1
Introduction

This chapter goes through an overview of the Standard model (SM) of electroweak interactions along with the motivation for the works performed in the thesis. We discuss about the limitations of the SM along with ways to probe new physics. Further, we provide the outline of the thesis.

1.1 The SM particle spectrum

The Standard Model (SM) of Particle Physics is a theory which describes three of the four fundamental interactions of nature, the electromagnetic, weak, and strong interactions, in term of gauge theories. The theories and experimental discoveries over a century have resulted in a remarkable insight into the fundamental structure of the universe. Our universe is made from a few basic building blocks called fundamental particles, governed by four fundamental forces. Our current understanding of how these particles and the interactions except gravity are related to each other is encapsulated in the SM of particle physics. The SM particle spectrum is depicted in Fig.1.1.

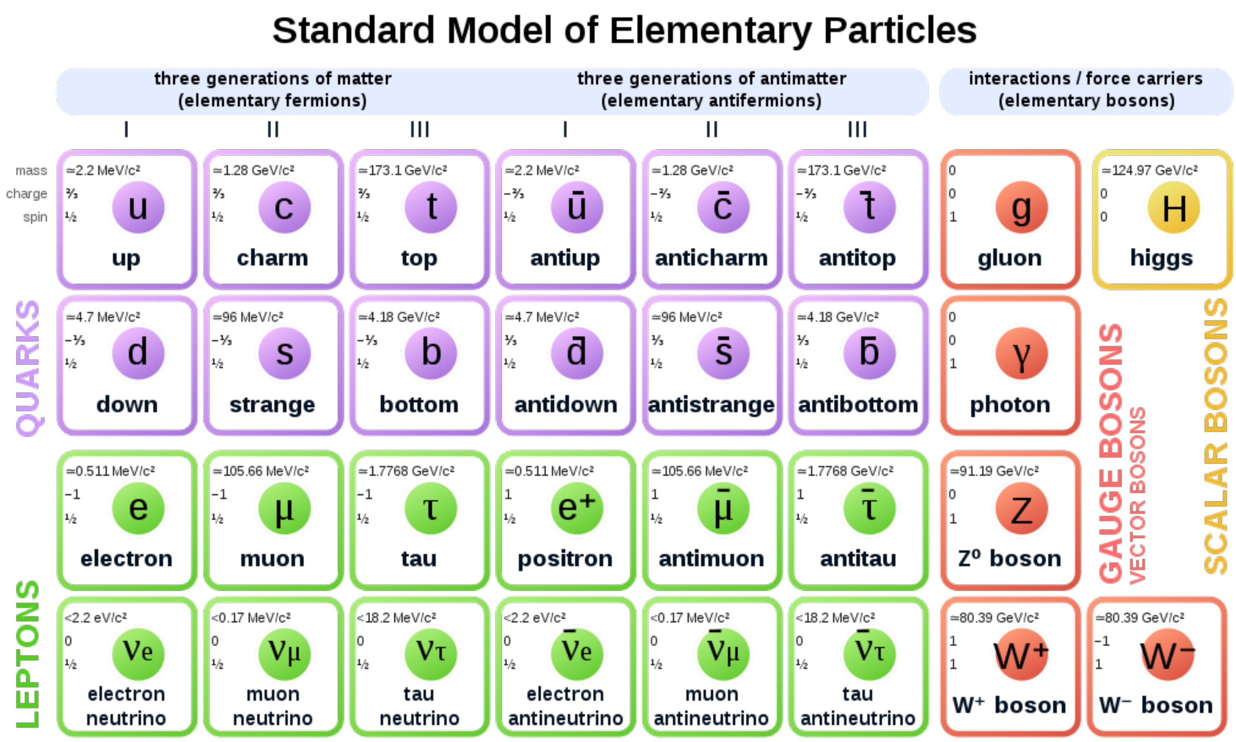

Figure 1.1: SM particle spectrum. Source: Wikipedia

The building blocks of matter are the fermions. This is due to the fact that they obey the Pauli exclusion principle. Elementary fermions are classified into quarks and leptons. There are six particles in each group along with their anti-particles. These fermions are represented in pairs or generation. The first generation particles make up all the stable matter; the heavier particles quickly decay to the next most stable ones. The six quarks are paired in the three generations: up (**u**) and down quarks (**d**), charm (**c**) and strange quarks (**s**) & top (**t**) and bottom (or beauty) quarks (**b**). Quarks appear in different "colours" also and combine only in such ways as to get "colourless" composite particles known as hadrons. Similarly, the six leptons are arranged in three generations: electron (**e**) and electron neutrino (ν_e), muon (μ) and muon neutrino (ν_μ) & tau (τ) and tau neutrino (ν_τ). The electron, the muon and the tau all have an electric charge and mass, whereas the neutrinos are electrically neutral and have tiny mass.

The interactions in the SM are mediated by following gauge mediators:

- the photon which carries the electromagnetic interaction. It is mass less and electrically neutral. The quantum gauge theory of electromagnetism is known as Quantum Electrodynamics (QED).
- the $W^\pm$ and Z bosons which mediate the weak interaction. This interaction affects all SM fermions. All of these mediators are massive. The mass of Z is higher than that of $W^\pm$ boson.
- the gluons which are mediators of strong interaction. Gluons are massless and like quarks, they carry color charge. This means that apart from quark-quark interaction which are mediated by gluons, the gluons can also interact amongst themselves. The quark-gluons interactions are described by the theory of quantum chromodynamics (QCD).

