in such machines results in conversion of the rotating energy into high vibrations and it is dangerous to the surrounding or the machine itself. The dangerous vibrations occur due to an interaction between the vibration characteristics of the system and the presence of the errors in the system or adverse environment. There are many sources of energy that produce vibration in the system. The prior understanding of the vibration behavior of the system under different imposed conditions is unavoidable in designing the rotating system for reliable and efficient working, and consequently for better understanding its performance and output. Further, continuous monitor of the system performance over its working life is also needed.

Errors in such systems are introduced either at the designing level or manufacturing level or assembly level or during a long-running of the system. One of the major errors is an unbalance which produces a harmonic load on the system during the running condition. The unbalance presence in the system may be due to an error in manufacturing/wear due to a long run or improper assembly of rotating parts or extra added mass due to welding or the bolting.

For an undamped system, the rotating speed near the critical speed at which the frequency of the unbalance force matches with one of the system's natural frequencies, causes infinite response amplitude of the system. As a result, the system may be subjected to a failure. Even if a zero-unbalance condition does not achieve, a proper insight and knowledge into dynamic behavior can be helpful to avoid the extreme critical conditions, which are prone to failure of the system. At critical conditions, the vibration behaviour of the rotating system is such that it cannot be predicted using linear theory. The unpredictable behaviour is mainly due to inherent nonlinear properties of the system which induce due to nonlinear interaction of the dynamic behaviour with the system characteristic.

In most of the high-speed applications such as turbo-pumps, jet engine etc, the rotor operates at a significantly high speed, especially at super-critical speed. At such a high speed, use of the lightweight rotating system can be very useful consideration to reduce the critical speed as the lightweight system enables lower critical speed of the system. At a high speed, the rotating system often undergoes large deformation under the effect of external disturbances and is found to be a major source of nonlinearity in the rotating system.



(a) (b) (c)

Fig.1.5: Accidents of rotating machines

Figure 1.5a (Ref: https://www.fostercove.com) is corresponding to the failure of a steam turbine generator due to high speed in Russia while Fig.1.5b (Ref: https://eandt.theiet.org) presents the catastrophic failure of Sayano Shushenkaya hydro plant's hydro-generators in Siberia. Figure 1.5c (Ref: https://www.enginelabs.com) shows the failed crankshaft of an automobile. These are well-designed machines for proper functioning but it is subjected to failure due to some unpredictable behavior and unavoidable conditions. So, it is necessary to extend research in a nonlinear area to predict and avoid such type of failures.

1.3 Application of present work

Literature review has been presented in the next chapter. There are many researchers have done work on vibrational behavior of rotating systems with geometric and inertial nonlinearities due to large deformation of the shaft. A few researches have attempted to analyses on modal characteristics of the rotating system with these nonlinearities. In this thesis, author has attempted to explore more the effect of the nonlinearities on a modal characteristic of the system by performing the parametric investigation extensively. The work has been extended to understand the combined effect of nonlinearities with different loading condition on bifurcation and stability of the system.

In most of the high-speed applications such as turbo-pumps, jet engine etc, the rotor rotates at a significantly high speed, especially at super-critical speed. At such a high speed, a lightweight rotating system can be useful to reduce the critical speed as lightweight system enables lower critical speed of the system. At high speed, the rotating system often undergoes large deformation under the effect of external disturbance. Major source of nonlinearities in the rotor-bearing system is due to the large deformation. This large deformation further leads to possible solid interaction between rotating parts and stator (such as casing). It gives rise to the rub-impact phenomenon in the vibration behavior. This rubbing imposes ambiguous excitations to the system such that the system may pass through route to chaos behavior on a slight change in its parameters. In other words, rubbing induces instability in the system and it may lead to system's poor performance or failure.

Along with these, there are unpredictable external excitation/uncertain means which affect the dynamic behavior and proper functioning of the system. One of the excitations is a base motion. In heavy industries, processing machines are installed with a rigid base support, but the vibration of surrounding machines gets transferred to the base of the machine and affects the system overall performance. Earthquake or loose/improper construction of the base is also another source of a base excitation. In automobiles, the rotating parts are always subjected to base motion during riding over bad roads or bumpers.

Unavoidable axial loading on a rotating system also appears in many rotating machines. The axial loading in the rotating machines are due to the use of helical gear train in the gearbox, load from the blower fan/compressor fan, hydraulic axial load in turbines, loading in drilling, milling, and boring operations, thrust force in jet engines and many special purpose industrial applications. The axial loading causes the parametric excitation of the system. It shows a different resonance condition than that of the conventional resonance such that the system vibrates at half of the excitation frequency.

We have seen in section (1.1.1), that most machines are composed of many disks on the shaft periphery. The examples are blades in gas turbines/steam turbines. The multiple cams on the camshaft in a multi-cylinder engine (Automobile, Ships, and Power Plants). These multidisc systems are complex structures and difficult to analyze their vibrational behavior. The presence of additional disks may alter the dynamics of the whole system. Therefore, it is inevitable to study the vibration analysis before put into working conditions for the proper functioning of the system.

Reducing the vibration in flexible mechanical elements/structures has always been an essential aspect. The light-weight and highly flexible structures are more prone to vibrations integrate with fatigue that often leads to a catastrophic failure. These unwanted sources of vibration have to be either eliminated or attenuated reasonably since they together can lead to poor performance and contribute to a premature damage. Most potential way of controlling and attenuating vibrations in rotating machines is either by using elements with damping or elastic properties and it can be provided using flexible bearings and /or bearing supports. The vibration in such systems can be attenuated using the damping property of viscous material. This is one of the passive vibration control strategies. It can help to reduce the severity of vibration by storing energy and dissipating in the form of heat.

