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LITERATURE REVIEW

2.1 Historical development with time (Ishida et al., 2012)

Research in rotor dynamics started developing with experimental analysis and practice than theoretical work. Rankine (1869) was the first researcher on this development. His topic was considered as the first research paper in rotor-dynamics. He was the first engineer cum researcher to report the term "whirling" in the field of rotor dynamics. He concluded that the shaft whirled around with considerable bent. Rankine model showed that the deflection of the shaft increases beyond the whirling speed without any restriction, but this is not true in reality. He did not consider the effect of Coriolis acceleration that caused a misleading conclusion. However, in 1883, De Laval built steam impulse turbine and operated near 42000 rpm. He incorporated a rigid rotor at the initial and then used a flexible rotor. He found the self-centering phenomenon of the disk beyond the critical speed. He showed that the speed above critical speed could be achieved by running at speed above seven times the critical speed. In the early development of the rotor dynamic theory, the major challenge before the researcher was building a formula for the critical speed. Dunkerley in 1894 formulated the concept of the lowest critical speed for a cylindrical shaft with several disk systems. He also incorporated the effect of the gyroscopic couple. He first introduced the word 'critical speed' for the resonance condition. Föppl (1895) put forth a theory of vibration behavior of a rotating system at a normal speed. He showed that the heavy side of the rotor flies out at a subcritical speed while fly in at a supercritical speed and confirmed the conclusion of the Laval rotor at high speed. Kerr performed an experiment in 1916 and provided the existence of second critical speed. This proved the misleading of the researcher for almost 50 years due to the conclusion of Rankine theory. In 1919, Jeffcott introduced a rotor model comprises a massless flexible shaft with centrally positioned a rigid disk and considered external damping. This model is also known as Laval rotor. He also verified the Föppl's prediction of stable supercritical speed. His theory recorded as the fundamental theory of rotor dynamics.

Prandtl (1918) was first to analyses a Jeffcott rotor with elastic asymmetry in the shaft. Holzer (1921) developed an approximate method for torsional vibrations to predict its characteristics terms such as mode shapes and natural frequencies. Stodola (1924) included all the developments in rotor-dynamic until the beginning of the twentieth century in his book. He explained almost all fields related to the dynamics of a steam turbine. In 1933, Smith formulated to calculate the threshold spin speed for the supercritical instability. This formulation included the effect of bearing stiffness and ratio of external to internal viscous damping. Newkirk (1924) and Kimball (1924) shown that the supercritical instability in a rotor occurs due to rotor internal damping. Then, Newkirk and Taylor (1925) found instability due to the oil whip phenomenon, which is a reason for the nonlinear behavior of the oil wedge in a journal bearing. Baker (1933) concluded that the contact between the rotating and stationary components imparts self-excited vibrations in the system. Stability analysis of the shaft disk system with the location of the disc at the mid and free end of the shaft was performed for transverse and torsional vibrations by Nikolai in 1937.

Kapitsa (1939) performed an analysis of a flexible shaft and concluded that the friction in the sliding bearing could be a reason for the instability. In 1959, Hori described the fundamental characteristics of oil whip by performing the dynamic analysis of the shaft motion for stability by taking into account pressure forces due to oil films. Thomas described the vibration behavior of a turbine due to the steam whirl in 1958. In 1965, Alford performed the same analysis for the compressors. Natanzon (1952) studied dynamic analysis of the shaft near critical speeds, and the effect of change in spin speed on the shaft vibrations was studied by Grobov (1953, 1955)

After 30 years, Crandall (1961) derived the formula for damping independently. Kollmann (1962) performed research on partially filling interior liquid cavities of rotors and demonstrated the occurrence of the instability. Ehrich (1967) pointed out the asynchronous vibration changes the shape of the resonance curve. The vibrations were developed due to trapped fluid interaction in the engine shaft. Stability analysis of the rotor in a compressor with the effect of a seal was performed by Jenny (1980), and he showed the destabilization of the rotor due seal.

As per the demand of high-speed machinery, the nonlinearity and its sources become an obstacle in design of the systems and predicting their behavior. In 1955, Yamamoto (1957) discussed the sub-harmonic resonance due to the effects of ball bearings. He also performed an investigation of the response of the system to combination resonances. Tondl (1965) investigated journal bearings to understand the nonlinear behavior due to the action of oil films. Ehrich (1966) pointed out the presence of the subharmonic resonances due to the strong nonlinearity in the radial clearance of squeeze-film dampers. This was observed in an aircraft gas turbine. Research on the use of the asymptotic method for nonlinear analysis of rotor behavior was started by Mitropolskii (1965). Lund (1964, 1974), Gunter's work (1966), Ruhl and Booker's (1972) made a great contribution to the field of rotor-bearing instability problems. Someya (1989), and Tiwari et al. (2004, 2005) performed numerical and experimental analyses corresponding to rotor dynamic parameters of bearings and seals. Self-excitation due to non-conservative forces imparts damage to the rotating system. To attenuate the self-vibrations, Tondl (1978, 1998, and 2004) developed a strategy of the anti-resonance phenomenon.

Eraslan et al. (2004) investigated the effects of different shaft speeds on the dynamic behavior of the rotating elastic-plastic solid and hollow shafts. They considered swift-type hardening law, Von Mises' yield criterion, deformation theory of plasticity in the development of the dynamic equation of the system. Dimentberg et al. (2005) analyzed an externally and internally damped Jeffcott rotor by considering Krylov–Bogoliubov averaging method in solving the equation of motion. They formulated the random coefficient of the internal damping using a parabolic approximation scheme. Eraslan et al. (2006) in their other research, studied the effects of the material non-homogeneity on nonlinear behavior of the functionally graded rotating solid shaft and rotating the solid disk. Wang et al. (2006) investigated the effect of bearing number and rotor mass on the vibration of a rigid rotor model, which was supported on the porous gas journal bearing support. Danesh et al. (2014) established the equation of motion for micro rotating disks by using the strain gradient theory and variation method to investigate the vibrational behavior of the system. Montielet.et al. (2014) considered the algebraic approach in online unbalance identification. Shahgholi et al. (2014) et al. developed the nonlinear governing equation of motion for simply supported shaft, considering the effect of the rotary inertia, spin speed, and damping. And they investigated nonlinear vibrational behavior of the system with the use of the multiple scale method. Sinou et al. (2015) incorporated the effect of cubic stiffness with uncertainties in the investigation of the rotors nonlinear behavior using Stochastic-HBM method. Przybylowicz (2016) investigated the effect of steady kinematic excitation on the vibrational behavior of the asymmetric rotor, supported by journal bearings. They performed analyses of the chaotic region behavior under the influence of damping and bearings mounting stiffness variation. Zou et al. (2016) developed coupled governing equation of a marine propulsion shaft to analyze the longitudinal vibration near resonance condition. They adopted the method of multiple scales to solve the nonlinear equations. Lee et al. (2017) employed the multiple shooting methods and pseudo-arclength continuation method to study bifurcation and stability of lateral- torsional vibrations of a rotating system under the effect of strong and weak eccentricity. Entezari et al. (2017) proposed Carrera unified formulation in developing a refined finite element model of the rotating system, and the model was analyzed by considering the different thickness of the disk.

They also concluded that the scheme had reduced computational cost. Chen et al. (2017) analyzed the vibrational response of the flexible rotor bearing system having geometrical and inertial nonlinearity under angular type base excitation by performing an experiment and discussed the response of rotor center orbits and spectra due to variation in rotating speed, base frequency, and its angular amplitude. Gaidai et al. (2018) performed a nonlinear analysis of the Jeffcott rotor under nonlinear restoring forces to study random vibration of the system. They considered a multidimensional dynamic system in the rotor modeling. Eftekhari et al. (2018) considered an unbalanced rotating shaft with an electromagnetic load to investigate the nonlinear behavior at the primary resonance condition. Jehle et al. (2018) performed nonlinear analysis on the gear models to investigate stability, and compared results for rigid and viscoelastic normal contact type.