Finally, the SM predicts the existence of a fundamental scalar particle known as Higgs boson which is supposed to provide mass to all elementary particles along with $W^\pm$ and Z bosons.

SM was developed in the early 1970s [1–3] and it has successfully accounted for all experimental results and precisely predicted a wide variety of phenomena. However, many of its important theoretical frameworks were developed in the 1960s. These include

- the development of non-abelian gauge theories by Yang and Mills [4],
- the Goldstone theorem regarding the spontaneous symmetry breaking [5, 6],
- the mechanism of *generating* masses of the SM fields, commonly known as the "Higgs mechanism" [7–12].

There are other important ingredients of the SM as well. These include

- the quark model [13]
- the concept of asymptotic freedom [14, 15],
- the proof of renormalizability of the SM [16].

On the experimental front, the discovery of the Higgs boson at CERN in 2012 [17, 18] marked the completion of the SM.

1.2 Gauge theory of electroweak interactions

The SM of particle physics is a non-abelian Yang-Mills gauge theory of electroweak interaction which is based on the gauge group $SU(2)_L \times U(1)_Y \times SU(3)_C$, where the subscripts C, L and Y denote color, left-handed chirality and weak hypercharge, respectively. This theory was formulated by Glashow [1], Salam [2] and Weinberg [3], and is known as GSW model. Strong interactions can be treated independently from the electroweak interactions. This is because the $SU(3)_C$ symmetry is unbroken and it does not mix with the $SU(2)_L \times U(1)_Y$ sector. The theory of $SU(3)_C$ color group which formulates the theory of strong interactions between quarks and leptons is known as QCD.

To build up electroweak theory, we start with the quantum field theory of electromagnetic interaction known as QED. The Lagrangian density of a free fermion field ψ with mass m is given by

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x). \tag{1.1}$$

This Lagrangian is invariant under a phase transformation $\psi(x) \rightarrow e^{i\alpha}\psi(x)$, where α is a global phase which leads to the global invariance of the Lagrangian. This phase transformation is a part of $U(1)$ gauge group which is abelian in nature and is the symmetry gauge group of QED. In case of local phase (or gauge) transformation, $\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$. Here gauge parameter $\alpha(x)$ is space-time dependent due to which the kinetic energy term is no longer invariant under this transformation. This is due to an additional term $\partial_\mu \alpha(x)$. In order to compensate for this additional term, the derivative ∂_μ is replaced with covariant derivative D_μ, $\partial \rightarrow D_\mu = \partial_\mu - ieA_\mu(x)$. Here a new vector field A_μ has been introduced.

