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CONCLUSIONS AND SCOPES FOR FUTURE WORK

6.1 General conclusion

In this work, the nonlinear analyses of the elastic and the viscoelastic flexible shaft disk systems with the different loading configuration have been studied. While the rotor system with nonlinearities due to large deformation and axial stretching has been considered, another system with the nonlinearities (such as geometrical and inertial nonlinearities) is also analyzed by taken into account of the inextensibility assumption. As well as, a multidisc rotating model is considered, and the effect of viscoelasticity is also investigated. The following six different rotating system configurations have been studied

- *Large deflection model: axial stretching*
  - Excitation of a base motion and an unbalance mass
  - Excitation of an axial loading and an unbalance mass
- *Large deflection model: In-extensible Condition*
  - Free vibration analysis of the nonlinear shaft disk system
  - Excitation of geometric eccentricity and extra added mass
  - Excitation of an unbalance with rub impact phenomenon
- *Large deflection model: multi-disk rotating system*
  - Viscoelastic shaft with base excitation

For all these types of rotating systems, the governing equation of motion has been developed by using the extended Hamilton principle and then the nonlinear temporal equation of motion has been obtained by using the generalized Galerkin's method. The temporal equation of motion contains the forcing terms, parametric terms, and nonlinear terms along with the geometric and inertial terms. While the linear forcing term is due to the support motion and unbalance, the parametric terms are due to the harmonically varying axial force. The nonlinear damping term exists due to the material loss factor in a case of the viscoelastic shaft. The nonlinear forcing term arises due to rubbing between the stator and the rotor. The nonlinear geometric and inertia terms arise due to the nonlinear curvature and axial stretching of the shaft.

To solve these temporal equations of motions, the first order method of the multiple scales has been used. Influences of the different system parameters on the instability regions and frequency response curves have been observed for different resonance conditions. The present numerical results have been compared with the previously published results and numerically solving the temporal equation of motion which are found to be in a good agreement.

It may be noted that carrying out expensive experiments and numerically solving the highly complex nonlinear temporal equation of motion are tedious and time consuming and hence, one may use the developed simplified mathematical expressions and the reduced equations in this work for finding the instability regions, frequency response and critical bifurcation points for the different resonance conditions.

When the frequency of the excitation is nearly equal to the natural frequency of the system, the system undergoes simple resonance condition and, the system may fail due to sudden jump at the saddle node bifurcation point in this resonance condition. Hence, the rotating system should be operated safely at a frequency less than that of the saddle node bifurcation point. As well as, the system may pass through behavioral transformations such as period doubling or route to chaos for the critical range of the parameters. The sudden change in the system behavior can be prone to failure or poor performance. Specific conclusions are given in the following subsections.

6.2 Specific conclusions

- **6.2.1 Large deflection model with axial stretching : Excitation of a base motion and an unbalance mass**

  The free vibration analysis of a base-excited elastically induced flexible rotating system has been performed to determine both the linear and nonlinear natural frequencies by introducing Campbell diagrams under the variation of design parameters. Further, the stability analysis and successive bifurcation point of the steady-state behavior have been investigated for three different resonance conditions. Fundamental interaction of various components such as mass imbalance, Coriolis, and gyroscopic effect, and geometric nonlinearity has been included to predict actual and appropriate dynamic behavior.

  - Substantial effects of the system parameters on the natural frequency of the system are detected using the Campbell diagrams by performing free vibration and frequency response characteristics analysis. It depicts hardening effect in the system characteristics due to the presence of the nonlinearity. This effect is also detected using the FFT plots. Along with it, the influence of the system parameter (such as disk position, mass of the disk and shaft radius) is also observed on the characteristic behavior of the system. It has been observed that not only a spin speed of the shaft but also other parameters like the magnitude of ground motion and mass imbalance strongly exhibit the route to chaos behaviors when the parameter value reaches to its critical one
  - It is noteworthy that the present nonlinear model formulated based on the Euler theory has been successfully examined and found to be adequate enough to predict the correct value of critical speeds and dynamic responses in contrast to the Timoshenko model under similar working conditions when the shaft length is more than 0.15 m.
  - The effect of geometric nonlinearity and other parameters like an amplitude of the excitation, mass unbalance, position of the disk, and mass of the disk on the system performance has been depicted and show a substantial effect on the nonlinear behavior of the system near the resonance conditions

  Under steady-state conditions, the catastrophic failure of the system due to sudden jump can be controlled successfully by altering the design parameters.