There have been enormous researches carried out by utilizing various reduced order models of rotor systems with understanding the variation of structural mechanics and controls. Rotor dynamic models always require careful consideration of many dynamic aspects in their theoretical or mathematical models such as flexibility due to elastic deformations in the rotor and bearing elements, un-symmetric stiffness, localized damping, gyroscopic couple effects, axial stretching and nonlinear relations between stress and strain. The present literature furnishes many mathematical progressions in these dynamic aspects from both these areas. However, most of these researches utilized the reduction models describing only nonrotating systems while very few are still applied to rotating systems. Successful addressing of inherent and dominant factors in rotor-dynamics problems delivers fundamental knowledge and working standard of various rotating equipment.

2.2 Nonlinearities in rotor-bearing structures

During a normal running condition of the rotating system, its dynamic behavior can be predicted using linear vibration theory, but unexpected loadings/excitations/manufacturing and assembly errors may cause the system to perform different phenomenal behavior which is highly uncertain using the linear models. These behaviors may include self-excited vibration, saturation, random vibration, modal interactions, subharmonic, super-harmonic, combination resonances, jumps, chaos and instability. The linear vibration theory is applicable mostly when the system subjected to small deformation. As a result, nonlinear phenomena cannot be explained using simple model. Hence, it is important to predict and understand the nonlinear phenomenon in the system under the different working and loading conditions and nonlinearities in developing a nonlinear model of the systems.

Nonlinearities in an analytical model of the system can be included using products of variables and their derivatives, variable boundary conditions, jumps, or discontinuity functions. Nayfeh et al. (2008) have described in detail different types of nonlinearities, as well as they have explained broadly in detail different approximation methods to analyze the effect of the nonlinearities. There is wide research in the identification and behavior of a dynamic system that has been going on for a century. Nonlinearities such as inertial nonlinearity, geometrical nonlinearities, due to damping, boundary conditions and material or physical nonlinearities are encountered in the applications of rotating systems.

2.2.1 Nonlinearities of Rotor/Shaft elements

Linear beam models such as Coulomb, Bernoulli, Euler, Kirchhoff, Rayleigh, and Timoshenko are also helpful today to analyze the system, which is subjected to small deformation. The large deformation of the vibrating system is prone to nonlinear behavior, so it is needed to include models of nonlinearities in the governing equation of motion. In 1951, Burgreen studied the effect of a compressive load in a vibrating beam having hinged ends. He included the effect of mid-plane stretching of the beam by introducing the nonlinear relationship of stress-strain in deriving a governing equation of motion. He saw the variation in vibration frequency with the amplitude of the system. Ray et al. (1969) extended the work of Burgreen by analyzing the mid-plane stretching effect with different boundary conditions i.e., simply-supported, clamped-clamped, and hinge-clamped cases with no movement of ends, experimentally. Atluri (1973) considered a hinged beam with no restriction in the movement of the one end in the axial direction. He included nonlinear terms due to inertia and geometry. He concluded that the system shows the softening effect due to inertial nonlinearities. Hodges et al. (1974) introduced terms of quadratic nonlinearities in equations of motion by considering the effect of large deformation in helicopter rotor blades. Rosen et al. (1979) considered few nonlinear terms with order three in governing equation of motion and his numerical results were in compliance with the results showed by Dowell et al. (1977) experimentally. Crespo da Silva et al. (1986) investigated the dynamics of helicopter rotor blade by considering cubic nonlinearity. The cubic term was included as geometrical nonlinearity. Moyer Jr. et al. (1984) investigated numerically the effect of pulse loading on the transient response of nonlinear beam. Liebowitz (1983) also considered geometrically nonlinear beams and analyzed under influence of impulse and impact loading. Pai and Nayfeh (1990) considered structural nonlinearities such as quadratic and cubic due to curvature and inertia while deriving nonlinear equations of rotating metallic beams. Pai et al. (1990) and Anderson et al. (1996) considered Crespo da Silva (1978) equations to investigate the dynamic behavior of cantilever beams with geometric and inertia nonlinearities. They reported that the system undergoes hardening phenomenon due to the geometrical nonlinearity at the first mode, and softening type behavior of the system found dominant at the second and higher modes due to the inertial non-linearity. Cveticanin (2005) performed the free vibration diagnosis on the Jeffcott rotor to understand the effect of cubic non-linearities with the help of Krylov-Bogolubov method (Atephor, 2008).

Eraslan et al. (2006) studied the effects of the material non-homogeneity on nonlinear behavior of the functionally graded spinning solid shaft-disk system. Hosseini et al. (2008) incorporated the effect of the nonlinearity due to large deformation in the development of flexible rotor's governing equations and investigated its steady-state behavior due to change in the damping coefficient and eccentricity of the system. Atephor (2008) investigated instabilities in rotor-dynamic due to mass unbalance and subsequently stabilized the performance of the flexible rotor-system using smart materials. Author developed a mathematical model considering translational and rotational inertia, bending stiffness and gyroscopic moment to analyze the dynamics of rotor-bearing system.Lu et al. (2008) utilized improved Wilson-θ method for theoretical treatment to analyze the nonlinear behavior of a flexible rotor system. They performed stability and bifurcation analysis using the Floquet theory for the periodic responses of the system. Rizwan et al. (2011) considered higher order deformation in bending to investigate the nonlinear behavior of the rotors. Shahgholi et al. (2012) developed a model for a nonlinear asymmetrical shaft having an unequal mass moment of inertia. They investigated the system behavior at primary and parametric resonance conditions. Shahgholi et al. (2014) et al. developed the nonlinear governing equation of motion for simply supported shaft, considering the effect of the rotary inertia, spin speed, and damping. And they investigated nonlinear vibration behavior of the system with the use of the multiple scale method. Sinou et al. (2015) incorporated the effect of cubic stiffness with uncertainties in the investigation of the rotors nonlinear behavior using Stochastic-HBM method. They performed analyses of the chaotic region behavior under the influence of damping and bearings mounting stiffness variation. Mahmoudi, A. et al. (2016) formulated a governing equation for a nonlinear rotating shaft with nonlinearity due to large deformation and investigated the system interaction with a non-ideal energy source. Chen et al. (2017) analyzed the vibration response of the flexible rotor bearing system having geometrical and inertial nonlinearity under angular type base excitation by performing an experiment and discussed the response of rotor center orbits and spectra due to variation in rotating speed, base frequency, and its angular amplitude. Gaidai et al. (2018) performed a nonlinear analysis of the Jeffcott rotor under nonlinear restoring forces to study random vibration of the system. They considered a multidimensional dynamic system in the rotor modeling. Eftekhari et al. (2018)

considered an unbalanced rotating shaft with an electromagnetic load to investigate the nonlinear behavior at the primary resonance condition. They considered the nonlinear curvature and gyroscopic effect in the formulation of governing equation with the in-extensional assumption in the longitudinal direction. Yang et al. (2019) analyzed electromechanical coupling effect and geometrical nonlinearity effect on the free vibration behavior of a rotating piezoelectric beam.

We, Phadatare et al. (2017) performed investigation of critical speeds and nonlinear free vibration analysis of a highly flexible rotor system. We developed governing equation of motion by taken into account rotary inertia and gyroscopic effect combined with inextensible geometric condition for pinned-guided shaft element. In this analysis, the nonlinear natural frequencies have been found to be higher as compared to the findings obtained via linear analysis. The system response has been observed to be either periodic or quasi-periodic depending on the spin speed.

2.2.2 Nonlinearities in Bearings

It has been found that the nonlinearities in support-bearings can influence substantially the vibration behavior of the rotating system. The nonlinear phenomenon of the bearings can induce chaos and instability in the system behavior. Yamamoto et al. (1976) studied the vibration analysis of a roller-bearing and proposed nonlinear spring characteristics due to dry friction and a clearance between the rolling element and ring. Yamamoto et al. (1981) and Ishida et al. (1990) reported that deep groove ball bearings have nonlinear forces in terms of the third power of the deflections. Nelson et al. (1988), Kim et al. (1990) and Goldman et al. (1994, 1994 and 1995) studied the nonlinear effect of the bearing clearance on the behavior of the rotating system. They showed that the existence of clearances in a bearing leads to many nonlinearities in the system such as discontinuous stiffness effects. Lee et al. (1993) investigated the effect of bearing spring constant on the frequency response characteristics. They showed the effect in inclination and jump phenomenon due to nonlinearity in the bearings. Azeez et al. (1999) observed catastrophic nonlinear instabilities due to small clearance in the bearings with large amplitude and chaotic behavior near the linear critical speed. Shabaneh et al. (2003) considered viscoelastically supported bearings in the rotating system. They reported that the shifting of resonance peak to a higher value of frequency occurs with increase in bearing elastic characteristics.