Keeping in mind above situations in a life of different rotating system, following configurations of rotating system have been addressed in this thesis to understand interaction of the nonlinearities on the dynamic response of the system.

- A shaft disk system with nonlinearities due to large deformation and axial stretching.
- A nonlinear shaft disk system with unbalance excitation
- A nonlinear shaft disk system with base motion
- A nonlinear shaft disk system with axial loading
- A nonlinear shaft disk system with unbalance excitation and rub impact phenomenon
- A viscoelastic shaft for a nonlinear multidisc rotating system.

1.4 Objectives

The linear analysis is insufficient to understand the behavior of the nonlinear system. Thus, nonlinear analysis of the rotating system is gaining importance nowadays. It is needed to design the system such that the working range of the system lies out of the instability region and its performance remain under control. It is a challenge to analyze and select the right configuration of the system under the influence of nonlinearities and different loading conditions. Even if the system falls under the instability range, the prior knowledge of the system behavior near the critical speed can be helpful to determine the different means to control it. Therefore, the major objectives of this research work are:

- ✓ Developing a large deflection models for a flexible shaft element with a rigid disk by including the nonlinearities due to the higher order deformation, axial stretching and inextensible condition.
- ✓ Carrying out the modal analysis to compute the modal parameters and its variation due to the influence of the nonlinearity and disk parameters using the vibration analysis tools such as Campbell diagram, time histories, phase portraits, and Fourier spectrums.
- ✓ Formulation of different large deflection models for representing the flexible rotating system in different loading conditions like an axial force, an unbalance mass, an eccentricity of the system, a base motion, the rotor-stator interaction, the multi-disc system and the viscous effect.
- ✓ Evaluating the approximate nonlinear steady state solutions in the various resonance conditions using the perturbation technique as Method of Multiple Scales which are raised due to the loading conditions to detect the instability regions.
- ✓ Validating the results obtained using the Method of multiple scales with the results obtained using the direct numerical integration method.
- ✓ Investigating bifurcation and stability of those obtained steady state solutions using the vibration analysis tools such as time responses, FFT, Phase portrait, Poincare's map, and frequency response characteristics
- ✓ Constructing bifurcation diagrams to show the locus of instabilities of periodic solutions and identify the route chaos or chaotic behaviour.
- ✓ Analyse the effect of different parameters such as rotating speed, eccentricity, unbalance mass, disk size, and disk location on the dynamic responses those can act as controlling means to attenuate/avoid unstable behaviour of the system.
- ✓ Investigating nonlinear behaviour of the system under the influence of base motion and the axial stretching due to a dynamical axial load.
- ✓ Investigating the effect of the axial load on the nonlinear behaviour of the rotating system.
- ✓ Investigating rotor-stator interaction in the large deflection model with an unbalance onto bifurcation and stability analysis
- ✓ Formulation of the rotating system with multi disks and performing nonlinear analysis of the system with base excitation to understand the effect of different parameters.
- ✓ Investigating the vibration characteristics by employing viscoelastic material along with nonlinearities to control the vibration of the system passively.

This research demonstrates both analytical and numerical approaches in identifying the regions of instability in the nonlinear rotating systems. Under different loading conditions, the system shows typical nonlinear phenomenon such as multiple solutions, jump phenomena, quasi-periodicity, period-doubling, torus-breakdown, and chaotic behavior. The parametric analysis can provide the controlling means to attenuate/avoid unstable behaviour of the system. Therefore, it demonstrates an importance of the system parameters in designing of the rotary equipment for reliable and safe operation for a better outcome.

1.5 Assumptions

Mathematical modeling generally provides insight into practical phenomena. While deducing the problem statement, some assumptions have been incorporated either to simplify the complex problem or eliminate some unessential factors which are negligible effect on the practice. However, the following assumptions are considered in the formulation of the rotating model.

- A uniform, continuous and flexible shaft is considered. The shaft and disk are isotropic materials.
- Euler beam theory is used in mathematical model the shaft. Thus, the effect of shear force and rotary inertia is neglected
- The disk is assumed to be rigid. Spring-damper system is used to equivalently represent the flexible bearings
- Forces due to gravity are neglected.

1.6 Outline and methodology

This thesis includes seven chapters. It starts with introduction in **chapter 1**. The introduction of a rotating system and its applications with examples, types of nonlinearity, significance of nonlinear analysis, Objective of this work is included in this chapter.

In chapter 2, the detailed literature review on the contribution of many researchers in developments of rotor-dynamic theory, nonlinearities in the rotating system, perturbation theory, and use of method of multiple scales to find solution of the nonlinear equations has been described. Along with this, research contribution on the free and forced vibration analysis for the nonlinear rotating system has been cited here.

In chapter 3, the flexible shaft disk system is mathematically modeled for free vibration analysis by taking into account the axial stretching and the large deformation of the shaft. In this chapter, forced vibration analysis of the rotating system under base motion and axial loading is discussed. The vibrations analysis tools such as time series, FFT, phase portrait, Poincare's section and frequency response curve are used to analyze the nonlinear behavior of the system.

Chapter 4 describes mathematical modeling of a rotating system with nonlinearities due to geometry and inertia by considering inextensibility assumption and large deformation. The effects of the nonlinearities on the characteristics of rotating system are widely discussed. As well as forced vibration analysis of the rotating system with excitation due to unbalance and rub impact phenomenon is also described.

Chapter 5 is devoted to analysis of multi-disk rotor system under an excitation due to base motion. Along with this, the shaft material having viscoelastic properties is considered and the effect of viscoelastic property on dynamic behavior the system is analyzed.

Chapter 6 highlights conclusion from thesis work in addition to the future work while the references have been included in **Chapter 7**.