- **6.2.2. Large deflection model with axial stretching: Excitation of an axial loading and an unbalance mass**

  This study exhibits a significant and inevitable theoretical development to understand the combine effect of unbalance and axial load on the overall system performance and subsequent critical working condition for the designing and developing of light-weight transmission systems.

  - Here, we considered a disk and disturbance parameters (i.e., axial force and mass unbalance) as control parameters and their influences on dynamic behavior of the proposed rotating system. Increase in the disk parameters value can reduce the hardening effect of the nonlinearity and helps to shift instability region to a high frequency. As a result, the bifurcation starts at a higher frequency and leads to a stable operating until the operating frequency reaches to one of the critical natural frequencies of the system. Here, the system loses its stability due to saddle-node and pitchfork bifurcation, respectively with sudden jump phenomena.

- In this proposed model, the nonlinear stiffness component gives a better stability than its counterpart while the pulsating axial load along with the mass unbalance shows appreciable effect on the dynamic behavior of the system such as increase in these positive effect cause rise in amplitude of the system vibration. Increasing the static axial load has been ensured the stability of the system since it increases the natural frequency of the whole system. Possibility of catastrophic failure as a sudden change in the amplitude due to jump up i.e., from unstable trivial solution to non-trivial solution gets high with increase in the disk-size with radius.
- The flexible bearings are considered to be substantially safe in working condition since it reduces the chances of failure due to shorter jump length comparatively. The vibration amplitude decreases remarkably while the jump length decreases with increase in the value of any these linear and nonlinear spring components. As a result of, the restoring force becomes more dominant over the externally influenced forces. From these critical observations, we can conclude with the evidence that monitor and control the vibrational characteristic and its behavior to avoid catastrophic failure can be successfully controlled with the adjustment of the design parameters. It helps further to design a system which can run in its operational speed range satisfactorily.

6.2.3 In-extensible Condition: Free vibration analysis of the nonlinear shaft disk system

A rotating shaft-disk system with rotary inertia, gyroscopic effect and nonlinear curvature has been analyzed for determining the natural frequencies and resulting free vibration response under the influences of various control parameters.

- In nonlinear free vibration, it has been seen that due to gyroscopic effect, when one plane is excited the other plane oscillates as well.
- The results obtained numerically from the numerical direct integration and perturbation analysis agreed well. The results for the system with disk are compared with that of the shaft alone and it has found to have lower natural frequency and first node appears before as it appears for the shaft for lower spin speed.
- Nonlinear whirling speed has observed to be higher about 5-8% as compared to the findings via the linear analysis.
- For lower spin speed of the shaft, the forward natural frequencies play a dominant role with respect to other frequency. Also, the system amplitude diminishes slowly for the combined rotor-bearing system. The linear forward frequency and backward frequency increase with angular rotating speed but rate of increase of the forward frequency is more compared to the backward linear frequency.
- The variation of the same with mass ratio showed that the unstable frequency decreases with increase in the mass ratio. Also, increasing slope of the forward frequency is less as compared to the backward frequency.
- The variation of linear frequencies with location of the disk denoted that with disk moving away from the centre, the rate of increase of the frequency increases. The rotor-bearing has been reduced to a simple shaft element by moving the disk nearest to either of the bearing ends. It is observed that the free vibration response portrays either periodic or quasi-periodic depending on the shaft spin speed.
- The nonlinear forward natural frequency has found to be increased whereas the nonlinear backward frequency decreases with increasing the angular speed and similar behavioural patterns are observed as linear part of the natural frequency.
- It has also been observed that the initial conditions play an important role for obtaining the nonlinear natural frequencies of the rotor-bearing system.

The present outcomes enable the insight of theoretical aspect of determining critical speeds and evaluating the free vibration behavior required in designing and developing flexible rotor-bearing performing under high speed operations.

- **6.2.4 In-extensible Condition: Excitation of geometric eccentricity and extra added mass**

Mathematical model of a shaft-disk system under the influence of external excitation due to eccentricity and unbalance mass is formulated with the nonlinear curvature and gyroscopic effect. A set of nonlinear algebraic equations have been derived from a nonlinear differential governing equation using MMS to further obtain the system responses and their stability. Based on the outcomes, the following observations have been depicted in the present work.

