2

Literature Review

2.1 INTRODUCTION

A comprehensive survey of the literature on the single-link, two-link, and multi-link flexible manipulators has been presented in this chapter. The emphasis is on the modeling methods, eigenanalysis, forced vibrations and control techniques adopted in the study of robotic manipulators. The manipulators can be assumed to have different configurations depending on the boundary conditions at both ends of the link of manipulators. The payload representing the end-effector of the manipulator can be modeled either as a point mass or an arbitrarily oriented mass having inertia and offset. The other end of the link is given either harmonic revolute motion or prismatic motion. Further, the end-effector is subjected to harmonically varying axial force imitating the working environment of the manipulator. The exact modeling and eigenanalysis of the robotic manipulator is essential for the appropriate vibration attenuation under forced/parametric excitations and control to achieve the desired trajectory. Firstly, the reviewed literature is categorized in three major sections based on dynamic formulations, eigenanalysis, forced vibrations and control techniques. Further, these sections are divided into three subsections based on the number of links i.e. single-link, two-link and multi-link manipulator.

2.2 MODELING

In this section, the literature regarding the techniques adopted for modeling the flexible manipulators in the existing works has been reported. The modeling of the any vibrating system can be accomplished by various methods such as Lagrange's principle, extended Hamilton's principle, Newton' method or lumped mass parameter system. The appropriate modeling of the manipulator incorporating the joint as well as the payload dynamics is indispensible perquisite for calculating the modal parameters and further analyzing the system for its nonlinear behaviors. Also, the number of links, payload and joint motions lead to great complexities in the dynamic modeling of flexible manipulator. Therefore, recent advances and a brief literature review regarding the dynamics and modeling of multi-link manipulators have been discussed in the following section.

2.2.1 Single-Link Manipulator

[Al-Bedoor et al., 2002] formulated the equations of motion for an inextensible beam, rotating around its hub centre and also moving against its flexible base by applying Euler-Lagrange's equations. Assumed mode method has been used to discretize to obtain the temporal equations of motion in terms of nondimensional terms which are further simulated to explore the effects of base flexibility on the dynamics of rotating flexible arms. [Bai et al., 2008] presented a dynamic model of a flexible hub beam system with tip mass using assumed mode method for discretization of governing equations of motion. The higher-order dynamic influence on the response of a single-link flexible manipulator subjected to rotational and translational motion has been investigated and simulated by [Basher, 2007]. Assumed mode method (AAM) with infinite number of modes has been used to develop the analytical model of the manipulator. [Dwivedy and Eberhard, 2006] presented a survey of literature available between 1974 and 2005 related to the dynamic analysis of flexible robotic manipulators. The classification of papers has been carried out according to the modeling, control and experimental studies. [Huang et al., 1999] derived the equations of motion for a cantilever beam attached to a translational/rotational base which included the velocity and acceleration of the translational motion of the base. The instability regions of solutions have been determined by using perturbation method after discretizing the governing equation of motions using Galerkin method. In [Shawky et al., 2013], the modeling and control of tip position of a single-link manipulator using Lagrange mechanics and assumed mode method has been presented.

[Emam, 2007, 2010] derived the equations of motion and boundary conditions of a nonlinear beam attached to the rotating hub on one end and carrying a tip mass at the free end using Hamilton's principle. [Khairudin, (2008)] presented the mathematical modeling of a single-link manipulator system based on Lagrangian approach in conjunction with the assumed mode method. The link has been considered as an Euler-Bernoulli beam and payload along with the actuator having inertia are included. The governing equation of motion of the link and joint dynamics of a single-link flexible manipulator having point payload has been derived by [Mishra et al., 2015] using Lagrange principle for small deformation model. The influence of payload mass, inertia and joint inertia on the system response has been demonstrated. The modeling of flexible single-link system considering tip mass and rotating hub having mass as well as inertia has been modeled by [Parks and Pak, 1991]. The influence of the payload and hub on the poles and zeros of the manipulator dynamics have been extensively studied graphically. [Piedboeuf and Moore, 2002] presented six different methods including Lagrange method, Hamilton's principle and Newton-Euler method for modeling an Euler-Bernoulli beam rotating in vertical plane. The payload has been considered as a point load having mass and inertia. [Saad et al., 2012] presented the mathematical model of a flexible link system using the Lagrange equations by taking into account of foreshortening effect. The linear and nonlinear kinematics has also been compared for small and large amplitudes. The nonlinear governing equation of motion of a flexible beam with prismatic joint is derived by [Tadikonda and Baruh, 1992] using Lagrange's equations. The system is designed to deploy a payload from a fixed base. [Xiao et al., 2007] established a nonlinear dynamic model of the system consisting of Euler-Bernoulli cantilever beam with tip point mass in a centrifugal field using Hamilton's principle. [Yigit et al., 1988] studied the flexural motion of a radially rotating flexible beam fixed to a rigid body. Extended Hamilton principle has been utilized to obtain the coupled governing equations of the rigid and elastic body motion.

[Cai et al., 2005] presented the dynamic modeling of flexible hub-beam with tip mass based on Hamilton principle and finite element discretization method and studied its dynamic characteristics. [Gunjal and Dixit, 2007] carried out the shape optimization of rotating beams. They studied the vibration analysis of the optimized beam considering the beam as Euler-Bernoulli beam based on finite element method. The tip mass is considered as a point mass attached at the free end of the beam. [Gurgoze and Zeren, 2011] studied the out of plane vibrations and eigenfrequencies of a rotating, internally damped beam carrying a tip mass using Frobenius method. The eigencharacteristics have been compared with the results of a conventional finite element modeling. The tip mass also possessed mass moment of inertia and has a centroid offset from the free end of the beam located along its extended axis. The simulation results considering two flexible single-arm manipulators with revolute and prismatic joint carrying payload have been presented by [Lai, 1994] using nonlinear finite element method. The dynamic influences of high speed and high payload on the gyroscopic inertia and geometric stiffening have been demonstrated in this work. [Naganathan and Soni, 1987] nonlinearly modeled the flexible link of robot manipulator using finite-element method considering Timoshenko beam theory i.e. including the effects of shear deformationa and rotary inertia. It is concluded that the displacement errors are significantly influenced by the nonlinear interactions. [Tohki eta al., 2001] and [Chung and Yoo, 2002] modeled the single-link manipulators using the finite element method. The dynamic characteristics of the manipulator such as tip displacement, velocity and settling time has been found theoretically and experimentally by [Tohki eta al., 2001]. The dynamic models of a single-link manipulator using finite element approach have been developed by [Martins et al., 2002]. The modal frequencies thus obtained have been compared with the experimental findings to validate the finite element modeling.