The Lagrangian now becomes

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x). \tag{1.2}$$

Under the phase transformation $D_\mu\psi(x) \rightarrow e^{i\alpha(x)}D_\mu\psi(x)$, the covariant derivative D_μ transform in the same way as field ψ. The vector field A_μ transforms as

$$A_\mu(x) \rightarrow A_\mu - \frac{1}{e}\partial_\mu \alpha(x). \tag{1.3}$$

The above transformation cancels the additional unwanted terms which spoils the gauge invariance of the Lagrangian. This field A_μ is known as gauge field which can be thought of as a physical photon field. Thus we see that the dynamics of the electromagnetic interaction is made gauge invariant by introducing a vector gauge field A_μ through the covariant derivative.

The entire system cannot be described by the Lagrangian (1.2). This is because the kinetic and mass terms for the gauge field A_μ are absent. Thus we need to add these terms for the gauge field A_μ which should be gauge invariant. The local gauge invariance insures that the photon field is massless. Including mass term for the field A_μ will violate the gauge invariance. The kinetic energy term for the gauge field A_μ is given by $\mathcal{L}_{kinetic} = -(1/4)F^{\mu\nu}F_{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor. Therefore the complete QED Lagrangian is given by

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{1.4}$$

Running on the similar lines, one can construct the gauge theory for electroweak interaction. This theory is based on the gauge group $SU(2)_L \times U(1)_Y$. The gauge theory based on the gauge group $SU(2)$ is non-abelian in nature as the generators of the theory do not commute with each other. By virtue of the $V - A$ nature of the weak interactions, only left-handed components of the fermion fields participate in the weak interactions. Therefore one identifies the weak isospin gauge group $SU(2)_L$ for weak interaction. The generators of group $SU(2)_L$ are expressed by the Pauli spin matrices ($\sigma^i, i = 1, 2, 3$) whereas the generator of the $U(1)_Y$ is the weak hypercharge (Y).

1.3 The SM Lagrangian

Running on the similar lines, one can construct the gauge theory for electroweak interaction. The most general renormalizable Lagrangian which is invariant under the local symmetry group $SU(2)_L \times U(1)_Y$

can be written as

$$\mathcal{L}_{\rm SM} = \mathcal{L}^{\rm gauge}_{\rm kinetic} + \mathcal{L}^{\rm fermion}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yuakawa}, \tag{1.5}$$

where

$$\begin{aligned}
\mathcal{L}^{\rm gauge}_{\rm kinetic} &= -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu},\\
\mathcal{L}^{\rm fermion}_{\rm kinetic} &= \overline{Q_{i,L}}i\not{D}Q_{i,L} + \overline{u_{i,R}}i\not{D}u_{i,R} + \overline{d_{i,R}}i\not{D}d_{i,R} + \overline{L_{i,L}}i\not{D}L_{i,L} + \overline{e_{i,R}}i\not{D}e_{i,R},\\
\mathcal{L}_{\rm Higgs} &= (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi),\\
\mathcal{L}_{\rm Yukawa} &= -Y^d_{ij}\overline{Q}_{i,L}\Phi d_{j,R} - Y^u_{ij}\overline{Q}_{i,L}\tilde{\Phi}u_{j,R} + Y^l_{ij}\overline{L_{i,L}}\Phi e_{j,R} + h.c.
\end{aligned} \tag{1.6}$$

The SM fermionic fields along with their quantum numbers are listed in Table 1.1.