- The system leads to a catastrophic failure due to S-N bifurcation point. The initial condition plays an important role to realize the actual system's response for its bi-stability presence.
- The behaviour of the system can be successfully controlled with an appropriate selection of the geometric eccentricity as well as the possibility of instability and catastrophic failure of the system can also be attenuated.
- With increase in the shaft speed causes sharp increase in the vibration amplitude when the disk is located away from the mid-span. For a moderately large value of the disk-mass, the region of instability improves with a smaller range.
- The amplitude and the instability range get smaller for lower value of the unbalance mass and the other way around. However, change in the unbalance mass does not affect the rate of change of amplitude $dv/d\Omega$.
- The system showed a decrease in the hardening effect with increase in the mass moment of inertia (M.I). Consequently, the system has smaller region of the instability for the higher value of the M.I.
- Finally, the eccentricity strongly exhibits chaotic behaviours when its value crosses to one of its critical value.
- The system with flexible bearing support shows less hardening effect but increase in the hardening effect can be observed by considering large value of the nonlinear stiffness coefficient of the bearings.
- An unbalance and nonlinear stiffness coefficient have substantial effect in vibrational nature of the system. These can also be considered as control parameters to regulate the system dynamical behaviour

- **6.2.5 In-extensible condition: Excitation of an unbalance with rub impact phenomenon**

A rub impact rotor system with the geometrical and inertial nonlinearities is analysed to investigate its stability and bifurcation behaviour.

- The bifurcation analysis is performed by regulating systems parameters such as spin speed, coefficient of friction, stiffness of the stator surface and unbalance.
- The substantial effect of variations in these parameters on the system dynamics has been observed. These variation causes route to chaos phenomena in the vibration behaviour of the system.
- For analysed range of parameters, the system shows dominance of the rubbing effect. This nonlinear nature of the system under effect of the rub and the unbalance can lead to instability and then to poor performance or may subjected to a failure.

- **6.2.6 Large deflection model with multi-disk: Viscoelastic shaft with base excitation**

A mathematical model of the rotor system with multiple disks under the influence of an external excitation due to the base motion is formulated with nonlinear curvature and inertia effect. A set of nonlinear algebraic equations have been derived from the nonlinear differential governing equation using the method of multiple scales to further obtain the system responses and their stability. Based on the outcomes, the following observations have been depicted in the present work.

- The behavior of the system can be successfully controlled with an appropriate selection of the base excitation frequency and magnitude as well as the possibility of instability and catastrophic failure of the system can also be attenuated.
- The increase in the number of discs causes an increase in the vibration amplitude and an expansion of the instability region. A viscous material with sufficient loss factor can attenuate the vibration amplitude and decreases the instability region, consequently it decreases the severity of the catastrophic failure.
- The amplitude and the instability range get smaller for lower value of the base excitation parameters (such as W_b) and the other way around.
- Finally, the systems parameters (N, δ, Ω_b and W_b) strongly exhibit the chaotic behaviors when its value crosses to one of its critical value.

Outcomes from this work enable significant theoretical understanding of forced vibration analyses which are of great practical importance for investigating the dynamic performance.

6.3 Scopes for future work

In the present work, the rotor system with nonlinearities due to large deformation and axial stretching has been considered, as well as another system with nonlinearities (such as geometrical and inertial nonlinearities) is also analyzed by taken into account the effect of the inextensibility assumption under effect of different excitations.

The following extensions may be carried out for this work.

- Here the analysis has been limited to the fundamental frequency of the system. Multimode analysis may be carried out to study the effect of different system parameters when the system operates at higher frequencies.
- Internal resonance conditions may be considered when the frequencies of different modes are commensurate.
- The combined effect of the nonlinearities due to large deformation and instability of fluid film bearings can also be explored for further work.
- For passively control of the vibration of the rotor, one may use a sandwich structure and shunted piezo-ceramics dampers.
- To actively reduce the vibration of the rotor one may use magneto-rheological elastomer where, the present analysis can be easily extended.
- Experimental analysis can also be carried out for the respective research models to explore more the nonlinear behavior of the system under different configurations and combined linear and nonlinear loading conditions. This will enhance the practical importance of the study.