MODAL ANALYSIS AND NONLINEAR DYNAMICS OF MULTI-LINK FLEXIBLE MANIPULATOR WITH GENERIC PAYLOAD MOUNTED ON A MOVING BASE

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Pravesh Kumar

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Conclusions and Scope of Future Works

6.1 THESIS CONCLUSIONS

In present work, the eigenanalysis, nonlinear behavior and dynamic characteristics of eight different models of flexible link manipulators with prismatic and revolute joint motions have been investigated. The analyses progressed from single-link manipulator to the multi-link manipulators for different end conditions for small deflection model. Considering the working environment and the payload usually lifted by the manipulator, the terminal end of the manipulator is assumed to be attached with sizeable mass with inertia, offset and orientation along with and without constraint pulsating axial force. The different configurations of the flexible manipulators considered for the analysis are as follows:

- Single-link manipulator
  - Rotating single-link flexible Cartesian manipulator incorporating a generic payload with axial pulsating force.
  - Single-link flexible manipulator with revolute pair
- Two-link manipulator
  - With point payload
  - With generic payload acted upon by axial pulsating constraint force.
  - Two-Link manipulator driven by harmonic revolute joints.
  - Two-link manipulator with harmonic prismatic and revolute motions.
  - Two-link manipulator with revolute pair incorporating extended payload.
- Multi-link manipulator with harmonic prismatic and revolute joints.

For all the manipulators mentioned above, the system dynamics have been defined using generalized coordinate system and the kinetic and potential energies are expressed in terms of a general point on the link and centroid of the payload. The extended Hamilton's principle is used to obtain the complex, nonlinearly coupled governing equations of motion in of the links and their corresponding boundary conditions along with the joint motions in space and time. The lengths of the links are assumed to be large as compared to the cross-sectional dimensions and hence the Euler-Bernoulli beam theory is used to model the links neglecting the shear deformations and rotary inertia. An appropriate dynamic model incorporating both link and joint flexibilities subjected to a harmonic motion at the joints has been developed. While harmonic joint flexibility has been modeled as a combination of torsional spring-inertia elements, actuator providing prismatic motion is assumed to be a linear spring and mass system. The kinetic energy of the manipulator comprised of the energy due to translatory motion of links, mass of payload, mass of actuator, mass of joints and rotational energy of payload. The potential energy constituted of strain energy due to elastic bending, stretching effect of the links, and strain energy due to elastic deformation of the joints. The system under external and internal exciation exhibit a peculiar behavior when the excitation frequency becomes nearly equal or equal to the system natural frequency. The vibratory behavior is also significantly affected by the change in system attributes and therefore it becomes necessary to study the effect of parametric variations of eigenspectrums. Hence, free vibration analysis is performed to obtain the eigenfrequency equation and mode-shapes of the manipulator and subsequently, the modal parameters of the manipulator in terms of nondimensional system attributes are tabulated and graphically illustrated. The qualitative and quantitative effect of ssts tem parameters on the eigenspectrums are demonstrated. The modal parameters have also been compared with the previously published works. In all cases the decrease in eigenfrequencies is noticed with the increase in payload mass, actuator mass, payload inertia, and beam mass density ratio. While, the eigenfrequencies increase with flexural rigidity ratio and joint/actuator frequency parameters.