In the rotor-bearing system with journal bearing, the fluid film-interaction with the rotor is almost nonlinear which influences substantially the dynamic behavior of the system. Holmes (1978) reported aperiodic behavior the rotating system with journal bearings. In 1994, Brown et al. considered the short bearing theory for journal bearing to develop a model of rotating system. They pointed out the chaotic behavior of the system when the gravitational load is smaller than the unbalance force. Yau et al. (2000) analyzed the vibration of a rotor-bearing system under the influence of the nonlinear suspension. They carried out the bifurcation analysis and stability check of the system responses to the various operating conditions using Poincare's maps, bifurcation diagrams, and power spectra. Lahmar et al. (2000) showed that low viscous fluid in high-speed journal bearings results in its turbulent flow while the turbulent flow affects substantially the dynamic behavior of the rotor-bearing system. Siew et al. (2001) considered different conditions in dynamic analysis of an unbalanced rotor having squeeze film damper and retainer springs. Karpenko et al. (2002) considered nonlinear bearing clearance effect in the nonlinear analysis of rotating system with excitation due to out of balance within the system. In another research, Karpenko et al. (2002) analyzed nonlinear response of a rotor system using two approximate methods and nonlinearity was considered due to contact between rotor and snubber ring. Harsha (2005) investigated nonlinear behavior of a unbalanced rotor with nonlinear effect of a roller bearing system. Wang et al. (2006) investigated the effect of bearing number and rotor mass on the vibration of a rigid rotor model, which was supported on the porous gas journal bearing support. Wiercigroch et al. (2006) performed experimental analysis of a nonlinear Jeffcott rotor model with the nonlinearity due to discontinuous stiffness of radial clearance between rotor and snubber ring. They verified results from approximate solution and numerical simulation with the experimental study.

Kakoty et al. (2007) et al. performed stability analysis of a rotor with porous oil journal bearings support. Sébastien et al. (2008) investigated numerically the nonlinear behavior of a rotating system with of gas foil bearings under excitation of an unbalance. Xia et al. (2009) investigated the effect of the nonlinear oil film force by deriving established a dynamic model of a rotor system. They employed the Hamilton principle and the finite element method in the development of system equations. Atepor (2008) presented the effect bearing flexibility on the dynamics responses under mass unbalance his thesis work. Gupta et al. (2008) analyzed unbalanced rotor with ball bearing support by considering nonlinearity due to clearance and Hertzian contact force deformation. Li et al. (2011) developed a nonlinear rotor-bearing-seal model considering Musznyska model. They also considered an unsteady oil film force model for the bearings and influence of the forces on the nonlinear behavior of the system. Karthik et al (2011) considered nonlinearity due to clearance in rolling element bearing in the development of a rotor bearing model. Upadhyay (2011) analyzed nonlinear behavior of an unbalanced rotating system with ball bearing support. Chouchane et al. (2012) analyzed bifurcation and stability of rotor bearing system with fluid film bearing support. Bhore et al (2013) analyzed flexible rotor with gas foil journal bearing support for the different system parameters. Zhou et al. (2013) proposed a dynamic model of rotor bearing system with ball bearing support including floating-ring squeeze film dampers. Chávez et al. (2013) investigated bifurcation analysis of non-smooth model of Jeffcott rotor with bearing clearance. Hou et al. (2014) considered a rub-impact rotor system with cubic stiffness in two general supports. They analyzed the effect of aircraft hovering flight on the rotating system with the illustration of amplitude power spectrum, orbit diagrams, Poincare maps, and phase trajectories.

Dakel et al. (2014) carried out dynamic analysis a rotor with hydrodynamic bearing support and a base motion. Chasalevris et al. (2014) investigated nonlinear behavior of a rotor bearing system with fluid film bearing support. Zhang et al. (2015) considered a rotating unbalance load and a bearing preload. They modeled nonlinear force model by considering ball deformation and dynamic displacement of the rotor. Chatzisavvas et al (2016) investigated nonlinear behavior of rotor bearing system with hydrodynamic thrust bearings. Przybylowicz (2016) investigated the effect of steady kinematic excitation on the vibration behavior of the asymmetric rotor, supported by journal bearings. They performed analyses of the chaotic region behavior under the influence of damping and bearings mounting stiffness variation. Navazi et al (2017) studied stability of a rotor with an unbalance and nonlinear supports. The supports had stiffness with cubic order. Cesar et al. (2017) investigated effect of impact between rotor and magnetic bearing on the nonlinear behavior of the rotating system having unbalance. Taghipou et al. (2018) analyzed dynamics of a Jeffcott rotor with nonlinear bearing forces and studied the system with different types of absorbers to reduce the vibration amplitude. Maraini et al. (2018) analyzed rotor bearing system with nonlinear bearing forces with clearance and Hertzian contact. They observe hardening and softening behavior with the clearance in the bearing and system with preload, respectively.

Investigation of linear dynamics of the rotor-bearing system under various loading/working conditions has been studied thoroughly by a number of researchers since last few decades, but very few researchers have tried to explore the effect of structural nonlinearity of shaft with/without considering other secondary effects such as mass imbalance, rotary inertia and gyroscopic effect for obtaining the critical speed of the rotating system. Thus, extending theirs work to analyze the effect of these nonlinearities on dynamics of a rotor bearing model can provide new insights in the analysis which is not yet explored.

We, Phadatare et al. (2019) considered a flexible rotating system mounted on flexible bearings which modeled as equivalent spring-damper system having linear and nonlinear stiffness elements. We studied initially the modal analysis to determine the modal parameters i.e., natural frequency and mode shapes prior to investigate the dynamics of the system and explored the effect of the flexible bearings consideration. Further, the effect of linear and nonlinear stiffness coefficient of the bearing on the stability of the system is analyzed and found noticeable change in stability performance of the system. This current research showed that flexible bearings stabilize the system as a result of increasing the restoring force.

2.3 Perturbation technique: approximate solutions

Research into utilizing the perturbation methods to find out the approximate solutions from the nonlinear problems started in the 17th-18th centuries. The necessity for a method came into light when the improvement in the Newton theory was demanded to solve an in-traceable problem related to planetary motion in the solar system. Pierre Simon Laplace (1749-1827) used the perturbation methods for developing an equilibrium solution of a large weightless drop on a plane. There are many techniques developed under the perturbation method, and these can be used to solve problems from many mathematical fields. Van Dyke (1964) presented perturbation methods with fluid mechanics in his book. In 1968, Cole (1968) described the perturbation methods with a mathematical approach. Aziz (1984) reviewed the use of the perturbation method with different applications in the heat transfer field. Nayfeh (1973), Bender et al. (1978), Lin et al. (1988), and Hinch (1991) have presented fundamentals with the applications in their books. Nayfeh and Mook (1979) provided different approaches of the perturbation method to solve the nonlinear problems of various fields. The methods such as Lind-Stedt-Poincaré, Incremental Harmonic Balance, Averaging, Method of Multiple Scales,Krylov-Bogolioubov, and Krylov-BogolioubovMitropolski, are widely discussed with examples. The perturbation methods such as Incremental Harmonic Balance, Lindstedt-Poincaré methods are used to obtain a periodic steady state solution directly, whereas Krylov-Bogolioubov-Mitropolski, Averaging, Krylov-Bogolioubov , and the Method of Multiple Scales methods use time scaling of the amplitude and phase of the response. Lau et al. (1982) investigated the nonlinear behavior of a viscously damped beam using an incremental harmonic balance method. Then in 1985, Pierre et al. used a version of incremental harmonic balance method in the nonlinear analysis of a viscously damped plate.

