Abstract

Symbolic dynamics originated in the late nineteenth century with the work of Jacques Hadamard, where he used the theory of symbolic dynamics to investigate the geodesic flows on surfaces of negative curvature. In 1938, Morse and Hedlund formally introduced symbolic dynamics to investigate the qualitative behaviour of a general dynamical system. Later, Shannon used symbolic dynamics to investigate some of the fundamental problems in communication theory. Since then, the theory has attracted attention of many researchers around the world and has found applications in various branches of applied sciences and engineering. Although the topic has gained attention and several interesting results have been obtained, many of the natural questions are still unanswered.

In this work, we investigate the nonemptiness problem and existence of periodic points for multidimensional shift spaces. For a two dimensional shift of finite type X, we characterize the elements of the shift space using an infinite square matrix. We prove that the multidimensional shift space is nonempty if and only if the characterizing matrix is of positive dimension. We prove that the matrix generated yields precisely the elements of the shift space X and hence characterizes the elements of the shift space. We introduce the notion of complementary sets to investigate the nonemptiness problem for the shift space X. We prove that a given shift space X is nonempty if and only if the set of indices of the characterizing matrix contains a nonempty complementary set. We derive sufficient condition under which the set of periodic points for the shift space is nonempty. We generalize the results obtained for a general d dimensional shift of finite type.

We also investigate the nonemptiness problem and existence of periodic points using finite matrices. We propose an algorithm to characterize the elements of given multidimensional shift space using a sequence of finite matrices of increasing size. We prove that the matrices generated characterize arbitrarily large allowed blocks for the shift space X and hence yield an element valid for the shift space X. Further, we prove that the algorithm precisely generates the elements of the shift space X and hence characterizes the elements of the shift space completely. We also investigate some of the structural and existential problems of periodic points for a multidimensional shift space. We prove that any point in a d dimensional shift space has a finite orbit if and only if its lattice of periods is of full dimension. We prove that any periodic point with infinite orbit in a multidimensional shift space can be represented as an arithmetic progressive arrangement of shifts of a lower dimensional infinite strip. We relate the dimension of the lattice of periods of the periodic point with the dimension of the lower dimensional infinite strip generating the given point. We also derive a necessary and sufficient condition for a periodic point with finite orbit to belong to the given multidimensional shift space. In particular, we prove that an element periodic for the full shift belongs to X if and only if it is a limit of a sequence of elements from $\mathscr{B}_n^X$. We extend our result to a general point in the multidimensional full shift.