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Introduction

This thesis is aimed at the study of various facets of nonclassicality in subatomic and quantum optical systems and their interplay with the underlying dynamics. The objective, in particular, is to analyze the behavior of various witnesses and measures of nonclassicality, both spatial as well as temporal, in different systems using closed and open system approaches. The problem of quantifying the degree of nonclassicality of quantum channels is also addressed and a measure is proposed for the same. Further, the thesis deals with the non-Hermitian Hamiltonians in the context of Parity-Time symmetry and their impact on various nonclassical properties.

The notion of nonclassicality is multifaceted in nature owing to the fact that there is no sharp boundary between the classical and the quantum world. Consequently, the characterization, quantification, and detection of nonclassicality is one of central problems of quantum theory. An operational approach to probe the nonclassicality of a system is by devising tests to simulate certain statistical properties of interest using a classical strategy. A failure of such a test implies that the concerned statistical properties cannot be simulated by the classical strategy. A classic example of such an approach is the Bell inequality [1], whose violation nullifies the existence of local hidden variable models. Another important example is the nonclassicality of a bosonic field formulated in terms of the Wigner function or the Sudershan-Glauber P function [2]. A negative value of these functions implies that one cannot represent the field in terms of probability distribution over phase space.

The study of nonclassical correlations has not only proved to be important in probing the basic features which make a quantum system different from a classical system, but has also provided potential resources for future quantum technologies. The nonclassical correlations can be quantified in many ways. Apart from the celebrated Bell inequalities [1] which serve as a test for the local realism, the well know quantum steering [3] allows one party to change the state of the other by local measurements. The nonseparability of the state of a system is probed by so called entanglement witnesses [4]. Quantum discord is another measure of nonclassical correlation which can even exist in systems which are not entangled [5, 6]. These spatial quantum correlations have been a subject matter of many theoretical [7–12] and experimental works [13–16]. Temporal quantum correlations, which exist between different measurements made on a single system at different times, have attracted lot of attention in recent years. Prominent among these are the Leggett-Garg inequalities (LGIs) [17]. In their different avatars, LGIs have been analyzed in various theoretical [18–28] and experimental works [29–35]. Based on the assumptions of *macrorealism* and *non-invasive measurability*, LGIs are considered as a test of macroscopic coherence [23, 36, 37]. Here, macrorealism means that the measurement of an observable $\hat{M}$ of a macroscopic system reveals a well defined pre-existing value and non-invasive measurability states that, in principle, one determines this value without disturbing the future dynamics of the system. These assumptions lead to bounds on certain combinations of two time correlation functions $C(t_i, t_j) = \langle \hat{M}(t_i)\hat{M}(t_j)\rangle$, which may not be respected by quantum systems. Since the notion of "realism" is often linked to the existence of hidden variable theories, the violation of LGI nullifies the existence of such theories [33, 38, 39].

One of the open questions is how a classical world could emerge out of the quantum world. In this connection the question arises whether *macroscopic superpositions*, e.g. the superposition of a Schrödinger cat in two distinct states, dead and alive, are possible? From the technical point of view macroscopic superpositions are suppressed by decoherence, namely the (unwanted) coupling to the degrees of freedom of the environment, see e.g., Ref. [40]. Other approaches are so called spontaneous collapse models [41] which propose an ontologically objective mechanism for a collapse in order to avoid the unobserved macroscopic superpositions. Recently, systems in high energy physics, including neutrinos, were studied and it was shown that these systems are very suited to come up with a conclusive test [28, 42–44].

The Bell inequalities could be considered as the spatial counterpart of LGIs. Both inequalities capture, if violated, a contradiction with the assumptions on which these inequalities are based. Refs. [8] and [9] show the violation of Bell–type inequality in the context of two and three flavor neutrino oscillations [45], respectively. In Ref. [46] a method was proposed for probing a simplified version of LGI, the Leggett-Garg type inequality, wherein the requirement of non-invasiveness is replaced by stationarity, within the framework of two flavor neutrino oscillations and it was shown that this inequality is violated in the MINOS experimental setup. Similar observations were reported recently for the Daya-Bay experiment in Ref. [47]. Both these analyses were done assuming only two flavor neutrinos, which means they cannot be sensitive to CP violation. Further, these analyses can be applied only to those neutrino oscillation experiments which measure survival probabilities. In Ref. [48] the authors studied the difference in the maximal violation of the LGI for a particular setup for two and three neutrino-flavor oscillations. The CP violation in the quark sector is shown to be a crucial ingredient in the violation of a Bell inequality for entangled K-meson pairs [49], namely only due to the CP violation the inherent nonlocality of the system is revealed for a certain setup. An interplay between various facets of quantum correlations and CP violation in the quark sector has been a topic of numerous interests, see for example [7, 50, 51]. A theoretical analysis of the impact of entanglement on neutrino oscillation wavelength and its possible implications on the mass squared splittings can be found in Ref. [52]. How the nature of neutrinos, e.g. if they are Majorana or Dirac particles, can be revealed by a geometric phase was discussed in Ref. [53]. This thesis includes works on various forms of LGI studied in three flavor neutrino system and decaying meson system [27, 28, 54–56].