Many researchers applied the method of averaging to a wide range of nonlinear problems. Mitropolsky (1967), Nayfeh (1973), Sethna (1965) and Haxton et al. (1972) applied this method to the equations with a term of quadratic nonlinearities in the analysis of the system's primary resonance behavior. The nonlinear analysis of a moving beam was carried out by Wickert (1992) using KrylovBogoliubov-Mitropolsky asymptotic method. A similar method was also used by Mockensturm et al. (1996) to investigate parametrically excited strings with an axial motion considering tension fluctuations in the strings. Siew et al. (2001) adopted two methods such as the modified iteration method and the modified harmonic balance method to analyse the unbalance response of the flexible rotating system with a squeeze-film damper) and retainer springs. Palacios et al. (2002) investigated the effect of quadratic nonlinearities in the dynamics of elastic foundation consisting of a portal frame with an excitation of a non-ideal energy source using the Bogoliubov averaging method. Yeh et al. (2004) considered the harmonic balance method to investigate the instability of a sandwich beam under the excitation of the axial force. The sandwich beam was composed of an electro-rheological fluid and constrained layers. Further to this Yeh et al. (2005) demonstrated the successful application of the IHB method by performing analyses of the instability region of beams with magneto-rheological material. Di-mentberg et al. (2005) analyzed an externally and internally damped Jeffcott rotor by considering Krylov–Bogoliubov averaging method in solving the equation of motion. They formulated the random coefficient of the internal damping using a parabolic approximation scheme. Sahebkar et al. (2011) adopted a perturbation method to investigate the effect of an axial motion and axial loading on the vibration behavior of a nonlinear drill string system. Perepelkin et al. (2013) used the modified Rauscher method and the harmonic balance method in the analysis of a disk rotor system. They considered the NNM approach to reduce the 8 DOF problem to 2 DOF nonlinear system. Sinou et al. (2015) incorporated the effect of cubic stiffness with uncertainties in the investigation of the rotors nonlinear behavior using Stochastic-HBM method. Zou et al. (2016) developed coupled governing equation of a marine propulsion shaft to analyze the longitudinal vibration near resonance condition. They adopted the method of multiple scales to solve the nonlinear equations. Heydari et al. (2017) adopted homotopy perturbation method to solve nonlinear second order equation of motion of a nonlinear rotor. Khorrami et al. (2017) analyzed dynamic behavior of cracked rotor disk system using a modified harmonic balance method. Yang et al. (2018) utilized harmonic balance (HB) method in solving the mathematical model of defective rotor bearing system. Chao Fu et al. (2018) investigated nonlinear response of rotor system with a crack and the harmonic balance method is used to solve the nonlinear equation of the system. Yang et al. (2019) investigated electromechanical coupling effect geometrical nonlinearity effect on the dynamic behavior of a rotating piezoelectric beam by adopting the method of multiple scales.

2.3.1 Method of multiple scales (MMS)

Researches such as Sturrock (1957, 1963), Cole et al. (1963), Frieman (1963), Sandri (1965, 1967), Nayfeh (1965, 1965, 1968, 1973), Kevorkian (1966a) and Cole (1968) have made substantial contribution towards the development of perturbation method in terms of multiple scales. The idea of using multiple scales was the expansion of responses presented as a function of the multiple independent variables. This method proved superiority over the Lindstedt-Poincaré method, and this method can be used for systems with significant damping. This method has been using mostly to analyze the stability of the problems. Ji et al. (1998) performed an analysis on a model similar to the Jeffcott rotor using the method of multiple scales. Lee et al. (1999) carried out multiple scales expansion up to second order to the problem of the spring pendulum with neglecting the zeroth order term. Authors considered the spring pendulum with a harmonic excitation. The resulted were pointed out to detect the instability boundaries and phenomenon of the route to chaos. They suggested the superiority of the second-order approximation over the first order using quantitative and qualitative analysis with the help of Poincare's maps and Lyapunov exponents. The slightly different version was used with the higher order expansions. Nayfeh (1985) presented the reconstitution method (MMS version I), in which effect of damping and forcing terms were considered at the same level as that of the effect of nonlinearities in primary resonance conditions. Rahman and Burton (1989) developed a newer version of MMS. They pointed out that the older version is incapable of fully describe the steady-state solutions, as well as produces solutions that do not comply with the system (such as a simple Duffing oscillator). Rahman and Burton proposed a series expansion of the excitation and damping terms with independent vanishing requirement of the time-scale derivative. The difficulty with this version II was in-capability of obtaining the unsteady-state solutions. Lee and Lee's (1997) modified MMS version II to evaluate the unsteady and steady-state solutions while El-Bassiouny et al. (2001) evaluated the instability boundaries for the nonlinear behavior of an oscillator by investigating second-order approximate analytic solution using method of multiple scales. They constructed bifurcation diagrams and investigated the presence of bifurcation types such as saddle-node, period-doubling, and rout to chaos. Evaluation of the method of multiple scales and the contribution of many researchers was described by Cartmell et al. in 2003. El-Bassiouny (2005) analyzed stability and steady-state response using the method of multiple scales with first-order the modulation of the amplitude and phase (Atepor, 2008).

Duchemin et al.(2006) evaluated the non-linear behavior of a rotor under base excitation. They used the method of multiple scales to compute the instability boundaries of the system. Pratiher et al. (2008) analyzed the dynamic behavior of a single link viscoelastic Cartesian manipulator under base excitation. They considered the method of multiple scales to treat the nonlinear equation of motion and investigated the effect of mass ratio, loss factor, and amplitude of base excitation. Hosseini (2009, 2011, 2013) used the method of multiple scale to analyze nonlinear characteristic and behavior of a nonlinear rotor considering large deformation. Shahgholi and Khadem (2012) developed a model for an asymmetrical shaft having an unequal mass moment of inertia. They investigated the system behavior at primary and parametric resonance conditions using the method of multiple scales. Ding et al. (2012) studied vibration behavior of the traveling viscoelastic beam under influence of support vibration. They used the method of multiple scales and a finite difference scheme to solve the nonlinear equation of the system in their analysis. Ghazavi et al. (2013) derived bifurcation equation using multiple scale method to analyze the nonlinear behavior of a rotating system with interaction of blade motion and whirling of the rotor. Arvin et al. (2013) employed method of multiple scales to nonlinear PDE of spinning Timoshenko beam to analyses modal characteristics of the system. Chen et al. (2014) applied the method of multiple scales to analyze the nonlinear behavior of a gyroscopic system having 4 degree of freedom system. Zou et al. (2016) developed coupled governing equation of a marine propulsion shaft to analyze the longitudinal vibration near resonance condition. They adopted the method of multiple scales to solve the nonlinear equations. Hou et al. (2016) studied dynamic analysis of aircraft rotor system using method of multiple scale. They considered a nonlinear but duffing type support with sine maneuver load. Eftekhari et al. (2018) considered an unbalanced rotating shaft with an electromagnetic load to investigate the nonlinear behavior at the primary resonance condition. They used the method of multiple scales to analyze the nonlinear equation of the rotating system. Yang et al. (2019) investigated electromechanical coupling effect geometrical nonlinearity effect on the dynamic behavior of a rotating piezoelectric beam by adopting the method of multiple scales. Kim (2019) adopted the method of multiple scales to treat the nonlinear equation of a rotor with asymmetric and nonlinear stiffness to analyze chaotic nature of the system.

It has been seen that the investigation of nonlinear dynamics of the rotor-bearing system require special treatment, many researchers have used different methods to analyze nonlinear behavior of the system. One of the most used method is Method multiple scales. Thus, Phadatare et al. (2017, 2019) considered this method to treat the nonlinear mathematical equation of motion to analyze stability of the nonlinear rotating system behavior under influences of the different forcing conditions.