The second order nonlinear temporal equations of motion of the links are obtained by discretizing the governing equations with the Galerkin's method using the obtained modal parameters. The temporal equations of the links consist of the terms due to geometric nonlinearities, inertial coupling between the links, damping, direct and parametric excitations. While the harmonic revolute and prismatic motions induce the linear as well as parametric excitation terms, the axial pulsating force can stimulate the linear or the parametric excitations depending on the orientation of the force with respect to the terminal link axis. The parametric nonlinear terms in the temporal equations of motion exist due to joint motions (revolute and/or prismatic) and the cubic geometric or inertial nonlinearity occurs respectively due to the axial stretching or large deformations in the links. As the temporal equation of the motion consists of the complex nonlinear terms, the closed-form solution is very difficult to obtain, and hence the second order method of multiple scales has been used to determine the steady-state solutions. The influence of essential system parameters and configurations on the frequency response curves and the system instabilities for different resonance cases have been investigated thoroughly. The analytical results thus obtained are in good agreement with those determined by numerically solving the equation of motions. The manipulator system undergoes primary resonance condition when the frequency of the prismatic and revolute motion of the joints becomes equal or nearly equal to the link's normalized frequency. Internal resonance is found to exist between the links of the manipulator due to the inertial coupling between the dynamics of the links. The system experiences sub-harmonic resonance conditions when the frequency of the pulsating axial constraint force becomes nearly twice to the link's normalized frequency. The type of the resonance conditions induced in the system depends on the orientation of the pulsating axial force with respect to the axis of the link. Saddle node and pitchfork bifurcations, depending on the resonance conditions, lead to the multiple solutions and jump phenomenon which may result in the catastrophic failure of the system. This can be avoided by operating the system in safe zones indicated in the frequency response curves.

Next, the assumed mode method is explored to develop a computationally efficient closed form of equations of links and joints which is simulated for smooth sinusoidal torque inputs at the joints. The performance evaluations of the manipulator for various system variables such as, payload mass, payload inertia, payload offset length, length of the link, joint masses and inertia have been graphically presented. The system responses typically, the link angular position, modal deflection and tip acceleration, are evaluated and graphically illustrated. Further, a model based inversion technique in conjunction with proportional-derivative (PD) controller is used to investigate the influence of the system variables such as link length, payload capacity, material of links, joint inertia and joint mass on the control parameters such as control torque, and modal displacements for the desired angular position of the joint angles. The work provides indicators for the selection of system parameters of flexible manipulator systems to achieve the required performance. The results of the different investigations are summarized in following subsections.

6.1.1 Single-link manipulator

With the objective of investigating the effect of a generic payload at the proximal end of the link on normalized modal parameters and dynamic behavior of a single-link manipulator with Cartesian or revolute motions, a nonlinear Euler-Bernoulli beam model has been derived.

- The linearly coupled longitudinal and transverse dynamics of the manipulator is obtained for the non-axial orientation of the generic payload which is solved simultaneously to obtain the system modal parameters.

- A sudden jump is noticed at the unit magnitude of the joint frequency parameter i.e. the frequency of joint spring-mass system becomes equal to the link natural frequency, owing to the fact that the joint dynamics gets decoupled from the link dynamics.
- The amplitude of manipulator at both the end i.e. actuator end and distal end are significantly affected by the attached masses at the respective ends and system tends to vibrates at higher mode of vibration for the hub frequency parameter greater than unity. The higher modes of vibrations are observed to be marginally effected by the variation of system parameters.
- The joint dynamics and type of payload, i.e., point or generic payload, significantly changes the mode shapes of the manipulator and it is observed that the effect of the longitudinal motion becomes prominent for the larger actuator mass.
- The angular position of the manipulator decreases with the increase in payload parameters, i.e., mass, inertia and offset, when the joint of the manipulator is being driven by smooth sinusoidal torque and the large rigid tip vibrations have been observed for the increased payload inertia.
- For subharmonic resonance case in Cartesian manipulator, the axial forcing frequency becomes nearly twice that of the system's normalized frequency and in combined resonance case, the frequency of harmonic transverse motion also becomes nearly equal to the system's normalized frequency. While in combined resonance case, jump phenomenon and multi-valued solutions are observed due to the existence of saddle node bifurcations, the system loses it stability through pitchfork bifurcations in case of subharmonic resonance case.
- The system tends to vibrate at larger amplitude and the bi-stable region increases with the increase in payload inertia, actuator mass, rotating frequency and amplitude of the axial force. The system also experiences behavior fluctuations from spring hardening to spring softening or vice-verse with the consideration of actuator mass, payload mass and offset.
- In flexible manipulator with revolute hub, primary and subharmonic resonances are respectively observed when the hub frequency becomes nearly equal and thrice to the normalized natural frequency of the link. A sudden change in amplitude of vibration has been noticed due to jump phenomena at sub-critical pitchfork point in subharmonic resonance and at saddle node point for combined and primary resonance.
- No behavior alteration is experienced in case of large deformation model and the system exhibits only spring softening behavior. The system is found to be vulnerable to catastrophic failure due to sudden change in vibration amplitude when it passes through the critical points, i.e., S-N and pitchfork bifurcation, either during starting or stopping of the prime mover.
- The sudden change with high jump length mostly occurs when the parametric variation of payload mass and inertia, actuator inertia, and offset ratio takes place. The safety of the manipulator system and the operator can be ensured by operating the manipulator in the safe zones indicated in the frequency response curves for different system parameters.