The study of quantum systems interacting with their environment helps in characterizing the behavior of the dynamics of the system. This is useful in many application of quantum mechanics and has lead to the field of open quantum systems [57, 58]. In many practical situations, the system-environment interaction brings in pronounced memory effects leading to the emergence of non-Markovian dynamics [59–65]. Recently, non-Markovianity has been a subject matter of various studies from quantum cryptography [66, 67], quantum biology [68–70], quantum metrology [71, 72] and quantum control [73]. It has been shown with ample evidence that non-Markovian channels can be advantageous over Markovian ones. In [74], it was reported that the non-Markovianity can enhance the channel capacity in comparison to the Markovian case. Non-Markovian behavior is a multifaceted phenomenon which can not be attributed to a unique feature of the system-environment interaction. Consequently, several different measures were introduced in order to quantify the non-Markovian behavior, viz., trace distance [59], fidelity [75], semigroup property [76] or divisibility [60] of the dynamical map, quantum Fisher information (QFI) [77], quantum mutual information [78]. In general, these measures are inequivalent and different predictions by these measures have been reported in [79].

The non-Markovian aspects become pertinent while dealing with quantum channels subjected to different types of environment. Another aspect of the system-environment interactions is the loss of the coherence and entanglement which is undesirable from the perspective of carrying out the tasks of quantum information. Therefore, this calls for the characterization of the quantum channels under the influence of different environments. Efforts have been made in this direction [80, 81]. Quantum coherence can be thought of as a resource [82–84] bringing out the utility of the quantum behavior in various tasks [85, 86]. As the system evolves under ambient conditions, modeled by the noisy channel under consideration, it has a tendency of getting mixed [87]. A pertinent question then to ask is the trade-off between the mixedness and coherence [88, 89]. The interplay between coherence and mixing in the context of non-Markovian evolution, has been studied [90]. Gate fidelity [91], which tell us about the efficiency of the gate's performance and channel fidelity [80], a measure of how well a gate preserves the distinguishability of states, and is thus connected to the Holevo bound of the channel, are two useful channel performance parameters. The performance of Lindbladian channels, such as the squeezed generalized amplitude damping (SGAD) channel [92] and the Unruh channel [93] have been studied using these parameters [80]. Here, these are used to characterize the two non-Markovian channels, RTN [94–96] and NMD [97]. This thesis contains a detailed study of non-Markovianity in these various channels using QFI-flow [98].

The non-Markovian nature of the dynamics brings subtleties in the analysis of LGI for such systems. It emerges from our work [99] that the intervening noise between two measurements is relevant for the evolution of the LG parameter. This can be understood equally well by absorbing the noise into the measurements, which can then be regarded as a noise-induced POVM [100], and no longer projective measurements. The non-Markovianity in general involves setting up system-bath correlations, even through the system and bath may be initially uncorrelated. Therefore, the intervention of measurement that is done to produce temporal correlations, will in general re-prepare the environment also, just as it re-prepares the system. Hence, correlations based on a subsequent measurement will be subject, in general, to a different noisy channel than the first measurement.

From a quantum information theoretic perspective, non-Markovianity has of late been studied by the (not always equivalent) criteria of (CP) divisibility and distinguishability [101]. In particular, non-Markovianity according to the former criterion manifests as the fact that the *intermediate map* (i.e., the dynamical map that propagates an intermediate earlier state to a later state) acting on the density operator is not-completely-positive (NCP) [95]. As a result, the intermediate time evolution of the density operator is no longer given by the Kraus operator-sum representation. Instead, the operator sum-difference representation must be employed [102], wherein the trace-preserving NCP map is represented as the difference of two CP maps. The failure of the quantum regression hypothesis (QRH) [103], which deals with multi-time correlation functions, also captures a traditional idea of quantum non-Markovianity [104]. In recent times, there have been a number of works that compute the two-time correlation functions for non-Markovian dynamics. For example, the evolution equations for the two-time correlation functions for non-Markovain evolution in the case of weak system-environment coupling was studied in [105], employing the full system-environment Hamiltonian. In particular, with regard to the question of the LGI violation in the context of non-Markovian noise, building on [105], the LGI violations for the Jaynes-Cummings model was discussed in [106]. The common theme in these works is to start from the full unitary evolution and then derive the evolution equations for the correlation functions using the appropriate limits.