2.4 Modal analysis

Modal analysis of the system is performed to define the vibration characteristic of system so the response of the system can be predicted under different loading conditions. The presence of nonlinearities affects modal characteristic of the system also. Such as, the geometrical nonlinearity induces hardening effect and the inertial nonlinearity induces softening effect in the system characteristics. There are many researchers who have contributed to defining the characteristic behavior of the rotating systems under the effect of nonlinearities; some of the recent researches are being discussed here. Grybos (1991) et al evaluated the vibration behavior of a rotor by incorporating the effect of shear deformation and rotary inertia to understand its effect on critical speeds. Choi et al. (1992) performed dynamic analysis of a rotating shaft under the influence of a constant compressive load by deriving a governing equation of motion and considering the flexural and torsional vibrations. Jei and Leh (1992) considered the effect of asymmetrical rigid disks and isotropic bearings in the modal analysis of a uniform asymmetrical Rayleigh shaft. Singh and Gupta (1994) proposed a model of composite cylindrical tubes using the beam and shell theory, and analyzed the free damped flexural vibrations of the system. Khulief et al. (1997) analyzed the dynamic response of a rotating system using two-reduction schemes. They considered the finite element approach for formulating the rotating system. Melanson and Zu (1998) developed a governing equation of motion of an internally damped rotating shaft to analyze free vibrations and subsequent stability analysis of the system. Ji and Zu (1998) used the method of multiple scales to analyze the nonlinear rotor bearing system behavior under free and forced conditions. They considered the effect of nonlinear bearings by using linear damper and a nonlinear spring.

Kim et al. (1999) presented the characteristic analysis of a tapered composite Timoshenko shaft. Shabaneh and Zu (2000) performed a dynamic analysis of a shaft disk system having elastic bearings to investigate the free and forced vibration behavior of the system. Ping et al. (2002) adopted the model synthesis and finite element approach to analyze the dynamic behavior of the rotor system. The nonlinear analysis of a rotating shaft was performed by Luczko et al. (2002). They incorporated gyroscopic effect, shear effects, Von-Karman non-linearity, nonlinear curvature effects, and large order displacements in the development of the mathematical model. Hwan et al. (2005) developed a new method to analyze the modal characteristic of a rotor. They used the transformation of periodically time-varying linear differential equations to the equivalent time-invariant linear differential equations. Villa et al. (2005) formulated a nonlinear rotor using the invariant manifold approach to evaluate the dynamic performance of the system. Won et al. (2007) adopted the Floquet theory and coordinate transformation for development of a complex modal analysis scheme for the rotating system. Qia et al. (2008) performed a modal analysis of a linear rotor system. They also used harmonic wavelet filtering, the Hilbert transform method, combined with the correction technique of spectrum analysis, and random decrement technique. Hosseini et al. (2009) et al. investigated the characteristic behavior of a rotating beam with random properties by considering the stochastic finite element method. Skjoldan et al. (2009) performed a modal analysis of a structure with bladed rotors using Floquet analysis. Hosseini and Khadem (2009) evaluated the free vibration behavior of a nonlinear rotating shaft by considering nonlinearities in the curvature and inertia. Chouksey et al. (2012) present the effect of shaft material damping and fluid forces on a rotor shaft system by conducting modal analysis. They studied stability of the speed, the direction of the whirl of the shaft in different modes, modal damping factors and the frequency response functions. Andrzej et al. (2016) performed a modal analysis of a rotating system experimentally. They determined modal parameters such as mode shapes and natural frequency using the modal hammer and laser vibrometer. Tamrakar et al. (2016) analyzed rotor behavior under the influence of rotary inertia and shear effects. They compared the results of the Euler-Bernoulli beam with the Timoshenko beam. Ankur et al. (2017) analyzed a geared rotor system to understand the effect of mesh damping and stiffness on the modal characteristics. Hassan et al. (2017) proposed a new approach for modal characteristic analysis of fill structure in rotor-dynamics problems. Ziyuan et al. (2017) investigated the vibration behavior of turbo-molecular pump rotor blades. They investigated the modal characteristics of the system theoretically and compared it with the experimental results. Roy et al. (2017) studied modal analysis of a viscoelastic rotor. In their study, free and forced vibration treatments are being performed by obtaining modes and FRF for the model. Mariusz et al. (2017) considered the Jeffcott model for a rotor to analyze the effect of crack presence on the modal characteristic of the system. Jie et al. (2018) investigated the effect of rub impact condition of the system on the modal characteristics by theoretical and experimental methods. Wedera et al. (2019) performed an experiment to investigate the vibrational analysis of a rotor-stator system. Castillo et al. (2019) investigated numerically and experimentally the modal characteristics of a submersible pump. They identified modal properties using the eigensystem realization algorithm method. Róbert et al. (2019) used a high-speed digital image correlation technique in the vibration analysis of a rotor system. They measured natural frequency and mode shapes using this method. Philip (2019) proposed an approach to derive a Campbell diagram of a rotating system based on the Jacobian matrix of the system.

We, Phadatare et al. (2017) determined critical speeds and investigated the nonlinear free vibration analysis of a highly flexible rotor system. This work showed the nonlinear natural frequencies have been found to be higher as compared to the findings obtained via linear analysis. In this research the authors also analyzed the effect of system parameters on the system characteristic using vibration analysis tool such as time series, FFT plot and Campbell diagram.

2.5 Forced vibration analysis

2.5.1 Unbalance/Eccentricity

As discussed in the previous chapter, an unbalance has a substantial effect on the dynamic performance of the rotating system while its presence is inevitable in the rotating system. Neilson et al. (1988) carried out analysis of a rigid rotor with nonlinear support under influence of the arbitrary imbalance excitation and the results are compared theoretically and experimentally. Kulkarni et al. (1993) considered the finite element approach to analyze the dynamic behavior of a rotor-bearing system with viscoelastic suspension. The analysis is performed to investigate the effect of a viscoelastic suspension on the unbalance response of the system. Neilson et al. (1994) analyzed two-disc rotating system with distributed imbalance. They considered coupling between the torsional, lateral and axial motions in the modeling of the system. Barr et al. (1995) obtained responses of a discontinuous nonlinear rotor system under excitation due to out of balance by numerically and experimentally. Here the nonlinearity was considered due to discontinuous stiffness in clearance between rotor and outer ring.Shortle et al. (1996) investigated an imbalance due to influence of the manufacturing processes and proposedprobabilistic model for the imbalance. Siew et al. (2001) considered different conditions in dynamic analysis of an unbalanced rotor having squeeze film damper and retainer springs. Karpenko et al. (2002) considered nonlinear bearing clearance effect in the nonlinear analysis of rotating system with excitation due to out of balance within the system.Wiercigroch et al. (2003) analyzed the influence of viscous damping and preloading on the dynamic response of Jeffcott rotor system with excitation due to out of balance. Eraslan et al. (2004) investigated the effects of different shaft speeds on the dynamic behavior of the rotating elastic-plastic solid and hollow shafts. They considered swift-type hardening law, von Mises' yield criterion, deformation theory of plasticity in the development of the dynamic equation of the system. Pavlovskaia et al. (2004) investigated impact interaction between Jeffcott rotor and snubber ring under out of balance excitation. Harsha (2005) investigated nonlinear behavior of a unbalanced rotor with roller bearing system. Hosseini et al. (2008) incorporated the effect of the nonlinearity due to large deformation in the development of flexible rotor's governing equations and investigated its steady-state behavior due to change in the damping coefficient and eccentricity of the system. Gao et al. (2008) investigated bifurcations analysis of a machine-tool spindle-bearing system with the effect of unbalance and bearing clearance numerically. Sébastien et al. (2008) investigated numerically the nonlinear behavior of a rotating system with of gas foil bearings under excitation of an unbalance. Gupta et al. (2008) analyzed unbalanced rotor with ball bearing support by considering nonlinearity due to clearance and Hertzian contact force deformation. Patel et al. (2009) attempted modeling and vibration analysis of an unbalanced rotor with fatigue crack and rub. Upadhyay (2011) analyzed nonlinear behavior of an unbalanced rotating system with ball bearing support. Shahgholi and Khadem (2012) developed a model for an asymmetrical shaft having an unequal mass moment of inertia and analyzed the system under the influence of an unbalance force and two parametric excitations to the system.