[Cao and Li, 2008] obtained the distributed parameter model to study the time varying constrained flexible manipulator system. The rigid motion is described by lumped parameter subsystem. The study of vibrations by virtue of singular perturbation method has been expressed through distributed parameter subsystem. [Ding and Selig, 2004] proposed a method to improve the efficiency of dynamic modeling based on Holzer method. [Esmailzadeh and Jazar, 1998] investigated a massless cantilever beam with a lumped mass attached to its free end while being excited harmonically at the base. [Kim, 2017] modeled a cantilever beam with a point tip mass as an equivalent spring-mass system using Rayleigh-Ritz approach. The influence of variation of ratio of tip mass to the beam mass on the values of the equivalent mass, stiffness, and angular natural frequencies has been demonstrated tabularly. [Nissing, 2000] demonstrated the vibration damping in a flexible single-link manipulator using a spring and damper attachment.

2.2.2 Two-Link Manipulator

[Abe, 2009] modeled the two-link manipulator with first link as rigid member and considering second link as flexible beam and joint as a point mass. The assumed mode method has been adopted to investigate the dynamics of the system and using the eigenfunction of a non rotating beam with payload. [Ding and Shen, 2017] demonstrated that assumed mode method have a good accuracy in predicting the displacement of the endpoint of the two-link manipulator by comparing the results obtained by assumed mode method and absolute nodal coordinate formulation. It has been concluded that the AMM is inefficient for analyzing the strength of robotic manipulator. [Luca, 1991] modelled the two-link flexible manipulator using Lagrange's equations and derived the boundary conditions considering only mass of the system by balancing the moments and shear forces. The dynamic equations of motion of a two-link flexible system representing two panels of solar array have been obtained using the Lagrangian and finite-element method by [Nagaraj et al, 1997]. The links undergo the locking which is modeled using momentum balance method and the vibrations induced by the locking in the system are studied theoretically and experimentally. [Oakley and Cannon Jr., 1989] developed the dynamic equations for a general two-link manipulator including geometric offsets and the modal parameters obtained using assumed mode method, lumped spring-mass model, and experimental findings have been compared. The dynamic modeling of two-link flexible manipulator based on finite-element/Lagrangian formulations is presented by [Usoro et al, 1986]. The tip responses of manipulator are reported and it is concluded that for the practicle purpose more efficient control strategies have to be developed in order to control the vibration modes. [Zhang et. al., 2004] compared the ODE and PDE modeling approaches for two-link manipulator and concluded that the PDE approach is theoretically rigorous with guaranteed stability properties. Independent boundary conditions along with the assumed mode method have been used to compare the ODE and experimental model with the experimental data of a flexible two-link manipulator. The nonlinear equations of motion have been developed for flexible manipulator arms consisting of rotary joints between the links by [Zhang, 2009]. Matrix representation for joint and deflection motion has been used and deflection transformations have been expressed in terms of summation of mode shapes with further comparison of presented formulation with the earlier reported cases of rigid link manipulator.

[Kalyoncu, 2008] studied the dynamics of a robot manipulator with a flexible arm sliding in a rotating-prismatic joint with a tip mass tracing a straight line through prescribed points. The Lagrangian principle has been used to obtain the governing equations of motion considering the rigid prismatic joint taking into account of rotary inertia, axial shortening and gravitation. [Khorrami, 1989] modeled the two-link flexible manipulator and derived the nonlinear governing equations of motion and boundary condtions. The asymptotic perturbation method is used to demonstrate the influence of rigid body motion on the flexture. The geometrically nonlinear differential equations of motions of interconnected rigid and flexible components undergoing large rigid body motions as well as elastic deflections has been derived using

Lagrange's equations by [Bakr and Shabana, 1986]. [Low, 1987] analytically formulated the equations of motion of mechanical manipulators with the elastic links using Hamilton principle. Hamilton's principle is utilized for the dynamic modeling of a rigid-flexible robotic manipulator with point paylaod by [Hu and Ulsoy, 1994]. Geometrically second-order beam theory is used to model the flexible link considering the axial shortening and investigated the effect of contact force on the bending vibrations. [Pedro and Tshabalala, 2015] modeled a two-link flexible manipulator using Lagrangian formulation and then actuator dynamics have been included to improve its real time control simulations. [Zhang and Liu, 2013] provided a brief modeling using extended Hamilton's principle and studied the boundary control for a flexible two-link manipulator with a changeable payload at free end.

The modeling of a manipulator carrying no payload having two flexible links and rigid joints using Hamilton's principle and finite element approach has been carried out by [Abedi et al. 2008]. The numerical solutions are performed to verify the validity of the obtained model. [Bayo et al., 1989] modelled the links of the manipulator as Timoshenko beam and finite element method has been used to discretize the equations of motions. [Castri and Messina, 2010] compared the analytical modal formulations for a two-link manipulator with the results obtained from finite element models. [Karagulle et. al., 2015] simulated the end point vibration signals by developing a Matlab code based on the finite element theory and Newmark solution for a two-link manipulator with flexible members. [Mayo et al., 1995] presented the nonlinear modeling of flexible multi-body system considering nonlinearities due to geometric elasticity, and geometric stiffening due to ceintrifugal foces. Flexible slider-crank mechanim is taken as an example and the responses of the system dueing four cycles of crank isresponses of the system dueing four cycles of crank are reported. [Sayahkarajy et al., 2017] exploited the Lagrangian and finite element discretization to study the vibration modes of a two-link manipulator with a rigid upper arm and a flexible forearm by employing Euler-Bernoulli beam model and lumped mass model. The links have been modeled based on Euler-Bernoulli beam theory, while the finite element model has been used to analyze the two-link manipulator by [Sayahkarajy, 2018]. The nonlinear modeling of single and two-link flexible manipulator using finite-element techniques is presented by [Shaker and Ghosal, 2006]. Nonlinear strain-displacement relations are used to derive the governing equations of motion of the system. The effect of axial and flexural stiffness on the tip responses and natural frequencies haveeffect of axial and flexural stiffness on the tip responses and natural frequencies has been graphically demonstrated. [Zebin, 2012] presented the theoretical modeling and characterization of a constrained two-link manipulator without payload using finite element method.

The possibilities of physical modeling of a simple rigid model of two-link manipulator in Matlab/SimMechanics have been explored by [Frankovsky et al. 2012]. The inverse dynamics to identify the manipulator movement under applied forces/moments and forward dynamics to calculate the forces causing a prescribed motion of the end-effector has been investigated. [Fung and Chang, 1998] derived equations of motions and corresponding boundary conditions of nonlinearly constrained two-link flexible manipulator with a tip mass for four flexible dynamic models i.e., Timoshenko, Euler, simple flexure and rigid body beam theories. Different control methods for endpoint tracking by a two-link robot manipulator neglecting payload inertia has been presented by [Green and Sasiadek, 2004] using the rigid dynamic model to obtain the system response and to ascertain the most suitable control scheme to further study the flexible dynamic model. [Low and Vidyasagar, 1998] derived the equations of motion for rigid and flexible links robot manipulators using Hamilton's principle resulting in nonlinear integro-differential equations. Also, performance of two-link manipulator considering one rigid and other flexible link has been studied. The influence of point payload on the relative motion in a two-link manipulator considering first link as rigid and second link as flexible, has been demonstrated by [Choura and Yigit, 2001].