6.1.2 Two-link manipulator

The objective of this segment of the work is to present a detailed mathematical modeling of two-link flexible manipulator with revolute and prismatic motions and detemine their eigen-frequencies from the free vibration analysis which is essential for the manipulator design. The nonlinear analysis has been conducted to demonstrate the effect of variation of system attributes on the stability of the system under time varying externally applied force, direct and parametric excitations due to harmonic joint motions and internal resonance occurring between the links due to inertial coupling. In order to facilitate the appropriate control design of the flexible manipulator and to understand the system performance when the revolute joints are driven by externally applied torques, the influence of system parameters on the angular position, modal deflections and tip accelerations have been reported.

- The eigenfrequencies tend to decrease with offset length if the payload inertia parameter is neglected. However, as soon as the payload inertia is included in the system, the lower eigenfrequencies tend to decrease with offset parameter. This is due to the fact that the inertia parameter has a tremendous effect on the higher eigenfrequencies and hence the fundamental mode of vibration should be controlled in order to achieve more accurate positioning of a heavy payload. The eigenfrequencies show a sudden jump to higher mode of vibration at unit magnitude of joint and actuator frequency parameter.
- The flexural rigidity ratio, beam mass density ratio, and joint inertia significantly influence the amplitude of lower modes of vibration and the amplitude of higher modes of vibration generally remains invariant with the change in system parameters. The unit magnitude of the joint and actuator frequency parameters represents the limiting condition, when the joint and actuator dynamics gets decoupled from the manipulator and, system starts vibrating at a higher mode of vibration and the joints behave like point masses.
- Multivalued solutions and jump phenomenon due to geometric nonlinearities and the existence of saddle node and pitchfork bifurcations are noticed. The analytical results obtained from the method of multiple scales have been found in good agreement with those obtained by numerically solving the governing equation of motion of links.
- Numerically, it is established that internal resonance exists between both the links and hence, primary, subharmonic resonance in both links and internal resonance in second link are investigated for the parametric variation of system attributes. 3:1 internal resonance occurs between the links of the manipulator while considering the joints and the payload as point mass and it has been found that as joint dynamics is included in the modeling, the links are in 1:1 internal resonance condition.
- The internally resonated second link exhibit only spring softening behavior for the variation of system parameters and the amplitude as well as the jump length increases with increasing excitation amplitude of the first link. The cubic nonlinearity coefficient associated with second link tends to decrease the peak amplitude of steady-state response of the system.
- For the two-link manipulator driven by harmonic revolute joints, an adverse phenomena of behavior change from spring hardening to spring softening or vice-versa is noticed for the variation of certain system parameters such as payload mass, beam mass density ratio, joint inertia and joint frequency parameters and this behavior alteration is prominent in case of first link.
- In case of two-link manipulator driven by harmonic prismatic and revolute joints, the system parameters, in general, have negligible influence on the frequency response curves of the first link, except for the actuator and joint frequency parameters where the behavior of both links change from the spring softening to spring hardening or vice versa. The peak amplitude and the jump length of the response curve of the second link increase with the increase in the system mass, beam mass density ratio, and flexural rigidity ratio, while the increase in the hub inertia parameters stabilizes the second link by reducing the unstable region and peak amplitude.
- Unlike the primary resonance case, in the case of sub-harmonic resonance condition arising when the end-effector is being exerted by the pulsating constraint axial force, the system experiences the pitchfork bifurcations due to parametric excitation terms. It is observed that in absence of the joint dynamics, the links exhibits softening type behavior due to the axial nonlinearities leading to multiple solutions and jump phenomenon.
- It is noticed that the inclusion of generic payload significantly increases the unstable region of the links and the peak amplitude decreases as compared to the no payload condition. The larger values of beam mass density ratio and the flexural rigidity ratio increase the unstable region in frequency response curve of the second link. The results