Fermionic field	SU(2) representation	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Left handed quark doublet	$Q_{i,L} \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$	$(3, 2, 1/6)$
Up-type right handed quark	$u_{i,R} \equiv u_R, c_R, t_R$	$(3, 1, 2/3)$
Down-type right handed quark	$d_{i,R} \equiv d_R, s_R, b_R$	$(3, 1, -1/3)$
Left handed lepton doublet	$L_{i,L} \equiv \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$(1, 2, -1/2)$
Right handed charged lepton	$e_{i,R} \equiv e_R, \mu_R, \tau_R$	$(1, 1 - 1)$

Table 1.1: SM fermions along with their quantum numbers in SM gauge group.

The covariant derivatives which drive the interactions between the gauge fields and matter are given as

$$D_\mu = \partial_\mu - ig_s G^a_\mu \frac{\lambda_a}{2} - ig\left(W^+_\mu T_+ + W^-_\mu T_-\right) - ieA_\mu Q - \frac{ig'}{\cos\theta_W} Z^0_\mu \left(T_3 - \sin^2\theta_W Q\right). \tag{1.7}$$

Here $T_\pm$, T_3 are the generators of $SU(2)_L$. The coupling constants g_s, g, g' and e correspond to $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ and $U(1)_Q$ couplings respectively. Also, the electric charge is $Q = Y + T_3$, where Y is the generator of $U(1)_Y$. Further,

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm iW^2_\mu), \tag{1.8}$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \tag{1.9}$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu, \tag{1.10}$$

where the Weinberg angle θ_W is defined as $\cos\theta_W = g/\sqrt{g^2 + g'^2}$ and $\sin\theta_W = g'/\sqrt{g^2 + g'^2}$.

Gauge Group	Gauge coupling	Gauge bosons	Generators
$SU(3)_C$	g_s	G^a_μ	a=1,....,8, gluons
$SU(2)_L$	g	W^a_μ	a=1,2,3, W bosons
$U(1)_Y$	g'	B_μ	B boson

Table 1.2: SM gauge groups, couplings, bosons and generators.

Moreover, several quantities used in the Lagrangian in Eqs. (1.6) are defined as:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^a_\mu G^b_\nu, \quad a, b, c = 1, ..., 8, \tag{1.11}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^a_\mu W^b_\nu, \quad a, b, c = 1, 2, 3, \tag{1.12}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{1.13}$$

1.3.1 Spontaneous symmetry breaking and Higgs mechanism

We know that the SM gauge boson and the fermions are not massless. However the mass term in the Lagrangian spoils the gauge invariance. Therefore one needs to fix this. One option is to assume beforehand that the gauge symmetry $SU(2)_L$ is not the symmetry of nature and can be discarded. However, a more logical and appealing approach would be that without discarding $SU(2)_L$ as a fundamental symmetry there should be a mechanism which spontaneously break this symmetry. The symmetry group $SU(2)_L \times U(1)_Y$ should be broken spontaneously in such a way that the electromagnetic symmetry remains as the true symmetry at the low energy scale, i.e.,

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{EM}.$$

The spontaneous breakdown of the symmetry is achieved by the *Higgs mechanism* [7–12] which requires that the theory should include a scalar field which would be a multiplet of $SU(2)_L$. The gauge bosons attains mass by symmetry breaking and coupling to a scalar field, called Higgs field. The interaction of the fermion fields with this Higgs field is responsible for the fermion masses.

To guarantee gauge invariance of the electroweak Lagrangian, the two complex scalar (or four real scalar) fields Φ_i where $i = 1, 2.$ must belong to $SU(2) \times U(1)$ multiplets.