2.2.3 Multi-Link Manipulator

[Book, 1884] developed the nonlinear dynamic equations of motion of flexible manipulator whose links are connected by rotary joints using Lagrangian formulation. The results have been compared with those of rigid manipulator. A linearized dynamic model of a general planar multi-link manipulator has been presented by [Chen, 2001]. While, the assumed mode method and Lagrange approach have been used to dynamically model the system, the established model has been verified by numerical simulations carried out for two-link manipulator. Modeling of N-flexible-link manipulator with revolute-prismatic joints is presented for the co-operative flexible multi mobile manipulator by [Korayem and Dehkordi, 2019] using assumed mode method to define the deformation of the flexible links. The links are assumed to undergo simultaneous rotary motion through joints and reciprocating motion through a rigid massless sleeve. An investigation of dynamics of multi-link spatial flexible manipulator using Lagrange method has been carried out by [Chen et al., 2012]. The links of the manipulator are driven by DC motor through the revolute flexible joints. The flexibility of the joint is modeled by linearly elastic torsional spring while the mass of the joint has also been taken into consideration. It has been proposed that the stiffening effects and flexible joints will have huge impact on the dynamic performance of the manipulators. Modeling and simulations of a multi-link planar flexible manipulator with only rotary joints in a vector matrix form has been performed by [Hwang et al., 1997]. Homogenous transformation matrices are used to describe the link deformation and the rotary joint motion. Lagrange-Euler equation along with the assumed mode method incorporating the mode shapes of clamped-clamped beam is used to formulate the dynamic equations of motion. [Benati and Morro, 1994] determined the dynamic equations of a chain of flexible links by exploiting Hamilton’s principle.

The dynamic modeling of planar multi-link flexible manipulators based on finite element formulations has been presented by [Farid and Cleghorn, 2014]. Euler-Bernoulli beam element approximating the curvature distributions by using its nodal curvatures has been utilized to model the links. The effect of link flexibilities and joint angles on the end deflections has been demonstrated graphically. [Chiou and Shahinpoor, 1990] used the assumed mode method to study the dynamics of multi-link flexible manipulator and compared the results with the corresponding rigid model. [Choudhury and Genin, 1989] developed a general formulation of kinematic model for N-degree of freedom multi-link manipulator system considering rigid body dynamics. The inverse kinematics problem illustrating a 16 degree of freedom redundant system is solved by minimizing an objective function that quantifies the joint displacement. The feasibility of the presented model is demonstrated by simulating both rigid and flexible models. [Giorgio and Vescovo, 2018] investigated the vibration control of a highly flexible planar multi-link robot arms considering each link as kinematic chain of rigid bodies and the bending resistance has been modeled as appropriate springs. [Na and Kim, 2006] analyzed the deployment of a multi-link beam structure undergoing locking and investigated the influence of slenderness ratio and shear on the response of the system. The rigid body model and Timoshenko beam model has been simulated and it has been concluded that the Timoshenko model follow the experimental data more approximately. The investigation of the joint angles and angular velocities, tip displacement, and velocity of each link to study the motions of the links at each time step has also been accomplished. [Subrahmanyan and Seshu, 1997] presented the finite element dynamic modeling of a planar five bar flexible manipulator. [Theodore and Ghosal, 1995] presented and comparised the dynamic modeling of flexible multilink manipulator by assumed modes and finite element method. It is concluded that the ciomputational time required for the finite element model is lesser as compared to the assumed mode method. The dynamic responses of the manipulator system isresponses of the manipulator system are also presented based on mode-based decoupling control law. A new dynamic modeling for planar flexible multi-link manipulator with rigid joints carrying a tip mass including the foreshortening due to bending and link material damping has been established by [Zhang, 2011]. The Lagrange’s approach has been used to obtain the governing equations of motion by expressing the kinetic and potential energies in terms of generalized coordinate system incorporating the gravitational forces.

2.3 FREE VIBRATION

The flexible manipulator undergoes some form of dynamic loading while performing specific tasks in various industrial applications which may cause unnecessary vibrations in the system. Moreover, the system may experience inadmissible vibrations when the frequency of the forces acting on the manipulator becomes nearly equal to the system's natural frequency. Hence, it is imperative to perform the free vibration analysis to identify the modal parameters and avoiding any critical failure of the system from large vibrations. The literature on the free vibration analysis of single-link manipulators is reviewed in next sub-section.

2.3.1 Single-Link Manipulator

[Anderson, 1978] considered a beam in which the centroid of the tip mass does not coincide with its point of attachment to beam and is located at an arbitrary distance perpendicular to the extended neutral axis of the beam. The coupled equations of motion for longitudinal and transverse deflections have been solved to obtain the natural frequencies of the system. [Auciello, 1996] presented an exact analysis of free vibration of flexibly constraint tapered cantilever beam with mass at the tip. The concentrated mass with rotary inertia having eccentricity has been incorporated in the model. [Bhat and Wagner, 1976] considered a cantilever beam with tip mass slender in axial direction and treated it in a more general manner. The frequency equation thus obtained for the problem has been solved by using perturbation procedure. Various values of the eigenfrequencies have also been tabulated for various mass ratios for the sake of comparison. [Coleman, 1998] analyzed the vibration eigenfrequency of a flexible slewing beam with a payload attached at one end using wave propagation method (WPM). The results showed that the higher frequencies are asymptotically identical to those obtained for the clamped free beam condition independent of the payload. [Coleman and McSweeney, 2004] discussed the Cartesian manipulator considering payload and actuator as the point mass, along with various modal behaviors of the manipulator arm. Results thus obtained by the exact equation are validated by those obtained by employing wave method and subsequently the behavior of the eigenfunctions has been investigated when the mass of payload and actuator is varied from zero to infinity. The natural frequencies of the single-link robotic manipulator of circular cross-section have been calculated using the finite element method by [Dixit et al., 2006]. The authors have considered two different models of the manipulator having prismatic and revolute joints with point mass payload.

Nonlinear equation of motion for a clamped-free flexible Euler Bernoulli beam rotating in a horizontal plane carrying a moving mass has been derived by [Fung and Yau, 2001]. Power series are used to solve the differential equations while frequency equation has also been derived in terms of nondimensional parameters. [Kojima, 1986] obtained the transient response of a beam-mass system fixed to a rotating body. The effect of flexibility of beam and rotational speed of body on flexural vibrations has also been investigated. The effect of variation of speed of spin, hub radius and aspect ratio of the beam on natural frequency has been studied thoroughly. The largely deformed free vibration analysis of an axially functionally graded beam of a tapered profile has been presented by [Kumar, Mitra et al. (2015)]. The unknown static displacements have been calculated from the static analysis and subsequently, the dynamic investigations have been carried out using the calculated displacements. The influence of the taper profiles and the boundary conditions on the mode shapes of the system has also been demonstrated. [Laura et al., 1974] calculated the natural frequencies and mode shapes of a clamped-free beam with a finite mass at the free end. Natural frequencies for varied ratios of the concentrated mass to the beam mass ratio have been tabulated. The variation of maximum dynamic stress with respect to the mass ratio has also been analyzed for the first mode of vibration. [Low, 1994, 1990] presented the free vibration analysis of a flexible rotating beam with a tip mass at the distal end. At the other end, the beam is attached to a rotating hub which has compliance in its linear and angular directions. The eigenanalysis of the model assuming it as an Euler-Bernoulli beam having a slender payload at the free end and being attached to a compliant, rotating hub at other end has also been presented. [Low and Ng, 1992] presented a study on the frequencies and mode shapes of a cantilever beam with a solid mass attached to the free end of the beam. Three dimensionless parameters viz. mass ratio, inertia ratio, and offset ratio have been varied to investigate their effect on the beam vibrations. [Malaeke and Moeenfard, 2016] studied the effect of tip mass on the dynamic behavior of the beam type structure. Large amplitude free vibration analysis has been carried out for an axially self-loaded, non-uniform beam carrying both transversely and axially eccentric tip mass having inertia. Natural frequencies and mode shapes of the system have been calculated numerically.