This thesis also addresses the problem of quantifying the degree of quantumness of channel [107, 108], which has both theoretical and practical significance in quantum information science [4]. The quantum channels are completely positive and trace preserving maps which describe processes like information transfer in a given environment [109]. Since quantum information is transmitted in the form of quantum states, it is important to quantify the degree to which a quantum state gets affected while subjected to a quantum channel [110]. The classical states are usually identified as those whose correlations can be described in terms of classical probabilities. This approach has lead to the quantification of some well known nonclassical correlations such as entanglement, discord and related quantities [111]. Alternatively, a different way of quantifying the quantumness of a single system is by exploiting the non-commutative algebra of observables, such that the mutual commutation of all the accessible states of the system identify with a classical system. This approach has advantages in that it make no reference to the correlations and no complicated optimization procedures are needed [112].

Noise is usually known for its negative role in reducing the degree of coherence in a system. However, they can show enhancement in nonclassical correlations for some states [92, 113, 114]. In [115–117], it was shown that local environments can enhance the average fidelity of quantum teleportation for certain entangled states. Enhancement in quantum discord by local Markovian (i.e., memoryless) noise channels was reported in [118, 119]. Quantum channels provide a platform for studying the interplay between quantumness of states and the underlying dynamics in presence of an ambient environment [58]. This has lead to several interesting observations. For example, in [120] it was shown that the quantum channels need not be decohering, but could have cohering power as well. The cohering power, that is, the ability of quantum operations to produce coherence, was given an operational interpretation in [121]. It was further shown that the cohering power of any quantum operation is upper bounded by the corresponding unitary operation. The entangling capabilities of unitary operations acting on bipartite systems was reported in [122], with the maximum entanglement being created with product input states [123]. The deteriorating effect of the environment on a quantum state has been studied in the context of coherence-breaking channels and coherence sudden death [124]. An interesting class of channels known as semi-classical channels Λ_{SC} map all the input states ρ to $\Lambda_{SC}(\rho)$, such that the later are diagonal in the same basis. Such channels are realized by complete decoherence after which only diagonal elements of the density matrix are non-zero [125]. Another well studied class of quantum channels are those based on Lindbladian evolution which focus on the dynamics at time scales well separated from that of the reservoir correlations. However, in a number of practical applications, this assumption is not true and one has to take into account the non-Markovian aspects of the underlying dynamics [63, 95, 97]. In [126–128], coherence of quantum channels was analyzed using Choi-Jamiolwski isomorphism.

The investigation of nonclassical properties of light forms part of this thesis and is based on [12, 129, 130]. Quantum optics has the reputation of providing a test bed for verifying various predictions of quantum theory, and has been used to design devices that can outperform their classical counterparts. This quantum power of devices is obtained by exploiting nonclassical states, i.e., states having no classical analogue and more technically, the quantum states having negative values of Glauber-Sudarshan P-function [2, 131]. Such states are not rare in nature, and entangled and steering states [132], squeezed states [133], antibunched states [134] are typical examples of nonclassical states. Existence of such states were known (at least theoretically) since a long time. In fact, squeezing [135], entanglement [3], and steering [136] were studied even before the pioneering work of Sudarshan [2] that provided a necessary and sufficient criterion of nonclassicality in terms of negativity of P-function. However, various interesting applications of these nonclassical states were realized only recently with the advent of quantum information processing [137–142] and various facets of atom optics and quantum optics [143, 144]. For example, squeezed vacuum state has been used successfully in detecting gravitational waves in the well known LIGO experiment [145, 146]; squeezed states are also used in continuous variable quantum secure and insecure communication [137, 138]; entanglement is established to be useful in both continuous and discrete variable quantum cryptography [137, 140], and in the realization of schemes for teleportation [141] and dense coding [142]. Additionally, the steerable states provide one-side device independent quantum cryptography [147]. Furthermore, powerful quantum algorithms for unsorted database search [148] to factorization [149], discrete logarithm problem [149] to machine learning [150] have repeatedly established that quantum computers (which naturally use nonclassical states) can outperform classical computer. In brief, in the last few years, on one hand, we have seen various applications of nonclassical states, and on the other hand, nonclassical features have been reported in a variety of physical systems [151–154], including but not restricted to two-mode Bose-Einstein condensates [153, 155], optical couplers [156, 157], optomechanical [154, 158] and optomechanics-like systems [154, 159], atoms and quantum dot in a cavity [160, 161]. Many of these systems involve different types of cavity which can be produced and manipulated experimentally [162–164]. Naturally, interest in such systems has been considerably enhanced in the recent past. Apart from the applicability of the nonclassical states, and the possibilities of generation and manipulation of these states, another interesting factor that has enhanced the interest in the nonclassical features present in these systems, is the fact that in contrast to the traditional view that quantum mechanics is the science of the microscopic world, these systems having nonclassical properties are often macroscopic [165].

