1
Introduction

The study of dynamical systems originated as a tool to investigate various natural and physical systems around us. In the process, several phenomena were modelled as discrete dynamical systems and their long term behaviour were approximated. One of the early studies were carried study by Johannes Kepler and Galileo Galilei during the early seventeenth century, where they investigated the qualitative analysis of planetary motion. Later, Newton formulated the fundamental laws of motion using the ordinary differential equations. Since then, mathematical tools have been extensively used to model various systems and predict their long term behaviour. The theory in the topological setting was first used by Henri Poincare in the late nineteenth century where he investigated three-body problem in a celestial mechanism. Birkhoff, in the early twentieth century used the qualitative theory of dynamical systems to investigate some of the fundamental problems in ergodic theory. In 1937, Pontryagin introduced the concept of structural stability for dynamical systems. He investigated the qualitative behaviour of dynamical systems under small perturbations. The investigations have answered some of the fundamental questions in this area and have found applications in various branches of sciences and engineering.

Symbolic dynamics originated as a tool to investigate the qualitative behaviour of general dynamical systems arising in various fields of science and technology. The simpler visualization and easy computability makes it an effective tool to determine the long term behaviour of the underlying system. In one of the seminal work in 1898, Jacques Hadamard used the theory of symbolic systems to study the geodesic flows on surfaces of negative curvature [Hadamard, 1898]. Claude Shannon used symbolic dynamics to develop the mathematical theory of communication systems [Shannon, 1948]. Since then, the topic has gained attention of several researchers around the world and has found applications in various branches of sciences and engineering. In particular, the topic has found applications in areas like control networks, biomedical engineering, anomaly detection in mechanical systems and computational modelling of gene networks. In [Hochma et al., 2013], the authors used symbolic dynamics to investigate the dynamics of Boolean control networks. In [Patankar et al., 2008], the authors used symbolic dynamics to develop a technique to detect and monitor failure precursors and anomalies in electrical systems. The technique resulted in robust detection and was observed to be superior to conventionally known techniques for anomaly detection. In [Khatkhate et al., 2006], the authors used symbolic time-series analysis for anomaly detection in mechanical systems. The method used principles of automata theory, information theory and pattern recognition to examine the efficacy of the proposed method. The proposed method was observed to perform better in many aspects when compared to some of the known methods in the literature. In [Mallapragada et al., 2008], the authors used symbolic dynamics filtering to investigate automated behaviour recognition in mobile robots. The work introduced the dynamic data driven method using symbolic dynamics for signature detection in mobile robots. The authors validated the proposed method by experimentation on a networked robotics test bed to detect and identify the type and motion profile of the robots under investigation. In [Voss et al., 2000], the authors used symbolic dynamics to characterize the dynamics of heart rate variability (HRV) and blood pressure variability (BPV). The authors used symbolic dynamics for risk stratification after myocardial infarction for characterization of different cardiovascular diseases and for phenotyping in genetic studies. In recent times, tiling spaces and their cohomology have found applications in investigating structural properties of quasicrystals. In [Forrest et al., 2002], the authors used quasiperiodic tilings to investigate properties AI-Mn type quasicrystals. They also use tiling patterns for indexing of the diffraction patterns. A detailed investigation on the relation between the properties of the tilings patterns and structural properties of the quasicrystals is available in the literature [Steinhardt and Ostlund, 1987; Janot and Mosseri, 1995; Axel and Gratias, 1995; Moody, 1997].

These studies highlight the importance of investigating the general theory of multidimensional symbolic systems. Although the topic has been investigated by many researchers around the world, many of the natural questions for multidimensional symbolic dynamics need to be answered. For example, can the elements of the multidimensional shift space be characterized by a square matrix? What is the minimal cardinality of the square matrix required to characterize the elements of the multidimensional shift space? In case the dimension of the desired matrix is infinite, can the elements be characterized by a sequence of finite matrices? Under what conditions does a multidimensional shift space posses a periodic point? When can the representation of a periodic (general) point for the given shift space X be obtained using elements of the full shift? Under what conditions, does a point of the full shift belong to a given multidimensional shift space X? In this thesis, we provide answers to some of the natural questions raised above. Before we move further, we first give some of the basic definitions and concepts required.