[Maurizi et al., 1990] derived the frequency equation of a cantilever beam of uniform cross-section with a tip mass, having its free end elastically restrained against rotation and translation. [Mitchell and Bruch Jr., 1988] modeled the single-link flexible manipulator as a continuous, clamped free beam with a payload at the distal end and supported by a flexible finite hub at other end as a representative model of flexible robotic arm. Parametric variation of first six eigenfrequencies of the system has been presented along with the mode shapes. [Naguleswaran, 2006] considered an Euler–Bernoulli beam with rigid bodies at the both ends, whose width is included in the analysis. The center of mass of the bodies has been assumed to be on the beam axis but away from the beam end. Different combinations of boundary conditions have been considered and the first three frequency parameters have been tabulated. [Oguamanam, 2003] examined the free vibration of Euler-Bernoulli beam with an offset tip mass. The beam experiences torsional deformation along with a planar elastic bending deformation. The effect of various parameters such as tip mass, offset, moment of inertia, length of the beam etc is examined using equivalent nondimensional terms. [Oguamanam and Arshad, 2005] carried out the free vibration analysis of a flexible manipulator with a rigid payload undergoing both out-of-plane and in-plane elastic flexural deformations in conjunction with torsional deformation. [Poppelwell and Chang, 1996] determined the natural frequencies of a single-link flexible manipulator when the center of gravity of the payload does not coincide with manipulator end. [Storch and Gates, 1983] studied the eigenvalue problem for two different cases i.e. a transversely vibrating cantilever beam with tip body and an unconstrained beam with rigid bodies at each end. The time-dependent frequency equations is derived and solved by [Theodore and Ghosal, 1997] for an axially translating beam driven by prismatic joint having payload of mass and inertia. The assumed mode method and Lagrangian formulation is used to obtain the closed-form dynamic model of the system and studied the influence of the translatory motion of beam on the system stability. [Teng and Cai, 2007] analyzed the frequency characteristics of an Euler- Bernoulli beam model with rotating hub considering intermediate point payload. [To, 1982] described a method to calculate the natural frequencies and mode shapes of a cantilever beam with a base excitation and tip mass whose center of gravity does not coincide with the point of attachment. Frequency equation has been solved by regula-falsi method and the roots of frequency equation are tabulated for different ratios. [Wang and Guan, 1994] established three types of dynamic models of a single-link manipulators. The influence of rotating inertia, shear deformation and tip load on the vibration behavior of the manipulator has also been studied. [Zhang et al., 2017] studied the natural frequencies of a flexible Euler-Bernoulli rotating beam with a concentrated mass at an arbitrary position. The effect of variation of the mass, rotating frequency and mass position on the natural frequencies has been discussed thoroughly. While the mass is considered as the point mass, the dynamics of the hub is neglected in the considered model. [Zhou, 1997] derived the exact and analytical expressions for the eigenfrequencies and mode shapes of a cantilever beam carrying a heavy tip mass with translational and rotational supports.

2.3.2 Two-Link Manipulator

A clamped-mass configuration of the two-link manipulator neglecting the joint dynamics has been assumed to calculate its eigen-parameters by [Ahmad et al., 2008]. The payload has been considered as point mass and the links are modeled as Euler-Bernoulli beam element. [Ata et al., 2012] computed the elastic deflection for each link of the two-link manipulator using the assumed mode method for four modes of vibrations. The boundary conditions lack the dynamics of the hub as well as the payload. The eigenfrequencies thus obtained are inadequate to explain the overall dynamics of the two-link manipulator. [Oakley and Cannon Jr., 1989] evaluated the natural frequency and mode shapes of a two-link manipulator system experimentally and the influence of foreshortening on the modal characteristics has been studied. [Castri and Messina, 2010] derived the analytical natural frequencies and mode shapes considering axial and transverse deflections of a two-link manipulator. [Cheong and Youm, 2003] studied the mode analysis of horizontal vibration for 3-D two-link flexible manipulator with a point tip mass. The analysis included both the bending and torsional vibrations and the compared the modal parameters for two different joint conditions. [Chiou and Shahinpoor, 1990] computed the eigenvalues of the linearized system of equations of a two-link robot manipulator. The cantilever and pinned boundary conditions are assumed to calculate the mode frequencies of the two-link manipulator having rigid and flexible links by [Green and Sasiadek, 2004].

[Damaren, 1998] studied the vibration modes of generic two-link flexible manipulators as a function of the link, rotor and payload dynamics. A simplified behavior of the torque to endpoint dynamics has been demonstrated using nonlinear arguments and illustrated with a numerical example. [Karagulle et. al., 2015] evaluated the natural frequency of a two-link manipulator model using finite element methods. [Mahamood and Pedro, 2011] modeled the links of the two-link flexible manipulator as cantilever beam and calculated the respective modal parameters of the links. [Milford and Asokanthan, 1999] derived the partial differential equations governing the system modes of a general two-link flexible manipulator by matching the boundary equations at the elbow. The eigenfrequencies of the system has been computed using the assumed mode method neglecting the joint dynamics and the influence of elbow angle on the eigenfrequencies has been demonstrated. [Reddy, et al. 1999] performed the free vibration analysis of a kinked cantilever beam with attached tip mass and analysed the influence of attached masses and kink angle on the modal frequencies. The first link is consided to be rigid and second flexible link is modeled as Euler-Bernoulli beam. In [Sayahkarajy et al., 2017] the first three natural frequencies and corresponding mode shapes of a two-link manipulator are evaluated and graphically demonstrated. The study remains inadequate to illustrate the effect of system parameters on the modal characteristics. [Sayahkarajy, 2018] presented a method for obtaining the global vibration modes of a two-link flexible manipulator and analyzed the complexities of dynamics of the system with further development of a control design based on the system modal parameters. The influence of payload variation on mode shapes has been investigated and performance of the proposed control strategy has been evaluated. [Yu and Elbestawi, 1995] calculated the eigenvalues of a two-link manipulator with a point payload by considering the links as lumped masses with inertia. The influence of the payload, the stiffness of the joint, friction, backlash and length of the links on the system dynamics is studied.