As mentioned above, the negativity of P-function provides us a necessary and sufficient criterion of nonclassicality. However, the P-function is not always well behaved, and there does not exist any general procedure that can be adapted to experimentally measure it. As a consequence, a set of operational criteria for nonclassicality have been developed over the years. There exists an infinite set of nonclassicality criteria involving moments of annihilation and creation operators that are equivalent to the P-function, but any finite subset of that would be sufficient only. We have used a few such moment-based criteria of noclassicality [166, 167], each of which is a sufficient criterion only. As none of these criteria provides any quantitative measure of nonclassicality (i.e., as they only provide signatures of noclassicality), in what follows, these sufficient criteria are frequently referred to as witnesses of nonclassicality. Through these criteria, different features of nonclassicality are witnessed under the influence of open quantum system evolution.

The effect of the ambient environment is a permanent fixture of nature and needs to be taken into account, especially in experiments related to nonclassical features which are known to be influenced appreciably by the environmental effects. As the present work aims to reveal the nonclassical features present in the system of interest, it would be apt to consider effect of environment in our calculation. Such effects are taken into account systematically by using the framework of open quantum systems [57]. Specifically, decoherence and dissipation are well known open system effects [87] and have been studied on myriad aspects of quantum information, such as in holonomic quantum computation [113], environmental deletion [168], noisy quantum walks [169], quantum cryptography [170] and the effect of squeezing on channel capacity [92]. We adapt open system effects on our system of interest by using the formalism of Langevin equations, which is basically the stochastic equations of motion approach [144]. Specifically, the equations of motion for each system mode in the Heisenberg picture are obtained by eliminating the environmental degrees of freedom. The obtained equations of motion for different system modes are usually coupled differential equations and are solved using various peturbative techniques. Here, we have used a perturbative technique that approximate all the higher-order correlations in terms of second-order correlations [171]. The technique has been recently used to study nonclassicality in Raman amplifier [172] and optomechanical oscillator [173].

This thesis also sheds light on some important observations noticed in Parity-Time ($\mathcal{PT}$) symmetric systems and are reported in [129, 130]. In textbook quantum mechanics, one of the fundamental axioms is that the physical observables are represented by the Hermitian operators which always possess real eigenvalues and conserve the probability [174]. In particular, the Hamiltonian H generating the time evolution of the system has real eigenvalues and the corresponding time translation operator $U = e^{-iHt}$ is unitary as a consequence of Hermiticity of H. However, a non-Hermitian Hamiltonian with the parity ($\mathcal{P}$) - time ($\mathcal{T}$) symmetry, often referred to as a $\mathcal{PT}$

symmetric Hamiltonian, can also possess real eigenvalue spectrum [175]. Such non-Hermitian Hamiltonians may undergo a spontaneous transition to $\mathcal{PT}$ symmetry broken phase [176]. The operators $\mathcal{P}$ and $\mathcal{T}$ are defined by their action on the dynamical variables $\hat{x}$ (the position operator) and $\hat{p}$ (the momentum operator), such that the *linear* operator $\mathcal{P}$ acts as $\hat{p} \to -\hat{p}$ and $\hat{x} \to -\hat{x}$, while the *antilinear* operator $\mathcal{T}$ acts such that $\hat{p} \to -\hat{p}$, $\hat{x} \to \hat{x}$. Further, $\mathcal{T}$ also flips the sign of $i = \sqrt{-1}$, i.e., it transforms $i \to -i$, such that the commutation relation $[\hat{x}, \hat{p}] = i$ is preserved. The $\mathcal{PT}$ symmetric systems can exhibit *exceptional points* (EPs) where the eigenvalues of the non-Hermitian Hamiltonian are degenerate.