Let $A = \{a_i : i \in I\}$ be a finite set and let d be a positive integer. Let the set A be equipped with the discrete metric and let $A^{\mathbb{Z}^d}$, the collection of all functions $x : \mathbb{Z}^d \to A$ be equipped with the product topology. Any such function x is called a configuration over A. Any configuration x is called periodic if there exists $u \in \mathbb{Z}^d$ $(u \neq 0)$ such that $x(v+u) = x(v) \quad \forall v \in \mathbb{Z}^d$. The set $\Gamma_x = \{w \in \mathbb{Z}^d : x(v+w) = x(v) \ \forall v \in \mathbb{Z}^d\}$ is called the lattice of periods for the configuration x. The cardinality of the largest rationally independent subset of the lattice of periods is called the dimension of the lattice of periods. It may be noted that the lattice of periods is closed under integral combinations. The function $\mathscr{D} : A^{\mathbb{Z}^d} \times A^{\mathbb{Z}^d} \to \mathbb{R}^+$ be defined as $\mathscr{D}(x,y) = \frac{1}{n+1}$, where n is the least non-negative integer such that $x \neq y$ in $R_n = [-n,n]^d$, is a metric on $A^{\mathbb{Z}^d}$ and generates the product topology. Note that if $d = 1$ then $A^{\mathbb{Z}^d}$ is the collection of all bi-infinite sequences over A. For any $a \in \mathbb{Z}^d$, the map $\sigma_a : A^{\mathbb{Z}^d} \to A^{\mathbb{Z}^d}$ defined as $(\sigma_a(x))(k) = x(k+a)$ is a d-dimensional shift and is a homeomorphism. For any $a,b \in \mathbb{Z}^d$, $\sigma_a \circ \sigma_b = \sigma_b \circ \sigma_a$ and hence $\mathbb{Z}^d$ acts on $A^{\mathbb{Z}^d}$ through commuting homeomorphisms. A set $X \subseteq A^{\mathbb{Z}^d}$ is σ_a-invariant if $\sigma_a(X) \subseteq X$. Any set $X \subseteq A^{\mathbb{Z}^d}$ is shift-invariant if it is invariant under σ_a for all $a \in Z^d$. A nonempty, closed shift-invariant subset of $A^{\mathbb{Z}^d}$ is called a shift space. A shift space X is called irreducible if for any pair of nonempty open sets U,V in X there exists $n \in \mathbb{Z}^d$ such that $\sigma_n(U) \cap V \neq \phi$. If $Y \subseteq X$ is a closed, nonempty shift-invariant subset of X, then Y is called a subshift of X. For any nonempty $S \subset \mathbb{Z}^d$ and any configuration $x : \mathbb{Z}^d \to A$, let $x|_S$ denote the projection of x on S. The map $\pi_S : A^{\mathbb{Z}^d} \to A^S$ defined as $\pi_S(x) = x|_S$ is the projection map and projects any element of $A^{\mathbb{Z}^d}$ to A^S. Any element in A^S is called a pattern over S. A pattern is said to be finite if it is defined over a finite subset of $\mathbb{Z}^d$. A pattern q over S is said to be an extension of the pattern p over T if $T \subset S$ and $q|_T = p$. The extension q is said to be a proper extension if $T \cap Bd(S) = \phi$, where $Bd(S)$ denotes the boundary of S. Let $\mathscr{F}$ be a given set of finite patterns (possibly over different subsets of $\mathbb{Z}^d$) and let $X = \overline{\{x \in A^{\mathbb{Z}^d} : \text{any pattern from } \mathscr{F} \text{ does not appear in } x\}}$. The set X defines a subshift of $\mathbb{Z}^d$ generated by set of forbidden patterns $\mathscr{F}$. If the set $\mathscr{F}$ is a finite set of finite patterns, we say that the shift space X is a shift of finite type. For the one dimensional case, a shift space is called M-step if it can be characterized by a finite collection of forbidden blocks of length $M+1$. We say that a pattern is allowed if it is not an extension of any forbidden pattern. We denote the shift space generated by the set of forbidden patterns $\mathscr{F}$ by $X_{\mathscr{F}}$. Two forbidden sets $\mathscr{F}_1$ and $\mathscr{F}_2$ are said to be equivalent if they generate the same shift space, i.e. $X_{\mathscr{F}_1} = X_{\mathscr{F}_2}$. A forbidden set $\mathscr{F}$ of patterns is called minimal for the shift space X if $\mathscr{F}$ is the set with least cardinality such that $X = X_{\mathscr{F}}$. It is worth mentioning that a shift space X is of finite type if and only if its minimal forbidden set is a finite set of finite patterns. For any $n \in \mathbb{Z}^d$, let $\mathscr{S}_n^X$ denote the set of all d dimensional cuboids of size n allowed in X. Let $\mathscr{B}_n^X$ denote the set of all multidimensional sequences obtained by repetitive arrangement of a fixed element of $\mathscr{S}_n^X$ (in each of the d directions). It may be noted that the shift space can equivalently be defined in terms of the allowed patterns. For a shift space X and any set $S \subset \mathbb{Z}^d$, let $\mathscr{L}_S = \{x \in A^S : x = \pi_S(y), \text{ for some } y \in X\}$. Then, $\mathscr{L}_S$ is the set of allowed patterns (for X) over S. The set $\mathscr{L} = \bigcup_{S \subset \mathbb{Z}^d} \mathscr{L}_S$ is called the language for the shift space X. Given a set $S \subset \mathbb{Z}^d$ and a set of patterns $\mathscr{P}$ in A^S, the set $X = X(S, \mathscr{P}) = \overline{\{x \in A^{\mathbb{Z}^d} : \pi_S \circ \sigma^n(x) \in \mathscr{P} \text{ for every n} \in \mathbb{Z}^d\}}$ is a subshift generated by the (allowed) patterns $\mathscr{P}$. For any multidimensional shift space X, the topological entropy is defined as