2.3.3 Multi-Link Manipulator

Timoshenko beam model has been adopted by [Castri and Messina 2011] to accurately model the dynamics of freely vibrating multi-link flexible manipulator incorporating the distributed link flexibilities along with the inertia loads typically mounted on the joints of manipulator. The influences of slenderness ratio and the angular configuration of the manipulator on the natural frequencies are investigated.

2.4 FORCED VIBRATION

The robotic system, when subjected to external forces or joint motions, the system is directly or parametrically excited and undergoes relatively large vibrations due to its low stiffness which is a major concern. Such large vibrations can be attenuated by the adequate understanding of the manipulator behavior under such circumstances and appropriate vibration suppression techniques can be adopted to suppress the inadmissible vibrations. The parametric study of the manipulator system under different forcing conditions and joint motions is very essential to avoid catastrophic failure and safe operation of the system. The literature review of the state-of-art research on nonlinear analysis of the flexible manipulators is discussed in the following text.

2.4.1 Single-Link Manipulator

[Cheng eta al. 2006] presented a frequency response analysis for a rotating Euler-Bernoulli cantilever beam with no payload and effects of rotating angular speeds have been investigated numerically. [Dwivedy and Kar, 1999] studied the steady-state response of a slender beam with an attached mass at an arbitrary position subjected to base motion through second-order method of multiple scales versions I and II while the payload has been assumed as a point mass. [Esfandiar and Daneshmand, 2012] investigated the dynamic modeling and state space approximation for flexible robotic manipulators. The link is modeled as Euler Bernoulli beam element with a point tip mass. The radially attached hub is considered to be rigid and they investigated the influence of joint flexibility and tip mass on the dynamic behavior of the rotating flexible arm. [Gandhi and Vyas, 2017] explored the nonlinear behavior of vibro-impacting cantilever beam with point tip mass with regard to frequency response analysis. The influence of taper parameters and tip mass on the frequency response curves has been obtained by numerical simulations and validated experimentally. [Hamdan and El-Sinawi, 2005] simulated the nonlinear dynamic response characteristics of an inextensible flexible beam clamped with a compliant hub rotating under the influence of an externally applied torque. [Hamed et al., 2018] derived the governing equations of the system modulation amplitude and phase for the case of primary resonance in a Cartesian manipulator carrying an intermediate point mass end-effector under mixed excitations. Further, the effect of nonlinearities and intermediate tip mass on the nonlinear behavior has been investigated. [Ju et al., 2017] investigated the influence of base disturbance while terminal load on the nonlinear response and dynamic stability of flexible Cartesian single-link manipulator. The manipulator beam is modeled as an Euler-Bernoulli beam and the terminal load is considered as point mass. [Lee, 1995] investigated effects of sinusoidal excitation of axial acceleration, velocity and displacement on the stability of the rotating beam and then examined the results by using Bolotin's method. [Liu et. al., 2015] investigated the vibration responses and power flow of a single-link flexible manipulator with moving base having a motion along with disturbances. The influence of disturbances on the vibration energy distributions, power flow of the system has also been presented. [Nayfeh and Arafat, 1998] investigated the nonlinear planar response of cantilever metallic beams to the parametric and external sub-combination resonances. The geometric and inertial nonlinearities have been included in the analysis. [Pai and Nayfeh, 1990] used Galerkin's method along with method of multiple scales to obtain the first order uniform expansion for the case of one-to-one internal resonance in a cantilevered beam subjected to lateral harmonic base excitation. The cubic nonlinearities arising due to curvature and inertia have been retained. The study concluded that the inertial nonlinearity is of softening type whereas; the geometric nonlinearity is of hardening type. The nonlinear response of a Cartesian single-link manipulator having the end-effector at an arbitrary position has been studied by [Pratiher and Bhowmik, 2012].

[Pratiher and Dwivedy, 2010] analyzed the Euler-Bernoulli beam with a point mass payload at free end subjected to both static and alternating magnetic field. The effect of payload on steady-state amplitude of system has been demonstrated. The nonlinear responses of a single-link manipulator having point payload under harmonically varying magnetic field and axial force have been studied by [Pratiher and Dwivedy, 2011]. [Yuan, 1995] derived the equations of motion for a hub-beam system using the Newton's second law in order to retain the simple physical structure of the problem. The rest–to-rest maneuver of a horizontally slewing torque driven beam undergoing elastic deflections is considered. [Reddy and Ghosal, 2015] discussed the nonlinear dynamics of rorating flexible single-link undergoing large deformation. Method of multiple scales is used for the detailed investigation of primary resonance and modal internal resonance. The study is focused on the influence of damping on the chaos exhibited by the system due to resonance phenomenon. Investigation of the nonlinear free vibration of an axially translating viscoelastic beam with an arbitrarily varying length and axial velocity has been carried out by [Wang and Hu et al. 2010]. The beam has been modeled as Euler-Bernoulli element while the extended Hamilton's principle has been employed to derive the temporal governing equations of motion of third order. The influence of beam extension, beam retraction and material viscosity on the vibration amplitude of beam has been demonstrated. The theoretical and experimental investigations of the nonlinear response of a slender cantilever beam carrying a lumped mass subjected to principal parametric base excitation have been accomplished by [Zavodney and Nayfeh, 1989]. The method of multiple scales is used to calculate an approximate solution to the equation of motion.

2.4.2 Two-Linkand Multi-link Manipulator

[Ahmad et al., 2008] presented the dynamic modeling of a two-link manipulator based on closed-form equations of motion. A clamped beam with end mass is considered for the development of boundary conditions and the hub dynamics has been neglected. A bang-bang torque has been used to demonstrate the effects of payload on the response of the manipulator. [Ata et al., 2012] investigated the effect of different sets of initial and boundary conditions on the joint torques of a two-link flexible manipulator. [Esfandiar and Korayem, 2015] examined their nonlinear dynamic analysis and determined the dynamic load carrying capacity in flexible manipulators. Timoshenko beam theory has been used to model the manipulator. The efficiency and the performance of the manipulator are evaluated both experimentally and theoretically. The dynamic modeling and characterization of a flexible two-link manipulator with payload has been investigated theoretically and experimentally by [Khairudin et al., 2014]. The influence of payload on the end point deflection and modal displacements has been studied by imparting bang-bang torques at the joints of the manipulator. The Hamilton's principle along with assumed mode method isare used to dynamically model the system and it is concluded that the payload decreases the angular positions of the links. [Matsuno et al., 1994] studied the two-link flexible manipulators in contact with constraint surface and developed dynamic equations of joint angles, vibrations of flexible links, and contact force. [Reddy, et al., 1999] reported the influence of attached masses and kink angle of a kinked cantilever beam on the tip responses. [Sato et al., 2016] derived the equations of motion of a two-link manipulator in consideration of characteristics of the driving source and examined the measurement technique for force of collision between link and object. The study concluded that it is possible to absorb the impact force by active motion and the results have been compared with those obtained experimentally.

2.4.3 Multi-Link Manipulator

The stability changes of equilibrium points of free link connected to passive joints using high frequency horizontal excitation of the first link in a three link under actuated manipulator carrying no payload has been analyzed by [Endo et al., 2017].

