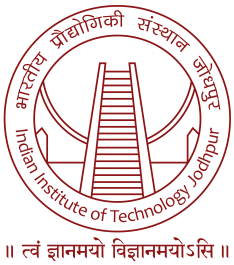
# Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies

*A Thesis submitted by*
**Aniket Dilip Monde**

*in partial fulfillment of the requirements for the award of the degree of*
**Doctor of Philosophy**




**Indian Institute of Technology, Jodhpur**
**Department of Mechanical Engineering**
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# Declaration

I hereby declare that the work presented in this thesis titled *"Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies"* submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of *Dr. Prodyut Ranjan Chakraborty*. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to any other Institute or University in India or abroad for the award of any degree or diploma.

*Aniket Dilip Monde*
*P15ME201*

# Certificate

This is to certify that the thesis titled *"Solidification and Shrinkage: Analytical and Numerical Model Development with Case Studies"*, submitted by *Aniket Dilip Monde (P15ME201)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University in India or abroad for the award of any degree or diploma.

*Dr. Prodyut Ranjan Chakraborty*
*Ph.D.Thesis Supervisor*

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*Ph.D. Student*

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# Nomenclature

| Variables | Description |
|---|---|
| $C$ | concentration |
| $C_e$ | eutectic concentration |
| $c_p$ | specific heat $(J/kgK)$ |
| $D$ | diffusivity $(m^2/s)$ |
| $d$ | diameter $(m)$ |
| $F_o$ | Fourier number |
| $f$ | mass fraction |
| $g$ | volume fraction |
| $g_a$ | gravitational acceleration $(m/s^2)$ |
| $H$ | height $(m)$ |
| $h$ | enthalpy $(J/kg)$ |
| $h_{sl}$ | latent heat $(J/kg)$ |
| $k$ | thermal conductivity $(W/mK)$ |
| $k_p$ | partition coefficient |
| $L$ | domain length $(m)$ |
| $l$ | length $(m)$ |
| $m$ | mass $(kg)$ |
| $Pr$ | prandtl number |
| $p$ | pressure $(Pa)$ |
| $T$ | temperature ($K$ or $^oC$) |
| $t$ | time $(s)$ |
| $T_c$ | cold boundary temperature ($K$ or $^oC$) |
| $T_e$ | eutectic temperature ($K$ or $^oC$) |
| $T_L$ | liquidus temperature ($K$ or $^oC$) |
| $T_m$ | melting temperature ($K$ or $^oC$) |
| $T_S$ | solidus temperature ($K$ or $^oC$) |
| $u, v, w$ | magnitude of velocity in x, y, and z directions, respectively $(mm/s)$ |
| $\vec{\mathbf{V}}$ | velocity vector $(m/s)$ |
| $v$ | specific volume $(m^3/kg)$ |
| $v_i$ | crystal growth rate $(m/s)$ |
| $W$ | width $(m)$ |
| $x, y, z$ | coordinate axes |
| $x_i$ | solid-liquid interface |

| Greek | Description |
|---|---|
| $\beta$ | solidification contraction |
| $\beta_T$ | thermal expansion coefficient |
| $\beta_C$ | solutal expansion coefficient |
| $\varepsilon$ | constant |
| $\rho$ | density ($kg/m^3$) |
| $\mu$ | dynamic viscosity ($kg/ms$) |
| $\gamma$ | surface energy ($J/m^2$) |
| $\alpha$ | thermal diffusivity ($m^2/s$) |
| $\lambda$ | relaxation factor |

| Subscripts | Description |
|---|---|
| $C$ | cavity |
| $i, 0$ | initial |
| $f$ | final |
| $int$ | interface |
| $l$ | liquid |
| $R$ | riser |
| $ref$ | reference value |
| $v$ | void |
| $s$ | solid |

| Superscripts | Description |
|---|---|
| $k$ | present time-step |
| $k-1, 0$ | previous time-step |
| $n$ | iteration step |

# List of Abbreviation

| Abbreviation | Full form |
|---|---|
| $2D$ | Two-Dimensional |
| $3D$ | Three-Dimensional |
| $S/L$ | solid-liquid |
| $ALE$ | arbitrary Lagrangian-Eulerian |
| $CCD$ | charge coupled device |
| $HRS$ | high rate solidification |
| $LMC$ | liquid-metal cooling |
| $LUDS$ | linear upwind difference scheme |
| $ND$ | non-dimensional |
| $NEE$ | non-equilibrium eutectic |
| $PCM$ | phase change materials |
| $PFT$ | pseudo-front tracking |
| $PIV$ | particle image velocimetry |
| $REV$ | representative elementary volumes |
| $RMS$ | root mean square |
| $SIMPLE$ | semi-implicit method for pressure linked equations |
| $SIMPLER$ | semi-implicit method for pressure linked equations revised |
| $TDMA$ | tridiagonal matrix algorithm |
| $TNT$ | trinitrotoluene |
| $VOF$ | volume of fluid |
| $VGF$ | vertical gradient freeze |