$$h_{top}(X) = \lim_{n \to \infty} \frac{|\mathscr{L}_{C_n}(X)|}{|C_n|}$$

where $C_n(X)$ $(\mathscr{L}_{C_n}(X))$ denotes the set of cubes (allowed cubes) of size n. A sequence (a_n) of integers is recursive if there is an algorithm T (formally a Turing machine) that, upon input $n \in \mathbb{N}$, outputs a_n. A set of integers is recursively enumerable if it is the set of elements of some recursive sequence. A subset $X \subseteq \{0,1\}^{\mathbb{N}}$ is effectively closed if its complement is the union of a recursive sequence of cylinder sets. A subset $X \subseteq (\{0,1\}^{\mathbb{N}})^{\mathbb{Z}^d}$ is an effectively dynamical system if it is effectively closed and invariant under the shift action [Quas and Trow, 2000; Ban et al., 2015; Beal et al., 2005; Boyle et al., 2010; Hochman and Meyerovitch, 2010; Hochman, 2009; Lightwood, 2003; Pavlov and Schraudner, 2015] for more details.

Let M be a square $0-1$ matrix (possibly infinite) with indices $\{\mathfrak{i} : \mathfrak{i} \in \mathfrak{I}\}$. We say that the index $\mathfrak{i}$ is u-related to $\mathfrak{j}$ if $M_{\mathfrak{j}\mathfrak{i}} = 1$. Let the collection of indices u-related to $\mathfrak{j}$ be denoted by $R^u_{\mathfrak{j}}$. We say that the indices $\mathfrak{j}$ is d-related to $\mathfrak{i}$ if $M_{\mathfrak{j}\mathfrak{i}} = 1$. Let the collection of indices d-related to $\mathfrak{i}$ be denoted by $R^d_{\mathfrak{i}}$. It may be noted that $\mathfrak{i}$ is u-related to $\mathfrak{j}$ if and only if $\mathfrak{j}$ is d-related to $\mathfrak{i}$. A nonempty subset K of the index set $\mathfrak{I}$ is said to be complementary if for each $\mathfrak{i} \in K$, there exists $\mathfrak{j}, k \in K$ such that $\mathfrak{j}$ is u-related to $\mathfrak{i}$ and k is d-related to $\mathfrak{i}$. For any two distinct k-tuples $(a_1, a_2, \ldots, a_k), (b_1, b_2, \ldots, b_k)$ over $\mathbb{R}$, we say that $(a_1, a_2, \ldots, a_k) < (b_1, b_2, \ldots, b_k)$ if $a_r < b_r$ where $r = \min\{i : a_i \neq b_i\}$. The relation defines a total order on $\mathbb{R}^k$ and is known as the dictionary order on $\mathbb{R}^k$. Let $O_{\mathscr{H}}$ (and $O_{\mathscr{V}}$) be the restriction of the dictionary order on the set of 2×2 matrices, when any matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is represented as (a, b, c, d) (and (a, c, b, d) respectively). Analogously, let $O_{\mathscr{H}}$ and $O_{\mathscr{V}}$ be the orders defined on the set of $r \times s$ matrices obtained by restricting dictionary order, when any matrix is represented as a tuple by reading entries row-wise (left to right) and column-wise (top to bottom) respectively.