2.5 CONTROL OF FLEXIBLE MANIPULATORS

In contemporary industries, the robots are being used for various tasks such as spraying, painting, welding, drilling etc., where the end-effector of the robotic system are required to maneuver to a specific position, or controlled to follow some required trajectories at faster speed. In precision industries such as medical, space, military, underwater explorations etc. the manipulator is appropriately controlled to suppress the end-effector vibrations to efficiently execute the required operations. Thus, the trajectory tracking and control of robotic manipulator has a great practical relevance in modern industries. The recent literatures from the existing research on the control and trajectory tracking of robotic manipulator have been briefly cited.

2.5.1 Single-Link Manipulator

The hybrid input shaping and PID control for vibration attenuation and trajectory tracking of a flexible single-link manipulator without payload is studied by [Ahmed et al., 2009]. The proposed schemes are investigated experimentally and effective vibration suppression and performance is achieved. [Al-Solihat et al., 2018] developed boundary and adaptive control laws to suppress the vibrations in a three dimensional Euler-Bernoulli beam with a point payload. A fuzzy logic controller is designed by [Botsali et al., 2010] for trajectory control and vibration suppression of a robot manipulator with rotating-prismatic joint with a tip mass tracing a straight line. Simulations have been carried out to evaluate the performance of the proposed control system. The nonlinear modeling and problem of trajectory tracking of single link flexible manipulator considering it as Euler-Bernoulli beam element is addressed by [Bolandi and Esmaeilzadeh, 2008]. The payload is considered to have mass as well as inertia. The hub angle of the manipulator link incorporating a payload is experimentally compared with the analytical results. [Bayo, 1987] calculated the torques required at the hub to imitate a desired motion at the tip of a single link manipulator. The link is modeled as Euler-Bernoulli beam without tip mass. [Cannon and Schmitz, 1984] performed the experiments to controlled torque at hub end of single-link manipulator without payload based on the required tip postion. It is concluded that the appropriate dynamic modeling is essential for the satisfactory tip control. [Cetinkunt and Yu, 1991] examined the influence of different boundary conditions on the design of feedback controller of a single-link flexible manipulator with payload. Euler-Benoulli beam model is used for the link. A robust nonlinear observer to estimate the variable representing the flexible and rigid motion of a single-link robotic manipulator with revolute joint carrying a payload has been developed by [Chalhoub and Kfoury, 2005]. The numerical investigations have been done to access the performance of the observer for the accurate estimation of rigid and flexible beam motions in presence of uncertainties in the system. [Choi, 2006] analyzed a smart structure modeled as an Euler-Bernoulli beam having a point payload featuring piezoceramic actuator. Further, a robust feedback control technique has been proposed for the active vibration control of the model subjected to actuator hysteresis and parameter variation. [Kaluarachchi et al., 2017] presented a lightweight tendon drive redundant manipulator designed with reduced joint torques using a single motor with no payload. [Li and Sankar, 1993] proposed different computationally efficient methods to dynamically model the flexible robot manipulators. Simulations of the single-link manipulator with point payload are demonstated to validate the posposed methods. [Luo, 1997] presented a shear force feedback control method for single-link manipulator with a revolute joint and investigated the stability of the closed loop feedback system. For vibration suppression and to move the flexible link to the target angle, a stable control strategy based on system's energy for a torque driven planar single-link flexible manipulator has been developed by [Meng et al., 2018]. The control objective of the system is achieved by designing the controller based on the Lyapunov function and the effectiveness of the control strategy is verified numerically. The centre of gravity of the mass is not coincident with the point of attachment.

An attempt has been made by [Qui, 2012] to suppress the vibration of a flexible Cartesian smart material manipulator driven by a ball screw mechanism using AC servomotor. The adopted method has been verified experimentally and comparative studies have been conducted using different control algorithms. Large and small amplitudes near the equilibrium point have been effectively controlled by the proposed controller. In [Shawky et al., 2013], the influence of open-loop control torque profile and payload on the tip position, hub position and control input is presented. A State Dependent Riccati Equations (SDRE) is utilized to develop model-based control law, which regulates the link displacement and drive the hub position to a desired set-point. [Spong, 1987] presented the modeling and studied the control of single-link manipulator witn no payload with flexible joint. The effect of joint flexibility on the angular response of the manipulator has been graphically demonstrated. Investigation of the lateral oscillatory motion of the flexible link carrying a payload in the longitudinal direction has been carried out by [Shin and Rhim, 2015]. A vibration controller using the input shaping technique has been designed to reduce the lateral vibration induced by the longitudinal motion. The effects of tip mass and damping on the dynamic behavior of the hub-beam system have also been discussed. [Tavasoli, 2015] presented the dynamic modeling and Lyapunov-based control of a hybrid Euler-Bernoulli beam with point tip mass undergoing both elastic and rigid motions. Further, nonlinear boundary control laws have been established to suppress the beam vibrations while driving the beam position and orientations at desired set points. [Yuan, 1995] provided a simple linear feedback law via Lyapunov- type method. It has been demonstrated that the uncoupled equations can lead to substantially incorrect natural frequencies.

2.5.2 Two-Link Manipulator

The feed-forward control scheme for attenuating the residual vibrations in a two-link flexible manipulator is presented by [Abe and Yoshida, 2012]. The parameters of dynamic equations are calculated experimentally to model the manipulator. It is found that the residual vibrations can be suppressed by rotating the hub angle along the optimal path. [Abe, 2009] proposed an optimal trajectory planning technique for suppressing residual vibrations in a two-link manipulator with rigid-flexible links and point mass joints. [Abe and Hashimobo, 2015] proposed a feed forward control technique for two-flexible links attached to one motor to suppress residual vibrations in a point-to-point motion. In this proposed method, an attempt is made to express the trajectory of the joint angle using a combination of cycloidal and polynomial functions, which enable the easy generation of a smooth motion. [Alhaddad et al., 2019] addressed the motion control of a two-link rigid robotic manipulator using an adaptive LQ-based computed-torque controller. In the proposed controller, because the adaptation rule continues to get the feedback signal from the tracking error, it has been established that the accuracy of the manipulator carrying unknown payloads will improve with time. [Ashayeri et al., 2008] demonstrated the boundary controllers designed by singular perturbation approach for trajectory tracking of two-link flexible arms. The payload considered in the analysis is having mass as well as inertia and the axial motion of the links has been neglected. [Bai et al., 1998] presented an adaptive state feedback control of planar two-link manipulator and it has been established that for both end-effector point-to-point and tracking control, there are still small steady deviations and tracking errors, respectively. It is suggested that in order to improve controller performance, either harmonic drive gear reducers or direct actuators should be used. [Chiou and Shahinpoor, 1990] analyzed the stability limitations for force-controlled two-link flexible manipulator and the effect of link flexibility, nonlinear effect due to discontinuous contact with environment and force sensor stiffness on the stability has been explored.