thus obtained shall be useful in the vibration attenuation of the flexible manipulator gripping a generic payload and working under the environment where the end-effector is subjected to pulsating force such as spraying, painting, grinding etc.

- From the results obtained for the single and two-link manipulator driven with prismatic joint , it is observed that the paylaod has marginal influence on the vibration amplitude for the case of primary resonance arising when the frequency of the actuator motion becomes equal or nearly equal to the normalized link frequency. However, for the case of primary and secondary resonance, respectively due to harmonic rotary joint motion and externally applied axial force, the consideration of payload changes the behavior of manipulator spring softening to spring hardening. Moreover, in case of manipulators where any kind of joint motions are not considered, and are internally or externally excited, no behavior changes are noticed but vibration amplitude increases with the change in payload mass. Hence, the payload has a significant effect on the vibration amplitude of the manipulator depending upon the type of motion and external excitations being input to the system.
- The payload mass, length of the second link and joint inertias decreases the angular tip positions for the same amount of torque at the joints. The variation of payload parameters such as payload inertia and offset length has a negligible effect on the angular tip position of the first link. However, the angular tip position of the second link decreases with the increase in payload parameters.
- The links experience large residual vibrations with the increase in joint inertias and the angular response along with the tip accelerations significantly increase for the manipulator with links made of aluminium as compared to steel or cast iron.
- The control problem for trajectory tracking described by a nonlinear model using inverse dynamics in conjunction with PD controller is addressed. The increased amount of torques is required at the joints of the manipulator as the payload capacity, joint inertia and length of the links is increased which in turn increases the power consumption by the manipulator.
- Increased end point residual vibrations are observed with the increase in joint inertias and length of the links which decreases the accuracy of the manipulator while reaching the desired set position. The required input torques and position accuracy decrease with the change in material of the links from steel to aluminium.

6.1.3 Multi-link manipulator

Here, the mathematical dynamic modeling of generalized multi-link manipulator with harmonic prismatic and prismatic and revolute joints mounted on a bi-directional moving base with subsequent modal analysis, nonlinear investigation and dynamic characterization has been accomplished. The present dynamic model can be reduced to 3R or 3PRR manipulator respectively by either neglecting or considering the actuator motion.

- The beam mass density ratio and flexural rigidity ratio of the second link and third link with respect to the first link significantly influence the higher mode shapes. This contrasts with the two-link manipulator where the mode shapes tend to clutter together for higher values of beam mass density and flexural rigidity ratio. The joint inertia parameters primarily affect the lower modes of vibrations and the system tends to vibrate at higher modes of vibrations for certain values of joint frequency parameters.
- The addition of payload mass decreases the angular tip position and settling time of the links. The influence of the payload becomes dominant on the terminal link and the tip accelerations decrease with the increase in payload mass due to the addition of inertia. The joint masses primarily affect the angular tip positions of the preceding links due to their role in the boundary conditions. The increase in joint inertia marginally affects the angular tip position of the first and second links. However, the tip of the all the links

rigidly vibrate and residual vibrations are visible which may lead to in accurate positioning of the respective links.