Let M be a $k^2 \times k^2$ matrix of the form

$$M = \begin{pmatrix} M_{11} & M_{12} & \ldots & M_{1k} \\ M_{21} & M_{22} & \ldots & M_{2k} \\ \vdots & \vdots & & \vdots \\ M_{k1} & M_{k2} & \ldots & M_{kk} \end{pmatrix}$$

where each M_{ij} is a $k \times k$ matrix. For a pair of matrices (M_{ij}, M), assign a $1 \times k^4$ matrix $(M_{ij} \bigotimes M)$ as

$$(M_{ij} \bigotimes M)_{1r} = (M_{ij})_{\alpha\beta} (M_{\alpha\beta})_{\gamma\delta}$$

where $\alpha, \beta, \gamma, \delta \in \{1, 2, \ldots, k\}$ and $(\alpha, \beta, \gamma, \delta)$ is the unique solution to the equation $r = \alpha k^3 + \beta k^2 + \gamma k + \delta - (k^3 + k^2 + k)$.

It may be noted that the if the $1 \times k^4$ row vector is visualised as k^2 groups of k^2 elements, then the first k^2 entries of the resultant $(M_{ij} \bigotimes M)$ are obtained by multiplying $(M_{ij})_{11}$ with k^2 entries of M_{11} (entries read row-wise), next k^2 entries are obtained by multiplying $(M_{ij})_{12}$ with k^2 entries of M_{12}, and so on. In general, for $n \in \{1, 2, \ldots, k^2\}$, the n-th group is determined by multiplying $(M_{ij})_{wz}$ by k^2 entries of M_{wz} where $(w, z) \in \{1, 2, \ldots, k\}^2$ is unique solution to the equation $n = (w-1)k + z$. Consequently, the operation $\bigotimes$ is well defined and assigns a k^4 row vector for any input pair (M_{ij}, M). It may be noted that although the operation is defined for the pair (M_{ij}, M) where M_{ij} is a submatrix of M, the operation is well defined for any pair (P, Q) where P and Q are square matrices of order k and k^2 respectively.

The above definitions provide some of the basic notions and concepts needed for investigating some of the fundamental problems for a multidimensional shift space. The topic has attracted the attention of several researchers around the globe and some interesting results have been obtained [Quas and Trow, 2000; Ban et al., 2015; Beal et al., 2005; Boyle et al., 2010; Hochman and Meyerovitch, 2010; Hochman, 2009; Lightwood, 2003]. In 1961, Hao Wang conjectured that if a plane can be tiled using a finite set of Wang tiles then there exist a periodic tiling for the given plane [Wang, 1961]. The conjecture implies the existence of an algorithm to decide whether a given finite set of Wang tiles can tile the plane. The algorithmic problem of tiling the plane with a given set of Wang tiles is known as Domino problem. In 1966, Robert Berger answered the problem in the negative and showed that no such algorithm for the problem can exist in general. In particular, he proved that for a multidimensional subshift, it is algorithmically undecidable whether an allowed partial configuration can be extended to a point in the multidimensional shift space [Berger, 1966]. Consequently, he observed that it is algorithmically undecidable to verify the nonemptiness of a multidimensional shift defined by a set of finite forbidden patterns. In [Robinson, 1971], the author gives examples to show that a multidimensional shift space may or may not contain any periodic points. These results unravelled the uncertainty associated with a multidimensional shift space and motivated further research in this area. In [Quas and Trow, 2000], the authors proved that multidimensional shifts of finite type with positive topological entropy cannot be minimal. In fact, if X is subshift of finite type with positive topological entropy, then X contains a subshift which is not of finite type, and hence contains infinitely many subshifts of finite type. In the same paper, the authors proved that every shift space X contains an entropy minimal subshift Y, i.e., a subshift Y of X such that $h(Y) = h(X)$. While [Ban et al., 2015] investigated the mixing properties of multidimensional shift of finite type, [Beal et al., 2005] investigated minimal forbidden patterns for multidimensional shift spaces. In [Boyle et al., 2010], authors exhibit mixing $\mathbb{Z}^d$ shifts of finite type and sofic shifts with large entropy. However, they establish that such systems exhibit poorly separated subsystems. They give examples to show that while there exists $\mathbb{Z}^d$ mixing systems such that no nontrivial full shift is a factor for such systems, they provide examples of sofic systems where the only minimal subsystem is a single point. In [Hochman and Meyerovitch, 2010], for multidimensional shifts with $d \geq 2$, authors proved that a real number $h \geq 0$ is the entropy of a $\mathbb{Z}^d$ shift of finite type if and only if it is the infimum of a recursive sequence of rational numbers. In [Hochman, 2009], the author improved the result and showed that $h \geq 0$ is the entropy of a $\mathbb{Z}^d$ effective dynamical system if and only if it is the lim inf of a recursive sequence of rational numbers. The problem of determining which class of shifts have a dense set of periodic points is still open. For two-dimensional shifts, Lightwood proved that strongly irreducible shifts of finite type have a dense set of periodic points [Lightwood, 2003]. However, the problem is still open for shifts of dimension greater than two.

