MODAL ANALYSIS AND NONLINEAR DYNAMICS OF MULTI-LINK FLEXIBLE MANIPULATOR WITH GENERIC PAYLOAD MOUNTED ON A MOVING BASE

A Thesis submitted by

Pravesh Kumar

in partial fulfillment of the requirements for the award of the degree of

Doctor of Philosophy

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Certificate

It is certified that the work contained in this thesis entitled **"Modal analysis and nonlinear dynamics of multi-link flexible manipulator with generic payload mounted on a moving base"** submitted by **Mr. Pravesh Kumar** to the Indian Institute of Technology Jodhpur for the award of the degree of Philosophy has been carried out under my supervision in the Department of Mechanical Engineering, Indian Institute of Technology Jodhpur. This work has not been submitted elsewhere for the award of any other degree or diploma.

The thesis, in my opinion, has reached the standard fulfilling the requirements for the award of the degree of Doctor of Philosophy in accordance with the regulations of the Institute.

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Declaration

I hereby declare that the work presented in this thesis titled *"Modal Analysis and Nonlinear Dynamics of Multi-Link Flexible Manipulator with Generic Paylaod Mounted on a Moving Base"*, submitted to Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Barun Pratiher. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to any other Institute or University in India or abroad for the award of any degree or diploma.

Pravesh Kumar
P15ME003

To my parents Late Shri G. Laxmanan Nair and Smt. Yashodha Devi, for instilling in me the belief that I could do this.

To my lovely wife Santoshi Goswami, whom I admire and love. I would like to thank you for your support, comprehension and love.

To my sweet brother Santosh Nair and grandmother who have been there to support me all through my life.

To my father who sacrificed a lot of his needs for my dreams.

Abstract

In the present work, the detailed mathematical modeling, modal analysis, nonlinear characteristics and trajectory analysis and response of multi-link flexible manipulators of various configurations are addressed. The flexible and light weight manipulators have many advantages over the conventional rigid and heavy manipulators such as lower power consumption, higher speed, higher payload to manipulator weight capacity, smaller actuators for manipulator movement, larger work space, and lower inertia. In case of flexible manipulators, the control objective is to achieve the desired end-effector trajectory and vibration attenuation of the end-effector. The control design of such manipulator system necessitates the accurate dynamic modeling to achieve the desired performance requirements. The multi-link manipulator are preferred over the single-link manipulator in various applications involved in production and manufacturing industries, pick and drop tasks, space explorations, nuclear power plant, medical operations, and so on due to their large work space, better maneuverability and safer applications. From the available literature it is evident that the single-link manipulator has been extensively studied but in case of multi-link manipulator the researchers have confined their attention towards the control and vibration suppression of manipulator to achieve the desired motion. Also, the robotic manipulators engaged in various operations such as grinding, drilling, painting, spraying etc where the manipulator is subjected to external forces when the tool attached to the end-effector comes in contact with the working surface. It is apparent from the literature review that there have been trivial studies regarding the multi-link manipulator subjected to external excitations. Hence, in this work, an attempt has been made to study the flexible multi-link manipulator and to develop some control strategy to attenuate the unwanted vibrations.

In this thesis, a systematic dynamic modeling of flexible manipulator with single, two and multi-link using Hamilton’s principle has been presented. The links have been modeled as Euler-Bernoulli beam element with payload at the distal end of the link. The cross-section of the links remain perperndicular to the bending axis and hence the rotary and shear deformation due to rotation of corss-section has been neglected. The payload is assumed to be either a point mass or an arbitrarily oriented sizeable mass having inertia and its centre of gravity is different than the point of attachment with the terminal link. The harmonic rotary and translatory motions have been imparted to the links of the manipulator is through the revolute and prismatic joints, respectively. The flexibilities have been induced in the revolute and prismatic joints by respectively, modeling them as torsional spring-inertia and linear spring-mass system. The coupled nonlinear governing equations and the boundary conditions are obtained by using extended Hamilton’s principle with subsequent modal analysis to determine the eigenfrequencies and eigenspectrums of the system. The modal parameters thus developed are further used in the nonlinear analysis and trajectory analysis of the manipulator system. The influence of nondimensional system parameters such as payload mass, payload inertia, offset ratio, offset angle, beam mass density ratio, flexural rigidity ratio, actuator mass and joint frequency parameter on the system eigenfrequencies and eigenspectrums have been tabulated and graphically presented.