[Choura and Yigit, 2001] considered the control of a two-link rigid-flexible manipulator and investigated the stability of the system when the payload mass is moving along the flexible link. Joint torque has been applied at rigid hubs for the motion control of both the links in addition to a force actuator which controls the relative motion of the payload and a PD-type controller that stabilizes the system maneuvering dynamics has also been proposed. The investigation of the optimal control of a two-link rigid manipulator performing under the action of forces at the joints in transporting a load from one point to the desired position for a specified time has been accomplished by [Demydyuk, 2019]. Subsequently, the interdependency of the suboptimal mode of motion of the two-link manipulator on the link configurations at the beginning and end of the operation has been demonstrated graphically. [Ding et al., 2014] presented the dynamic modeling of a closed chain mechanism in a forging manipulator applying screw theory and reduced order system model. It has been demonstrated that the actuator forces as well as the dynamic equations of the multi closed loop mechanism can be obtained by the proposed method. The motion of a two-link flexible arm with non uniform cross-section has been considered by [Dogan and Morgul, 2010] and a control scheme has been proposed consisting of a dominant control law together with parallel controller in order to achieve the given desired angles and suppress the link vibrations.

[Dou and Wang, 2013] addressed the control issue in the synchronization motion of a multiple two-link manipulators and developed the robust adaptive control laws for motion and force in order to converge to the desired trajectories. [Gawronski et al., 2008] presented the solution of dynamics, inverse dynamics and control problems associated with the flexible manipulators. Subsequently, the rigid body inverse kinematics has been used to obtain the corresponding deflections and comparison is made with flexible body configuration. A new approach for a model predictive control dynamics of a two-link rigid manipulator with a payload by linearizing a nonlinear dynamic model of the system using a feedback linearization control has been presented by [Guechi et al., 2018]. The objective of the developed method is to control the manipulator arm from initial to final configuration using a predictive control approach and lastly, the efficiency of the proposed method is also investigated through simulations. The use of a decentralized vibration control scheme for vibration suppression of a multi-link robotic manipulator system having piezoelectric transducers undergoing large rigid body motion has been investigated by [Halim et al., 2014]. The manipulator has been modeled using multiple coordinate systems to describe the motion of manipulator links. A two-link manipulator has been used to demonstrate the effect of transducer on the vibration suppression and it is proposed that the present methodology can be extended to the multi-link manipulator. [Henmi et. al., 2011] proposed a control method for an under-actuated two-link manipulator and presented the numerical simulations to show the effectiveness of the proposed method. [Hermle and Eberhard, 2007] proposed a hierarchical control concept for fast and precise point-to-point motion with acceptable vibration characteristics and the accurate trajectory tracking of the end-effector of flexible manipulator. [Jun et. al., 2018] established the mechatronics model of 2-DOF parallel manipulator in a 5-DOF hybrid machine tool on bond graph and investigated the coupled interactions between the mechanical subsystems and control subsystems. [Khorrami et al., 1994] developed a control scheme derived from an asymptotic expansion of system dynamics of a two-link flexible manipulator which reduces the frequency variations arising due to geometric configuration of arm. [Kim et.al., 2001] presented brief modeling and a control strategy for the position and force control of planar two-link manipulator system incorporating piezoceramic actuators and piezofilm sensors at each link and employing servomotors mounted at the hub. The first mode of vibration has been used to discretize the nonlinear equation of motion. The structural vibration controllability of a two-link, three joint-type manipulator neglecting the residual modes of vibrations is discussed by [Konno et al., 1997]. The influence of actuators on the structural vibration modes considering the horizontal motion of flexible manipulators is demonstrated. [Korayem et al., 2009] investigated the influence of point payload on the the residual vibrations of a two-link flexible manipulator. The error in the end-effector holding different payload is calculated.

[Lee and Wang, 1988] derived the dynamic equations of a two-link flexible robot arm moving in the vertical plane. The control strategy and general procedures to construct a linear observer and to formulate a control law has also been discussed. [Lochan and Roy, 2015] presented two low chattering SMC control techniques for position tracking and tip deflection control of a two-link flexible manipulator. [Lochan et al., 2016] presented a survey on two-link manipulator based on the existing works till 2016. The classification of manipulators is done based on dynamic analyses, complexities involved and control strategies used. It also mentioned whether the work conducted has been solely based on simulation and it has also been validated through experiments. [Lochan et al., 2016] designed a controller for two-link flexible manipulator to track the chaotic signal in presence of bounded disturbances and to regulate the tip deflection to its desired value close to zero. It is shown that the manipulator dynamics follow the desired trajectory with good tracking performance in presence of bounded disturbances. Lyapunov stability criterion has been used to achieve the stability of the sliding surface and convergence of error dynamics. Tip trajectory synchronization of a pair of identical two-link flexible manipulator has been presented by [Lochan et al., 2017]. A second order PID terminal SMC control technique is used to evaluate the robustness of the controller in presence of payload variation. Assumed mode method has been used to model the two-link flexible manipulator. The reduction in tracking error by inverse dynamic control strategy for a class of multi-link structurally flexible manipulators is attempted by [Moallem et al., 1997]. The results are illustrated for the case of single and two-link manipulators. It has been concluded that the control of flexibility effects of the second link by controlling the input of first link is hardly feasible and hence, stability issues may arise.

The problem of control of two-link manipulator with unknown parameters has been presented by [Moradi and Malekizade, 2013]. [Oh and Kong, 2015] analyzed the dynamics of two-link manipulator by using rotating coordinate system with the biarticular actuator coordination and designed a disturbance observer based on the derived dynamics to nominalize the actual dynamics and to reject the undesired disturbances. [Ower and Vegte, 1987] used the Lagrangian dynamics approach to model the planar motion of a manipulator consisting of two flexible links and two rotary joints. The equations have been linearized and represented by a transfer matrix. With an objective to reduce the production time and cost, a semi-active parameter control technique has been numerically investigated for a two-link robot manipulator involved in turning process by [Ozer et al., 2013]. The joint stiffness has been varied in a synchronized mode with the spindle speed to achieve the control of chatter. The stability and effectiveness of the proposed method has also been investigated. [Perez et al., 2012] employed the adaptive recurrent neural control for the trajectory tracking control problem for a rigid two-link manipulator without payload. The Lyapunov control functions have been used to analyze the stability of the tracking error while the control law has been obtained based on PID control. The proposed approach is experimentally verififed for a two-link manipulator. Recently, a regularized approach for modeling the frictional impact dynamics of flexible spatial manipulator arms has been presented by [Qian et al 2018]. Lagrangian method has been used for general recursive simulations with further analysis of double pendulum. The study of spatial manipulator arms colliding with targets to observe the switching between stick/sliding and forward-backward sliding of the contact pair has been accomplished. [Sato et al., 2013] derived the equation of motion of a two-link manipulator and also theoretically and experimentally investigated the dynamic characteristics of two-link system controller based on the trajectory for saving energy. A trajectory tracking method has been proposed for a two-link manipulator with lubricated revolute joint, where the joint has been modeled based on short journal bearing by [Sun, 2016]. The influence of clearance on the accuracy of the proposed method is discussed.