Before we move further, we give some of the notations used in the literature Let X be multidimensional shift space and let $\mathscr{F}_n(X)$ denote the set of forbidden cubes (for X) of size n. Let $X_{\mathscr{F}_n(X)}$ denote the shift space generated by set of forbidden blocks $\mathscr{F}_n(X)$. For a two dimensional shift space, let $\mathscr{F}_{m,n}(X)$ denote the set of forbidden rectangles (for X) of size $m \times n$ and let $X_{\mathscr{F}_{m,n}(X)}$ denote the shift space generated by forbidding the elements of $\mathscr{F}_{m,n}(X)$. Let $\mathbb{M}_n^1(X)$ denote the set of minimal forbidden patterns of size n (i.e. set of forbidden patterns of size n such that each subsquare of size $n-1$ is allowed). An element x is called *strongly periodic* of period $n > 0$ if $\Gamma_x = n\mathbb{Z}^2$. An element x is *1-periodic* of period $u \in \mathbb{Z} \times \mathbb{N}$ if $\Gamma_x = u\mathbb{Z}$. An element x is *horizontally periodic* of period $n > 0$ if n is the least positive integer so that $n\mathbb{Z} \times \{0\} \subseteq \Gamma_x$. We now give some of the known results for symbolic dynamics.

Theorem 1.0.1 *[Vries, 2014] Let X be a shift space and $x \in A^{\mathbb{Z}}$. Then $x \in X$ if and only if all blocks occuring in the point x are in the language $\mathscr{L}(X)$ of shift space X.*

Theorem 1.0.2 *[Lind and Marcus, 1995] If $X \in A^{\mathbb{Z}}$ is shift of finite type, then there exists an integer $M \geq 0$ such that X is M-shift of finite type.*

Theorem 1.0.3 *[Lind and Marcus, 1995] A shift space $X \in A^{\mathbb{Z}}$ is $M-$ step shift of finite type if and only if whenever $au, ub \in \mathscr{L}(X)$ and $|u| \geq M$ then $aub \in \mathscr{L}(X)$.*

Theorem 1.0.4 *[Berger, 1966] A domino problem is decidable or undecidable according to whether there exists or does not exist an algorithm which, given the specifications of an arbitrary domino set, will decide whether or not the set if solvable.*

Example 1.0.1 *[Schmidt, 2001]* ***(The Chessboard Problem):*** *Let $n \geq 2$ and $A = \{0, 1, 2, \cdots, n-1\}$ be the given set of colours. Let X be the collection of all configurations in which adjacent lattice points have different colours. As the shift constructed can be described by a finite set of forbidden blocks (adjacent placement of same colours), X is a shift of finite type. While X consists of precisely two points for $n = 2$, X is uncountable for any higher value of n. Also, as any finite allowed square can be extended periodically, the set of periodic points is dense in X for any value of $n (\geq 2)$. It is worth mentioning that for $n \geq 3$, as any adjacent position (to be filled) has more than one option, the shift generated contains both periodic and nonperiodic points (and hence the plane can be tilled in both periodic and nonperiodic manner using the given colours).*

Example 1.0.2 *[Robinson, 1971]* ***(Shift of finite type without periodic points)****: In [Robinson, 1971], the author introduced a set T' of six polygonal tiles (Figure 1.1). Consider each of the tiles as a unit square with bumps and dents on the edges and on the corners of each tile. Let T be the set of all tiles obtained by allowing horizontal and vertical reflections as well as rotations of elements in T' by multiples of $\frac{\pi}{2}$. Taking the mirror images of all the tiles, we obtain a new set consisting of 8 tiles, any valid arrangement of which (translation and rotation allowed) is nonperiodic. If only translation is allowed, the new set consists of 32 tiles any valid arrangement of which is again nonperiodic. Thus, the set $W_T \subset T^{\mathbb{Z}^2}$ consisting of all tilings of $\mathbb{R}^2$ by translates of elements of T (aligned to the integer lattice) is a shift of finite type and has no periodic points.*

