APPENDIX

A.1 Displacement relation, Euler angle and Taylor expansion (Nayfeh, 2004)

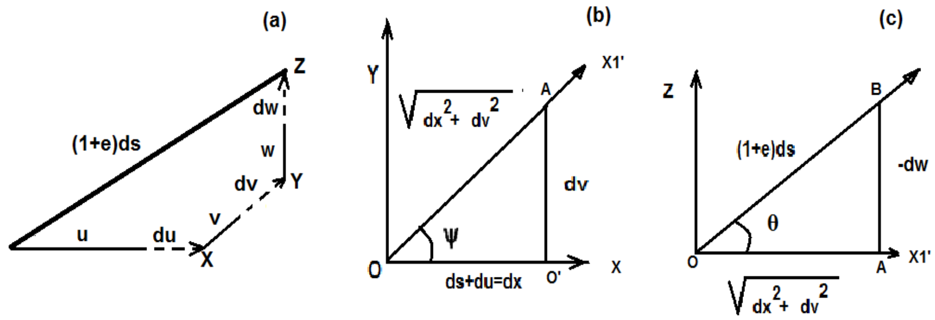

Fig. A.1: Displacement relation and Euler angle

From Fig. A.1, the element length after a deformation can be

$$(1+e)ds = \sqrt{(ds+du)^2 + dv^2 + dw^2}$$

Thus, strain along the element can be

$$e = \sqrt{(1+u')^2 + v'^2 + w'^2} - 1 \approx u' + \frac{1}{2}(1-u')(v'^2 + w'^2) + ... \tag{A.1}$$

And, the relationships of the angles and the displacements can be written as

$$\cos\psi = \frac{(1+u')}{\sqrt{(1+u')^2 + v'^2}} \approx 1 - \frac{1}{2}v'^2 + u'v'^2 + ...$$

$$\sin\psi = \frac{v'}{\sqrt{(1+u')^2 + v'^2}} \approx v' - u'v' + u'^2 v' - \frac{1}{3}v'^3 + ...$$

$$\psi = \tan^{-1}\left(\frac{v'}{1+u'}\right) \approx v' - u'v' + u'^{2v'} - \frac{1}{3}v'^3 + ...$$

$$\theta = \tan^{-1}\left(\frac{-w'}{\sqrt{(1+u')^2 + v'^2}}\right) \approx -w' + u'w' - u'^2 w' + ...$$

$$\sin\theta = \frac{-w'}{\sqrt{(1+u')^2 + v'^2 + w'^2}} \approx -w' + u'w' - u'^2 w' + \frac{1}{2}u'^2 w'^2 ....$$

$$\cos\theta = \frac{\sqrt{(1+u')^2 + v'^2}}{\sqrt{(1+u')^2 + v'^2 + w'^2}} \approx 1 - \frac{1}{2}w'^2 + u'w'^2 + ... \tag{A.2}$$

Considering the inextensible condition the shaft along its length (i.e. e=0), Eq. (A.1) can be expressed as

$$u' = -\frac{1}{2}(v'^2 + w'^2)$$

Thus,

$$u = -\frac{1}{2}\int_0^x (v'^2 + w'^2)dx. \tag{A.3}$$