- The manipulator links exhibit saddle-node bifurcations for the internal resonance existing between the links due to inertial coupling and primary resonance induced in the system because of harmonic motions being imparted to the joints. The system exhibits the multiple solutions and jump phenomena due to the existence of cubic geometric nonlinearities arising because of axial stretching.
- For 1:1:1 internal resonance between the links of 3R manipulator, and as the excitation amplitude is increased, the jump up phenomena in the second and third links occurs at a much lower frequency. The peak amplitude of the second link decreases and third link increases with increase in payload mass and second beam mass density ratio. The jump length and the stability region increases for the second link and decreases for the third link with increase in flexural rigidity ratios, and first beam mass density ratio. No behavior change is noticed for any system parameter variations.
- In the case of primary resonance due to imparted harmonic joint motions, only the first link demonstrates a behavior change from spring hardening to spring softening or vice-versa with the change in payload mass, first beam mass density ratio, joint inertias, and first joint frequency parameter. For other the parameters, while first link exhibit spring hardening behavior, the second and third links exhibit spring softening behavior. All the links display this adverse behavior for the change in the third joint frequency parameter.
- Based on the present work, the critical amplitude and frequency of the joint motions depicted in the frequency response curves under different operating conditions can be determined for vibration control of the multi-link manipulator.
- The manipulators involved in long reach applications in various industrial operations can be analyzed based on present modeling to provide satisfactory performance and safety.

6.2 RECOMMENDATIONS FOR FUTURE WORKS

In present work, the modal analysis, nonlinear dynamics and vibration characteristics of different manipulator models having revolute and prismatic motions incorporating generic payload subjected to bi-directional base movement have been accomplished. The links are modeled as Euler-Bernoulli beam element, payload of sizeable mass with inertia, offset and orientation. The joints and actuators have been treated as appropriate spring-mass or inertia elements. For the enhancement of the present model, various other considerations can be incorporated such as:

- The manipulators used in industries are driven by various joint combinations in-order to perform the required tasks. Here, we have considered few such combinations and therefore the present study can be extended to estimate the modal parameters and perform the nonlinear analysis of other manipulators for example, Cartesian manipulator with both prismatic joints and manipulator with first joint as revolute and second joint as prismatic.
- Here, the links of the manipulator has been assumed as a slender beam and hence, Euler-Bernoulli beam theory is used. In future, the shear deformation and rotary inertia by modeling the links as Timoshenko beam elements or considering Reyleigh beam theory for more realistic model can be considered to develop the modal parameters.
- Practically, in industries, the robotic manipulators are used in various applications undergoing three dimensional motion such as in space industries spatial manipulators are employed. In present study we have considered the planar motion of manipulators, and hence, the three dimensional motion of the manipulator with or without generic payload can further be analysed.

- In nonlinear analysis, only the fundamental eigenfrequency has been considered for the discretization of the governing equations. The multi-modal investigations may be implemented to illustrate the influence of system attributes when the manipulator performs at higher frequencies.
- The combination of resonances other than primary and sub-harmonic can also be investigated for small as well as large deflection model of the multi-link manipulators.
- The joints can be imparted with multi-frequency torques to determine the nonlinear behaviors of the multi-link manipulators.
- In present study, only the flexibility is considered in the joints while modelling the manipulator by considering linear spring-mass system. While, it is known that the joints may behave nonlinearly which indicates the consideration of linear spring-mass system is not sufficient. Hence, the joints can be modelled as nonlinear spring and having material damping.
- In modern times, various kinds of materials have been used to manifacture the robotic manipulators due to their various advantages over traditional materials. Viscoelastic and magnetoelastic material can be used for the links which provide an advantage of passive vibration control.
- Micro and nano scale analysis can be performed for the multi-link manipulator for their applications in precision industries.
- In case of multi-link manipulator, the analyses for 3R manipulator and dynamic model of 3PRR manipulator have been demonstrated. The present methodology can be extended to investigate the vibrational characteristics of 3PRR, 3PPR, 3RPR, 3PRR and 3P manipulator
- The active control of the multi-link manipulator can be achieved by using piezoelectric material and thus the vibration characteristics and nonlinear behavior can be further studied.
- The experiments and model based simulations should be performed to validate the analytical and numerical results.