The nonlinear analysis is very imperative to determine the behaviors of a system under external and/or parametric excitation as the system exhibits inadmissible vibrations when the forcing frequency becomes equal or nearly equal to the system’s natural frequency. Hence, it is the prime requirement for the design engineer to inhibit such vibrations by operating the system in safe zones or by manipulating the system parameters accordingly. Now, the governing equations of the links are discretized using the Galerkin’s method along with the mode shapes of the system to obtain the nonlinear temporal differential governing equations of the motion of the links which are further nondimensionalized with respect to the system parameters. The governing temporal equations of motion of the links contain many nonlinear terms including the cubic nonlinearities due to axial stretching, nonlinear terms due to inertial coupling, damping terms, force and parametric excitation terms. The forcing and parametric excitation terms respectively arise due to the directly applied force at the end-effector and the harmonic revolute or cartesian motion of the manipulator. The closed form solution of the complex nonlinear temporal equations of motions is sought by using method of multiple scales as one of the perturbation techniques. The governing equations of motions are reduced to the autonomous set of first order equations in terms of the amplitude and the phase and finally the frequency response equations of the respective links are obtained for steady-state conditions. The effect of essential system parameters on the stability of steady-state solutions and bifurcation diagrams have been demonstrated through graphical illustrations. The system behavior under forced resonance, parametric resonance, internal resonance and combined resonance conditions have been studied using time response, phase portraits, FFTs and frequency response curves.

Further, to analyze the system parameters on the system responses such as angular tip positions, modal deflections and tip accelerations, numerical simulations have been carried out. Here, the joints of the manipulator are given a smooth sinusoidal torque as an input and system responses have been plotted. The significant influence of system parameters on the vibration characteristics provides a comparative analysis of performance and accuracy of manipulator. Finally, to control the trajectory tracking of flexible manipulator, a model based controller using stable inversion technique in conjunction with proportional-derivative controller is developed to examine the effect of system attributes on the control parameters.

Initially, the single-link and two-link manipulator with harmonically varying revolute or prismatic motion incorporating a generic payload with constraint pulsating axial force has been dynamically modeled with subsequent investigation of free vibration analysis. The influence of the payload and joint variables on the eigen-parameters has been investigated. The joints have been provided with harmonic motion causing the jump phenomenon and multiple steady-state solutions due to the primary and secondary resonance phenomena which may cause the catastrophic failure of the system. Further, due to the increased application of long reach manipulator in various industrial applications, the investigation of multi-link manipulator has been pursued. Five different models of two-link manipulator have been taken into considerations. Initially, the basic model of a two-link manipulator with the links connected with motors and payload designed as point mass is considered. In second model, the end-effector has been modeled as a generic payload with mass, inertia, offset and orientation and a harmonic axial force imitating its working environment is exerted. Further, in order to accomplish a realistic model of two-link manipulator, the prismatic and revolute motions are considered and the influence of their motions on the vibrational behavior of the system is demonstrated graphically. Forward and inverse dynamics with proportional-derivative controller of two-link manipulator with revolute pairs is explored to understand the consequence of parametric variation of system attributes on the system responses such as angular tip positions, modal deflections, control torques, and link accelerations. Finally, the mathematical modeling of a multi-link manipulator with bi-directional base motion which can incorporate any number of links and joints has been presented. The modal parameters along with the nonlinear behaviors and vibration characteristics have been thoroughly investigated. The results obtained in this thesis will alleviate the better design and manufacturing of flexible manipulators involved in various industrial applications and facilitate the efficient active/passive control or suppression of end point vibrations of manipulators engaged in precession operations. The analysis presented in the thesis can be extended to robotic manipulators with different configurations and joint dynamics involved in pick and drop operations, long reach manipulators where human interaction is not possible, manipulator involved in medical and nuclear industries

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Pravesh Kumar
IIT Jodhpur, 2020.

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List of Symbols

Symbol	*Description*
α_m	Mass parameter.
α_{mC}	Payload mass parameter.
α_{ma}	Actuator mass parameter.
α_M	Beam mass density ratio.
α_L	Link length parameter.
α_{Ih}	Joint inertia parameter.
χ	Flexural rigidity ratio.
α_c	Offset ratio.
α_{Ic}	Payload inertia parameter.
α_{ah}	Actuator inertia parameter.
γ	Offset angle.
E	Young's modulus of material of link.
I	Moment of inertia of link.
I_C	Mass moment of inertia of generic paylaod.
k_a	Stiffness of the Cartesian joint.
k_θ	Stiffness of the revolute joint.
ρ	Mass density of the material of link.
A	Area of cross-section of the link.
m_C	Mass of generic paylaod.
m_a	Mass of actuator.
x	Abscissa of general point on the undeformed link.
y	Ordinate of general point on the undeformed link.
(X,Y)	Inertial co-ordinate system.
$\vartheta(t)$	Horizontal base motion.
$\eta(t)$	Vertical base motion.
δ	Eigenfrequency of the manipulator.
ω_m	Natural frequency of manipulator.
n	n^{th} mode of vibration.
θ	Angular motion of joint.
u	Axial deformation of link.
w	Transverse deformation of the link.
I_h	Mass moment of inertia of the revolute joint.
I_a	Mass moment of inertia of actuator.
Ω_a	Frequency parameter of actuator.
Ω_h	Frequency parameter of the revolute joint.

List of Abbreviations

Abbreviation	*Full form*
cc	*Complex Conjugate*
PD	*Proportional Derivative*
PID	*Proportional Integral Derivative*
MMS	*Method of Multiple Scales*
FFT	*Fast Fourier Transform*
IED	*Improvised Explosive Device*
FFT	*Fast Fourier Transform*