Montielet et al. (2014) considered the algebraic approach in online unbalance identification. Shahgholi et al. (2014) studied a nonlinear rotating system with a mass eccentricity. They investigated effect of nonlinear energy sinks on the vibration attenuation of the system. Liu et al. (2015) proposed a dynamic model of a rotating system with rub-impact considering disk–drum–shaft type rotating system. The system is analyzed numerically for the parameters as stator radial stiffness, rotating speed and disk mass eccentricity. Zhang et al. (2015) considered a rotating unbalance load and a bearing preload. They modeled nonlinear force model by considering ball deformation and dynamic displacement of the rotor. Hu et al. (2016) analyzed an asymmetric double-disk rotor bearing system to understand its nonlinear behavior due to the effect of rub-impact and oil-film instability by producing results numerically and experimentally. The study was focused on the nonlinear responses of the rotating system due to the effect of eccentricity and the stator stiffness. Vervisch et al. (2016) investigated the effect of rotating damping experimentally to predict the stability threshold of a rotating machine. Harmonic base excitation and dynamic imbalances were considered as sources of excitation in this system. Yang et al. (2016) developed a model of geometrically nonlinear rotor system with an unbalance-rub fault by taking into account the surface coat on the disc and casing.

Lee et al. (2017) employed the multiple shooting methods and pseudo-arclength continuation method to study bifurcation and stability of lateral- torsional vibrations of a rotating system under the effect of strong and weak eccentricity. Entezari et al. (2017) proposed Carrera unified formulation in developing a refined finite element model of the rotating system, and the model was analyzed by considering the different thickness of the disk. They also concluded that the scheme had reduced computational cost. Navazi et al (2017) studied stability of a rotor with an unbalance and nonlinear supports. The supports had stiffness with cubic order. Cesar et al. (2017) investigated effect of impact between rotor and magnetic bearing on the nonlinear behavior of the rotating system having unbalance. Maraini et al. (2018) analyzed nonlinear rotor bearing system with rotating unbalance. Behzad et al. (2018) investigated effect of unbalanced induced rubbing on dynamic behavior of the rotating system. Eftekhari et al. (2018) considered an unbalanced rotating shaft with an electromagnetic load to investigate the nonlinear behavior at the primary resonance condition. They considered the nonlinear curvature and gyroscopic effect in the formulation of the governing equation with the in-extensional assumption in the longitudinal direction. Sanches et al. (2018) performed theoretical analysis on two disks- rotor system with an unbalance and shaft bow using the finite element approach, and they also carried out an experimental analysis to identify the faults using the correlation analysis. Feng et al. (2019) analyzed vibration behavior of electric spindle by considering a Jeffcott rotor model with an eccentricity.

We, Phadatare et al. (2019) performed nonlinear dynamic analysis of a light-weight flexible rotor-disk-bearing system with geometric eccentricity and mass unbalance. The large deflection model has been used to study the bifurcation, stability and route to chaos. The system stability has been studied by investigating the bifurcation and route to chaos upon changing the design parameters such as geometric eccentricity, mass unbalance and disk parameters under the resonance conditions. Authors have given special attention to predict its rich dynamics to highlight the route to chaos.

2.5.2 Base excitation

There are unpredictable external excitation/uncertain means which affect the dynamic behavior and proper functioning of a rotating system. One of the excitations is a base motion. In heavy industries, processing machines are installed with a rigid base support, but the vibration of surrounding machines gets transferred to the base of the machine and affects the system overall performance. Earthquake or loose/improper construction of the base is also another source of a base excitation. In automobiles, the rotating parts are always subjected to base motion during riding over bad roads or bumpers

There are many researchers have contributed in the analysis of the effect of base excitation on rotating systems. Such as, Lin et al. (2003) performed dynamic analysis of an unbalanced rotor system in a maneuvering aircraft. In another research, Lin et al (2004) analyzed cracked rotor system in a maneuvering aircraft. Driot et al. (2006) analyzed the effect of support excitation on the vibration behavior of the rotating system by performing a theoretical and experimental study. Duchemin et al. (2006) investigated the effect of the support movement on the vibration stability of the rotor. They validated the theoretical results by performing an experimental analysis. Duchemin et al. (2006) also employed a theoretical and experimental approach for understanding the effect of base excitation on the dynamic behavior of a rotating system. Marx et al. (2007) developed a mathematical model of a rotating system with magnetic bearing support. They analyzed nonlinear behavior of the system under excitation of the base. Pratiher et al. (2008) analyzed the dynamic behavior of a single link viscoelastic Cartesian manipulator under base excitation. They considered the method of multiple scales to treat the nonlinear equation of motion and investigated the effect of mass ratio, loss factor, and amplitude of base excitation. Jing et al. (2009) analyzed effect of base excitation on the nonlinear behavior of a cracked rotating system.

Driot et al. (2009) analyzed the effect of support excitation on the uncertain parameters of rotating machines by considering the constant speed of rotation using stochastic methods. Das et al. (2010) considered a flexible rotor shaft system and analyzed the effect of base motion on the vibration behavior of the rotating system with electromagnetic control force. El-Saeidy et al. (2010) investigated effect of mass unbalance and base excitation on the dynamic behavior of a rigid rotor with nonlinear spring support. Saha et al. (2011) analyzed effect of platform precession and an unbalance on a rotating system. Dakel et al. (2012) considered an on-board rotating system with elastic bearings support to analyze the vibration behavior under influence of rigid support movements. Ding et al. (2012) studied vibration behavior of the traveling viscoelastic beam under influence of support vibration. They used the method of multiple scales and a finite difference scheme to treat the nonlinear equation of the system in their analysis. Han (2014) studied effect of time variable base motion on geared rotating system. They considered yawing, rolling, and pitching motions of the base. Dakel et al. (2014) carried out dynamic analysis a rotor with hydrodynamic bearing support and a base motion. Han et al. (2015) investigated the effect of a base angular motion on the nonlinear behavior of a rotating system.

Hou et al. (2016) studied dynamic analysis of aircraft rotor system using method of multiple scales. The considered a nonlinear but Duffing type support with sine maneuver load. Vervisch et al. (2016) investigated the effect of rotating damping experimentally to predict the stability threshold of a rotating machine. Harmonic base excitation and dynamic imbalances were considered as sources of excitation in this system. Przybylowicz (2016) investigated the effect of steady kinematic base excitation on the vibration behavior of the asymmetric rotor, supported by journal bearings. They performed the analyses of the chaotic region behavior under the influence of damping and bearings mounting stiffness variation.

Chen et al. (2017) analyzed a rotating system under a base excitation. They found nonlinear effect in the model due to the base motion and further investigation carried out for base angular motion. Shahgholi et al. (2019) investigated effect of base motion on vibration behavior of symmetrical and asymmetrical rotating shafts with nonlinearities due to a large deformation. Jarroux et al. (2019) analyzed effect of the support motion on the nonlinear behavior of a rotating system with active magnetic bearing support. They analyzed combined effects of an imbalance, a support motion and contact nonlinearities

As of now to the best of the authors' current knowledge, no research has been yet attempted to study the nonlinear motion characteristics of a highly flexible rotor-bearing system under base motion. Hence, here authors have made an effort exhaustively to understand a deep perception of vibration phenomena of rotor dynamics under moving support and to find a way of eliminating the possible vibration amplitude by alleviating the causes of vibrations. We, Phadatare et al. (2017) investigated the bifurcations and chaotic behavior of an elastically induced flexible rotor-bearing system subjected to a harmonic ground motion. Nonlinear vibration analysis has been carried out to determine the critical speeds, i.e., Campbell diagram followed by demonstrating the inherent nonlinear signatures through the illustration of time history, Fourier spectrum and Poincare's map upon varying the system design variables. Authors showed not only the spin speed of the shaft but also other parameters like the magnitude of ground motion and mass imbalance strongly exhibit the route to chaos behaviors. Under steady-state conditions, the catastrophic failure of the system due to sudden jump can be controlled successfully by altering the design parameters.

2.5.3 Axial Loading

Unavoidable axial loading on a rotating system also appears in many rotating machines. The axial loading in the rotating machines are due to the use of helical gear train in the gearbox, load from the blower fan/compressor fan, hydraulic axial load in turbines, loading in drilling, milling, and boring operations, thrust force in jet engines and many special purpose industrial applications.