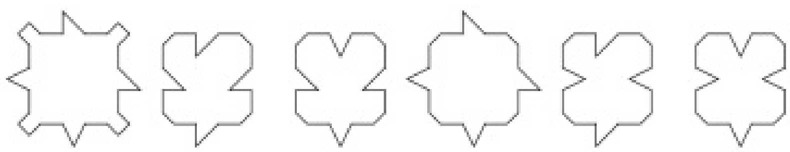

Figure 1.1 : Robinson Tiles

Theorem 1.0.5 *[Quas and Trow, 2000] Let X be a shift of finite type. If $h(X) > 0$, then X contains a proper subshift of finite type. In particularly, X is not minimal.*

Theorem 1.0.6 *[Quas and Trow, 2000] If X is a shift of finite type, and Y is a subshift of X which is not of finite type, then X contains infinitely many subshifts of finite type.*

Theorem 1.0.7 *[Quas and Trow, 2000] Every shift space X contains an entropy minimal subshift Y with the property that $h(X) = h(Y)$.*

Theorem 1.0.8 *[Quas and Trow, 2000] If X is a shift of finite type and $h(X) > 0$, then X has a subshift Y which is not of finite type.*

Theorem 1.0.9 *[Beal et al., 2005] The set $\mathbb{M}^1(X)$ is a set of forbidden patterns for X, that is $X = X_{\mathbb{M}^1(X)}$.*

Theorem 1.0.10 *[Beal et al., 2005] A shift space X is of shift of finite type if and only if $\mathbb{M}^1(X)$ is finite.*

Theorem 1.0.11 *[Hochman and Meyerovitch, 2010] For $d \geq 2$ the class of entropies of $d-$dimensional shift of finite type is the class of non-negative right recursively enumerable numbers.*

Theorem 1.0.12 *[Hochman and Meyerovitch, 2010] For $d \geq 2$ the class of entropies of $d-$dimensional sofic shifts is the same as that of $d-$ dimensional shift of finite type.*

Theorem 1.0.13 *[Hochman and Meyerovitch, 2010] The entropy of an irreducible shift of finite type is computable.*

Theorem 1.0.14 *[Hochman and Meyerovitch, 2010] The entropy of every sofic shift is right recursively enumerable.*

Theorem 1.0.15 *[Hochman, 2009] Fix $d \geq 2$. Then a real number $h \geq 0$ is the entropy of a $\mathbb{Z}^d$-shift of finite type if and only if it is the infimum of recursive sequence of rational numbers.*

Theorem 1.0.16 *[Hochman, 2009] For each $d \geq 1$, a real number $h \geq 0$ is the entropy of a $\mathbb{Z}^d$-effectively dynamical system if and only if it is the infimum of recursive sequence of rational numbers.*

We now provide a brief summary of work done in subsequent chapters of the thesis.

In chapter 2, we characterize the elements of multidimensional shift space using infinite strips of fixed height. In the process, we address the nonemptiness problem and existence of periodic points for multidimensional shift of finite type. We prove that any two dimensional shift of finite type can be characterized by a square matrix (possibly of infinite dimension). We prove that the elements of the shift space can also be characterized by limits of periodic configurations arising from allowed cubes for the shift space X. We extend our result to a general d- dimensional shift space X. We also give a sufficient condition ensuring the existence of periodic points for a multidimensional shift space X.

In chapter 3, we address the problem of characterizing the elements of two dimensional shift space of finite type X using finite matrices. In particular, we provide an algorithm for characterizing the elements of the shift space using a sequence of finite matrices of increasing size. The algorithm generates arbitrarily large cubes for the shift space X and hence generates any element valid for X (as a limit of arbitrarily large allowed cubes). Consequently, the algorithm generates precisely all the possible elements of shift space under investigation and hence determines the shift space completely. We also extend our algorithm for a general d dimensional shift space.

In chapter 4, we address some of the questions related to structural and existential properties of a periodic point in a multidimensional shift space. We prove that any periodic point in a d dimensional shift space has a finite orbit if and only if its lattice of periods is d dimensional. We prove that in a multidimensional shift space, any periodic point with infinite orbit can be represented as a repetitive arrangement of shifts of lower dimensional strips (of infinite length). We also derive the relation between the dimension of lattice of periods of periodic points and the lower dimensional infinite strip. We derive necessary and sufficient conditions for a periodic point of the full shift to belong to a given multidimensional shift space. We extend our result to a general point of the multidimensional full shift.