The dynamics and control of a redundantly actuated parallel manipulator are investigated by [Wu and Wang et al., 2009]. It is observed that, with the same trajectory, the contour error of the redundant manipulator is similar to that of the corresponding non-redundant manipulator. By combining the redundantly actuated parallel manipulator with a feed worktable, a hybrid machine tool has also been created. The vibration control of a two-link flexible manipulator having no payload carried by a translational stage has been investigated in which the joint motor has been modeled as torsional spring by [Wu and Tang et. al, 2018]. A direct feedback from the joint angle attenuates the fundamental-mode vibrations and the results are verified experimentally. [Yang et al., 2015] considered trajectory tracking control of a two-link flexible manipulator model in space. Method of back stepping control has been used to design the controller of the nonlinear system. In nonlinear and control analysis, [Yeon et al., 2012] proposed a robust control using recursive design method for robot manipulators with flexible joints. The performance of the proposed control has been verified through simulations with uncertainty in the inertia and joint stiffness. [Yun and Su, 2014] proposed a method for designing a disturbance observer to decouple the joint interactions in two-link robot dynamics with nonlinearity. The links of the two-link manipulator are modeled to be rigid and no payload condition has been considered. [Yu and Elbestawi, 1995] investigated the dynamic characteristics of a two-link planar flexible manipulator with large joint angle motions. The parametric studies demonstrated the important relations between the system parameters and the manipulator dynamics. [Zhang et al., 2005] transformed the partial differential model of a flexible two-link manipulator to an appropriate form for the development of stable control strategies. A detailed dynamic modeling of the manipulator including the hub dynamics has been presented and also, stable control strategies using Lyapunov and passivity techniques have been developed and validated experimentally.

2.5.3 Multi-Link Manipulator

[Bayo et al., 1989] presented a procedure for the inverse kinematics and dynamics of multi-link flexible manipulators and validated the same experimentally. The joint torque is calculated by the proposed method producing a specified end-effector motion. The issue of joint trajectory control with oscillation cancelling for planar multi-link flexible manipulators has been addressed by [Benosman and Vey, 2002]. The effect of fast and slow motion of joint on the tracking performance has been demonstrated and the scheme is validated numerically and experimentally. A manipulator system composed of multiple flexible links and revolute flexible joints mounted on a mobile platform has been considered for kinematic and dynamic analysis by [Korayem et al., 2012]. The flexible links are assumed as Euler-Bernoulli beam elements and the joints are considered to have both stiffness and inertia. The manipulator equations and modal responses are verified and discussed for a two-link manipulator, along with a proposal for indirect solution of open loop optimal control problem to optimize the path in order to achieve the predefined objective.

A compliant motion control framework for multiple contacts distributed over multiple links has been presented by [Park and Khatib, 2005]. An adaptive output feedback control for trajectory tracing of a flexible multi-link planar manipulator is proposed by [Rahmani and Belkheiri, 2018]. The active attenuation of flexural vibration of a multilink planar robotic system obly by joint control inputs is addressed by [Theodore and Ghosal, 2003]. A model based control algorithm is proposed for the trajectory tracking and its effectiveness is illustated using a manipulator having one rigid and two flexible links. [Wang and Pai et. al., 2017] presented a new motion and vibration synthesized control system designed as linear quadratic regulator/strain rate feedback controller with adaptive disturbance attenuation for a multi flexible link mechanism. The mechanism is subjected to uncertain harmonic disturbances with arbitrary frequencies and unknown magnitudes. In order to eliminate the shaking forces and shaking moments in a high speed parallel manipulator due to the base vibration, an approach for the design and evaluation of high speed dynamically balanced parallel manipulators is presented by [Wijk et al., 2013]. The balance and unbalanced manipulator are compared experimentally and the dynamic model of the manipulator is derived and validated. It is found that the balanced manipulator has significantly lower shaking forces and moments as compared to the unbalanced manipulator. [Wu and Gao et al., 2017] investigated the workspace and dynamic performance evaluation of a 3-DOF parallel manipulator in a spray painting equipment. Based on the kinematics, four working modes is analyzed. [Wu and Haug et al., 2007] presented a variational formulation of constrained dynamics of flexible multibody systems using body reference frames to define the global position and orientation of individual bodies in the system.

2.6 SUMMARY

In the present chapter, the existing literature regarding the flexible manipulators based on the mathematical modeling, modal analysis, nonlinear investigations and control is presented to discern the recent developments of flexible robots. The comprehension of modal parameters, vibration characteristics and nonlinear behaviors of a robotic system is imperative for its safe operation and satisfactory performance. Hence, available research regarding the flexible robotic manipulators has been substantially reviewed and the observations are further summarized as follows:

- The flexible robotic manipulator systems can be modelled as interconnected Euler-Bernoulli beams, with revolute and prismatic motions respectively. The revolute hub and actuator are assumed as torsional spring-inertia and linear spring-mass system. Also, the payload being lifted by the manipulator can be expected to be arbitrarily oriented mass having inertia with centre of gravity different than the point of attachment to the link.
- It is perceived that while a significant amount of studies related to the dynamic modeling, nonlinear analysis and control of single-link manipulator are available, mostly considering the point payload. A few works examined the effect of a sizeable payload and joint dynamics on the modal parameters of flexible manipulators. While, in case of nonlinear analysis, the effect of payload and joint attributes on the system stability remains absent.
- A very few literature is concerned with the calculation of modal parameters of flexible two-link manipulator. Most of the authors have considered the joints of the manipulator as point mass and the joint dynamics have been neglected. Also, most of the studies considered predefined beam boundary conditions such as cantilever, simply suppoted, clamped-clamped condtions. Such boundary conditions are inappropriate because the coupled end conditions, joint dynamics as well as payload parameters play a significant role in the determination of modal parameters which directly affects the control and nonlinear dynamics of manipulator. Moreover, the studies remain unaccounted for the parametric influence of system attributes and payload characteristics on the eigenfrequencies and mode shapes.
- The long reach manipulators with prismatic motion find extensive applications in modern industries. The modeling and vibration analysis of such manipulators have been accomplished either for the single-link flexible manipulator or while considering the rigid links.
- In case of two and multi-link manipulators, while most researchers have mainly focused their attention on the modeling and control, very little work is accomplished regarding the investigation of nonlinear vibrations and stability of such robotic systems. Subsequently, negligible studies have been reported related to the parametric influence of system parameters of multi-link manipulators on its nonlinear stability and bifurcations arising due to different joint motions as well as external excitations.
- In various industrial applications such as spraying, painting, grinding, drilling etc., the end-effector of the manipulator is subjected to pulsating constraint axial force which may instigate the direct or parametric excitations in the system depending on its orientation with respect to the end-effector. The nonlinear behaviors of such robotic systems must be investigated for the safety of the operator involved and avoidance of catastrophic failure of system.
- In precision industries, the end point vibration control remains the major concern. Hence, various efficient control strategies have been developed by the researchers. However, the studies did not examine the influence of the system attributes and its configurations on control parameters such as input torque, modal displacement and angular acceleration when the robotic system is required to follow a specific trajectory.