Numerous researchers have contributed in the analysis of an axial loading effect on the rotating system. Such as, Shih et al. (1988) analyzed the effects of an axial force and a magnetic field on the vibration behavior of the rotating system. They utilized time history and phase portrait for dynamic analysis of the system. Liu et al. (2005) investigated the influence of the transverse axial loads, magnetic fields, and external force on the dynamic behavior of a magneto-elastic beam. Young et al. (2007) performed the vibration analysis of a rotating system to understand the effect of random axial forces at both ends on the vibrations of the system. Ratko et al. (2008) considered a Voigt–Kelvin model for viscoelastic material of a rotating shaft. They performed a stability analysis of the system under the influence of an axial load, and the force is considered as a time-dependent stochastic function. Driot et al. (2009) employed the stochastic approach to investigate the effect of support motion on uncertain parameters of the rotor. Pei et al. (2010) assumed the Timoshenko theory with the finite element method in the mathematical formulation of a rotating system having an unsymmetrical rigid disk and analyzed the effect of a periodic axial load on stability of the system by employing the harmonic balance method. Sahebkar (2011) adopted a perturbation method to investigate the effect of an axial motion and axial loading on the vibration behavior of a nonlinear drill-string system

Bai et al. (2012) formulated the stochastically model for the axial load and investigated its effect on a rotor systems model. Bartylla (2012) used a harmonic axial force as an excitation to the non-symmetric rotor and analyzed the vibration behavior of the system at its critical speed. Sofiyev et al. (2015, 2017) analyzed the effects of time dependent periodic axial compressive loads on simply supported cylindrical and conical shells. Chatzisavvas et al. (2016) investigated influence of axial loading on nonlinear behavior of a rotor bearing system with hydrodynamic thrust bearings. Ebrahimi et al. (2016) considered active magnetic bearings support in the formulation of non-linear equation of motion for a coaxial rotor. The also considered the effect of contact with auxiliary bearings. Zou et al. (2016) developed coupled governing equation of a marine propulsion shaft to analyze the longitudinal vibration near resonance condition under longitudinal excitation. Shaban et al. (2017) used Galerkin’s principle, and multiple scales approach to investigate the dynamic behavior of a composite spinning shaft that has geometrical nonlinearities and an unbalance mass. Sadath et al. (2017) analyzed the effect of an axial load excitation on stability characteristics of a flexible tube while Qadri. Et al. (2018) performed bifurcation and stability analysis on a rotating shaft by considering the effect of parametric and external excitations. Yang et al. (2018) discussed the effect of coating hardness and support stiffness of the casing on a geometric nonlinear rotor casing system with coupling fault of the axial load and the radial rub. Dai et al. (2018) investigated effect of a periodic axial load on the periodic stability of a spinning cylindrical shell.

From the recent literature, studies to investigate the stability and bifurcation of a nonlinear rotor-disk supported by flexible bearing under a combined effect of mass unbalance and pulsating axial force are highly inadequate. Study of the combined effect of axial load and mass imbalance onto system performance stability renders a useful progress in rotor dynamics fault diagnosis and prognosis. Hence, deep understanding of the bifurcation and its possible consequences due to the combined effect of axial load and mass unbalance (i.e., a common machinery fault in rotating machinery) onto the system stability uncovers the path of reducing the chances of failure and guides a safe and smooth functioning of the system. We thus, attempt to investigate the synergetic effect of an axial load and mass unbalance on vibration behavior of the nonlinear spinning rotor-disk-bearing system. The mathematical model which comprises flexible shaft, flexible bearing and a rigid disk is derived using Hamilton principle and then solved using method of multiple scales to obtain steady state nonlinear solutions. We further analyze the stability of the solutions and critical operating condition. Steady state responses have been validated with the exact solutions by solving the governing equation and found to be in good agreement.

2.5.4 Rotor-stator interaction: Rubbing-impact phenomenon

At high speed, the rotating system often undergoes large deformation under the effect of external disturbance. This large deformation further leads to possible solid interaction between rotating parts and stator (such as casing). It gives rise to the rub-impact phenomenon in the vibration behavior. This rubbing imposes ambiguous excitations to the system such that the system may pass through route to chaos behavior on a slight change in its parameters. In other words, rubbing induces instability in the system and it may lead to system's poor performance or failure.

There many researches have contributed in the analysis of the rub-impact phenomenon in vibration behavior of rotating systems. Such as Barr et al. (1995) analyzed response of a discontinuous nonlinear rotor system under excitation due to out of balance by numerically and experimentally. Here the nonlinearity was considered due to discontinuous stiffness in clearance between rotor and outer ring. Chu et al. (1998) analyzed the dynamic characteristic of the Jeffcott rotor under the rub-impact condition. They performed bifurcation and stability analysis of the system using the Floquet theory and Fourier series. Karpenko et al. (2002) analyzed nonlinear response of a rotor system using two approximate methods and nonlinearity was considered due to contact between rotor and snubber ring. Pavlovskaia et al. (2004) investigated impact interaction between Jeffcott rotor and snubber ring under out of balance excitation. Zhang et al. (2006) developed a Jeffcott micro-rotor system model with the rub-impact phenomenon, and they investigated the stability of the rotor system under different system parameters such as the rotating speed, unbalance, damping, and friction coefficient and impact stiffness. Wiercigroch et al. (2006) performed experimental analysis of a nonlinear Jeffcott rotor model with the nonlinearity due to discontinuous stiffness of radial clearance between rotor and snubber ring. They verified results from approximate solution and numerical simulation with the experimental study. Gao et al. (2008) investigated bifurcations analysis of a machine-tool spindle-bearing system with the effect of unbalance and bearing clearance numerically. Patel et al. (2009) attempted modeling and vibration analysis of an unbalanced rotor with fatigue crack and rub. They investigated the responses of a rotor with the interaction between rub and crack. Khanlo et al. (2011) studied the effect of Coriolis and centrifugal forcing effects on the response of the rub-impact rotor model illustrated with time series, phase plane portrait, power spectra, Poincaré map, and bifurcation diagrams. Hou et al. (2014) considered a rub-impact rotor system with cubic stiffness in two general supports. They analyzed the effect of aircraft hovering flight on the rotating system with the illustration of amplitude power spectrum, orbit diagrams, Poincare maps, and phase trajectories. Peletan et al. (2015) analyzed the dynamic behavior of the rotor-stator contact. For this analysis, they developed a quasi-periodic harmonic balance method (HBM) coupled along with an algorithm of the pseudo-arc-length continuation. Liu et al. (2015) proposed a dynamic model of a rotating system with rub-impact considering disk–drum–shaft type rotating system. They used an elastic impact model and Coulomb's friction law for the disk–stator contact and the drum– stator contact. The system is analyzed numerically for the parameters like stator radial stiffness, rotating speed and disk mass eccentricity. Ma et al. (2016) explained a review on the vibration characteristics of rotor, blade, and casing as well as the mechanism of coating wear and blade–casing interaction/rubbing with/without coating. As well as, a review of the experimental results corresponding to the blade–casing rubbing for the bare and coating casings respectively included. Yang et al. (2016) investigated the effect of random excitation on a model of rub impact rotor system. They used Taylor series expansion and Chebyshev polynomial approximation method to develop the stochastically model of the rotating system model. Hu et al. (2016) analyzed an asymmetric double-disk rotor bearing system to understand its nonlinear behavior due to the effect of rub-impact and oil-film instability by producing results numerically and experimentally. The study was focused on the nonlinear responses of the rotating system due to the effect of eccentricity and the stator stiffness.

Yang et al. (2016) developed a model of geometrically nonlinear rotor system with an unbalance-rub fault by taking into account the surface coat on the disc and casing. However, they considered a contact force model as the impact force and the Coulomb model for the frictional force. The effects of different parameters on the equivalent model have been investigated using whirl orbit, bifurcation diagram, and Poincaré map. Xiang et al. (2016) formulated a mathematical expression of an asymmetric two-disc rotor-bearing system with the effect of nonlinear oil-film force and rub-impact force by assuming rub-impact as a Hertz contact and a Coulomb friction model. In another study, Xiang et al. (2016) performed a study of parametric instability of a rotor-bearing system to understand the vibration behavior of the system considering time-varying crack stiffness, a rub impact force and a nonlinear oil-film force. Ma et al. (2016) proposed a finite element model of the rotor blade system with single and four blade rubbings by accommodating the influence of the rigid disk swing and stagger angles of the blades. Chen et al. (2017) developed a new rub-impact contact model with the cylindrical shell using the Sanders shell theory while Xiang et al. (2018) established a nonlinear dynamic model of a multi-fault rotor system by considering the interaction of time-varying stiffness and nonlinear forces due to rub-impact and oil film effect. They analyzed the rotor system for crack detection using orbit morphological characteristics by performing experimental and theoretical analysis. Yang et al. (2018) discussed the effect of coating hardness and support stiffness of the casing on a geometric nonlinear rotor casing system with coupling fault of the axial load and the radial rub. Hou (2018) investigated the complex bifurcation behavior of an aircraft rub-impact rotor system due to flight maneuvers by taking into account duffing type nonlinearity. They derived approximate periodic solutions of the system by using the harmonic balance method combined with an alternating frequency/time domain procedure under the constant excitation. Ebrahim et al. (2018) analyzed the vibration analysis of a rotating system with the tilting pad journal bearing support and by taking into account of rub–impact. Hong et al. (2019) established a modified Jeffcott rotor model by considering the rub impact effect and analyzed the stability of the system with the additional effects of the model characteristics.