The $\mathcal{PT}$ symmetric Hamiltonians belong to a more general class of pseudo-Hermitian systems [177]. The eigenfunctions of a system Hamiltonian in $\mathcal{PT}$ symmetric phase are also the eigenfunctions of the $\mathcal{PT}$ operator, i.e., all eigenfunctions are also $\mathcal{PT}$ symmetric. However, in the PTB phase, some or all the eigenvalues become complex and not all the eigenfunctions of the Hamiltonain possess $\mathcal{PT}$ symmetry. With these interesting properties, non-Hermitian quantum mechanics has attracted a lot of attention, leading to the exploration of $\mathcal{PT}$ symmetric systems in different domains. The *phase lapses* observed in the experiments with Aharonov-Bohm rings remained a puzzle until the phenomenon was explained using non-Hermitian Hamiltonian [178, 179]. The non-Hermitian Hamiltonians have been used to describe the laser induced continuum structures in atoms [180, 181]. In [182], a scheme based on *resonance coalescence* to achieve vibrational cooling was proposed. The extension of $\mathcal{PT}$ symmetric quantum mechanics to quantum field theory with cubic interaction was reported in [183]. Further, the role of non-Hermicity in open quantum systems has been explored in [184–186]. The dominance of Lyapunov exponent over the non-Hermicity parameters leads to real eigenvalues in Hatano–Nelson non-Hermitian Anderson model for disordered systems [187]. Based on Lagrangian principles a formalism was developed to describe coupled optical $\mathcal{PT}$ symmetric systems [188]. In [189], it was demonstrated that the $\mathcal{PT}$ symmetric potentials can exhibit phenomena such as double refraction, power oscillations, and secondary emissions. The existence of solitons in optical $\mathcal{PT}$ symmetric systems was reported in [190]. These solitons were found to be stable over wide range of potential parameters. The concept of *pseudo* $\mathcal{PT}$ symmetry was introduced in [191], where it was shown that one can manipulate $\mathcal{PT}$ symmetry properties in periodically modulated optical systems with balanced gain and loss. The quantum phase transition and its connection with geometric phase was studied in non-Hermitian $\mathcal{PT}$ symmetric Ising model [192, 193]. A non-Hermitian, $\mathcal{PT}$ symmetric model of dimerized spin chain was introduced in [194] and its (anti-) ferromagnetic quantum phase transition was analyzed. The $\mathcal{PT}$ symmetric systems with spontaneous generation of photons and superradient emission of radiation were also investigated in [195, 196].

From a practical point of view, $\mathcal{PT}$ symmetry has found many important applications such as the single-mode $\mathcal{PT}$ lasers [197, 198], unidirectional reflectionless $\mathcal{PT}$ symmetric metamaterial at optical frequencies [199]. Based on $\mathcal{PT}$ symmetry, many new photonic devices have been designed and fabricated [200, 201]. The $\mathcal{PT}$ symmetric periodic structures act as unidirectional EIT (electromagnetically induced transparency) devices near the exceptional point [202]. A $\mathcal{PT}$ symmetric coupler under appropriate conditions can act as an optical switch [203]. Further, $\mathcal{PT}$ symmetry has made the notion of *loss* useful, which was otherwise considered as a detrimental physical effect [204]. The above mentioned features and applications of $\mathcal{PT}$ symmetry and the potential application of entanglement in quantum computing and communication have motivated us to look for a $\mathcal{PT}$ symmetric physical system which can be realized experimentally and which can generate entanglement. Effective Hamiltonians in quasi-open systems related to microwave cavities have been investigated in [205] with regard to PT symmetric sub-configurations and effective PT phase transitions. It is worth mentioning here that the non-Hermitian Hamiltonian, in the context of open quantum systems [58], are often referred to as *effective* Hamiltonians H_{eff}, governing the dynamics in a restrictive subspace of the quantum system and appear as von-Neumann type evolution in the Master equations [206–208]. Thus, the notion of $\mathcal{PT}$ symmetry has proved to be a useful tool in probing the behavior of dynamics of the systems described by effective Hamiltonians which correspond to non-Hermititan systems. Since, the degree of quantumness of a system is controlled by the underlying dynamics, this naturally invites one to explore the interplay between nonclassicality and $\mathcal{PT}$ symmetry in such systems [129, 209].

Before going into the detailed description of various aspects of this thesis, an introduction is provided to some of the important tools and concepts used throughout the thesis.