Based on the existing research works on mechanical rub phenomena, identification of mechanical rub in a large deflection continuous model of shaft-disk system under mechanical unbalance has not been yet explored. Most of the researchers used either conventional linear models or lump parameter model as spring–mass-damper system. Outcomes obtained from these models offer highly erroneous and inadequate statistics to correctly predict and detect the state of structural stability, especially when the system is being operated at high speed. Thus, in this thesis, the author has made one attempt made to highlight the state of rub impact occurred between casing and rotor-disk under mechanical unbalance. The dynamic state of present system is being evolved with extreme conditions of non-conservative forces due to mechanical rub-impact and mass unbalance. The identification and detection of rubbing is being investigated by bifurcation diagrams i.e., enabling route to chaos scenario, obtained numerically with the variation of the design parameters as control parameters. The present system has been shown an interaction of stator and disk of structurally nonlinear rotating system due to large deformation in bending.

2.5.5 A rotating system with viscoelastic effect

Reducing the vibration in flexible mechanical elements/structures has always been an essential aspect. Most potential way of controlling and attenuating vibrations in rotating machines is either by using elements with damping or elastic properties and it can be provided using flexible bearings and /or bearing supports. The vibration in the systems can be attenuated using viscous material instead of elastic material due to its high viscous effect.

There are many researchers have been working on the controlling the vibration amplitude of a system using the damping property of a viscous material. Such as Kulkarni et al. (1993) considered the finite element approach to analyze the dynamic behavior of a rotor-bearing system with viscoelastic suspension. The analysis is performed to investigate the effect of a viscoelastic suspension on the unbalance response of the system. Kurnik (1994) analyzed stability and bifurcation analysis of a self-excited rotating shaft under the influence of transverse loading. The shaft was modeled by taking into account elastic and viscoelastic nonlinearities. Sturla et al. (1996) performed free and forced analysis of a viscoelastic Rayleigh shaft by using a three-parameter solid model for the viscoelastic effect. Shabaneh et al. (2000) considered a Kelvin–Voigt model for viscoelastic suspension of a rotor-bearing system and investigated the effect of the viscoelastic properties on vibrational characteristics of the system. Abolghasemi et al. (2003) derived a system of equations of motion for a viscoelastic spinning beam having a variable pitch angle. They used the perturbation techniques, Poincare′ map to analyze the nonlinear behavior of the system Ratko et al. (2008) considered a Voigt–Kelvin model for viscoelastic material of a rotating shaft. They performed a stability analysis of the system under the influence of an axial load, and the force is considered as a time-dependent stochastic function. Friswell et al. (2010) considered frequency dependent viscoelastic material model to analyze the vibration behavior of the system using the Augmenting Thermodynamic Fields approach. Ding et al. (2012) studied vibration behavior of the traveling viscoelastic beam under influence of support vibration. They used the method of multiple scales and a finite difference scheme to treat the nonlinear equation of the system in their analysis. Deng et al. (2016) derived a governing equation in the form of a stochastic differential equation of the viscoelastic rotating system and performed stability analysis using moment Lyapunov exponents. Roy et al. (2016) formulated a governing equation of motion for a multi-disk composite rotating system. They used a lumped mass and the finite element approach to analyze the system. They used viscoelastic material for the rotor in this analysis. Roy et al. (2017) developed a higher order finite element model for a viscoelastic rotor to investigate the effects of the various asymmetries, arises due to the presence of internal damping, gyroscopic couple and fluid film forces of journal bearings. Ganguly et al. (2016) presented a finite element approach in the investigation of vibration behavior of the viscoelastic rotors. They considered the Maxwell-Wiechert model for the viscoelastic material. Jehle et al. (2018) performed nonlinear analysis on the gear models to investigate stability, and compared results for rigid and viscoelastic normal contact type.

To the best of the author’s knowledge, it has been observed that a limited number of researchers studied the dynamics of rotating system under base motion while few authors have used viscoelastic material instead of elastic material to model flexible rotating shaft. However, incorporating multi-disks into the dynamics has not been explored thoroughly in the recent past. Notwithstanding, the analysis of vibration aspects and nonlinear behavior of a viscoelastic rotating system with multiple disks mounted on a moving support by considering large deflection model theory is found to be neglected in the literature. Further, implementing the concept of multi-disk rotor system under moving platform has huge necessity in the modern industrial applications. Thus, in this thesis, an attempt has been made to investigate the evolutionary dynamics of a high-speed rotating system given the emphasis on bifurcations, multi-stability and route to chaos phenomena. The system offers an extensive understanding of operating dynamics and critical functioning ambiences of a rotating system mounted on a movable platform. The dynamic model encounters large elastic deformation especially experienced at high speed during its running condition like jet and aircraft engines. This study highlights the investigation of vibration behavior of a viscoelastic rotor with multiple disks which is mounted to a moving support. The nonlinearity in governing equation of motion of the system has been inducted due to the consideration of large elastic deflection for the shaft element. Assessment of bifurcation and it’s out-turn on the system stability has been investigated by using identification and prognosis tools as frequency response characteristics and bifurcation diagrams for different design parameters. These tools along with time series, phase portrait and Poincare's map have been demonstrated to study the multi-stability situation and route to chaos phenomena.

2.6 Summary

The research review of a rotating system with nonlinearities in a shaft-bearing system, mathematical approaches to solve nonlinear problems, and different loading configurations (such as unbalance, axial load, base motion, rub impact phenomenon) has been described thoroughly. It is observed that while a number of works related to rotating system have been available, very few literatures are there for a lightweight shaft disk system with geometrical and inertial nonlinearities. Hence, the works related to the lightweight shaft disk system is critically reviewed and one may summarize the works as follows.

- A shaft disk system can be modelled by considering nonlinearities due to higher order deformation in bending and axial stretching.
- Investigation of linear dynamics of the rotor-bearing system under various loading/working conditions has been studied thoroughly by a number of researchers since last few decades, but very few researchers have tried to explore the effect of structural nonlinearity of shaft with/without considering other secondary effects such as mass imbalance, rotary inertia and gyroscopic effect for obtaining the critical speed of the rotating system. However, dynamics of rotor-bearing system under the support motion has been encountered in few researches without considering any nonlinearity while a nonlinear mathematical model considering geometric nonlinearity has been derived in along with other secondary effects existing in rotor-bearing system.
- Few authors have studied the vibration analysis of nonlinear shaft and performed parametric study by considering nonlinearities due to geometry and inertia, but the effect of disc parameter on the characteristic behaviour of the nonlinear system has not yet been explored.
- In many applications, rotating system subjected to different loading condition and suffers through unavoidable circumstances. Hence many authors have performed research on rotating system with different loading conditions such as unbalance force, axial loading, and base motion and rub impact phenomenon, but the effect of these loading conditions on the stability and bifurcation of the rotor-disk with nonlinearities due to curvature and inertia has not yet been explored.
- In practical applications, a rotor is composed of multi disc mounted along the shaft length. A few authors have explored research in such applications considering nonlinearity due to inertia and geometry. And, many authors have worked on vibration attenuation of the rotating system using viscoelastic materials but use of the viscoelastic material for shaft modeling to attenuate vibrations in a multidisc rotating system has not yet been explored.