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} . \tag{1.14}$$

In 1967, Weinberg suggested that the most economical choice would be to arrange these four fields in an isospin doublet with weak hypercharge $Y = 1$. With this choice of hypercharge $Y = 1$, charges of these two complex scalar field can be calculated using Klein Nishijima formula: $Q = I_3 + \frac{Y}{2}$, where I_3 and Y are the components of weak isospin and hypercharge, respectively. For field Φ_1, $I_3 = +\frac{1}{2}$ and $Y = 1 \Rightarrow Q = +1$. Similarly for field Φ_2, $I_3 = -\frac{1}{2}$ and $Y = 1 \Rightarrow Q = 0$. The above multiplet can now be represented as

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} , \tag{1.15}$$

where superscripts show charge of complex fields as Φ^+ and Φ^0 are called positive and neutral scalar fields, respectively. Every complex field can be represented by two real scalar field. So finally the Higgs field can be described by a doublet in $SU(2)_L$ with four degree of freedom as follows

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} , \tag{1.16}$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are four real scalar fields.

The Lagrangian for the scalar Higgs field can be given as

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) , \tag{1.17}$$

where he Higgs potential $V(\Phi)$ has the following form

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2 = \frac{\lambda}{2} \left(\Phi^\dagger \Phi + \frac{\mu^2}{\lambda} \right)^2 + \text{const.} \tag{1.18}$$

The minimum of above potential energy can be obtained by writing $\Phi^\dagger \Phi$ as

$$\Phi^\dagger \Phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \frac{1}{2} \sum_{i=1}^{4} \phi_i^2 . \tag{1.19}$$

We then have

$$V(\Phi) = \sum_{i=1}^{4} \left[\frac{1}{2}\mu^2\phi_i^2 + \frac{\lambda}{8}\phi_i^4\right], \tag{1.20}$$

and the condition of the extremum

$$\frac{\partial V(\Phi)}{\partial \phi_i} = 0, \tag{1.21}$$

gives

$$\phi_i \left[\mu^2 + \frac{\lambda}{2}\sum_{i=1}^{4}\phi_i^2\right] = 0. \tag{1.22}$$

This implies that the extremum of $V(\Phi)$ would occur at

$$\phi_i = 0 \ (i = 1, 2, 3, 4), \tag{1.23}$$

or at

$$\sum_{i=1}^{4}\phi_i^2 = -\frac{2\mu^2}{\lambda}. \tag{1.24}$$

The second order differential

$$\frac{\partial^2 V(\Phi)}{\partial \phi_i^2} = \mu^2 + \frac{3\lambda}{2}\sum_{i=1}^{4}\phi_i^2, \tag{1.25}$$

which for Eq. (1.23) becomes

$$\frac{\partial^2 V(\Phi)}{\partial \phi_i^2} = \mu^2, \tag{1.26}$$

and for Eq. (1.24) becomes

$$\frac{\partial^2 V(\Phi)}{\partial \phi_i^2} = -2\mu^2. \tag{1.27}$$

For a physical system, we must have $\lambda > 0$ otherwise there will not be a stable vacuum. However, there is no such restriction on the other parameter μ^2 which can be positive as well as negative. First, we consider $\mu^2 > 0$. In this case we can see from Eqs. (1.26) and (1.27) that the minimum of potential energy is for $\phi_i = 0$, and the symmetry of the Lagrangian density (1.17) is not broken. In this case μ^2 can be simply interpreted as the square of mass of the particle with field Φ.

Now, we consider $\mu^2 < 0$ for which the mass parameter in Higgs theory μ^2 becomes a negative quantity and symmetry breaking happens spontaneously. In that situation, $\langle\Phi^\dagger\Phi\rangle = 0$ becomes a maximum and the global minimum gives rise to a Higgs vacuum expectation value (*vev*), v: $\langle\Phi^\dagger\Phi\rangle = |\langle\phi_1\rangle|^2 + |\langle\phi_2\rangle|^2 + |\langle\phi_3\rangle|^2 + |\langle\phi_4\rangle|^2 = -2\mu^2/\lambda = v^2$ (from Eq. (1.24)). This corresponds to an infinity of degenerate minima but choice of a particular minima, say, $\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0$ and $|\langle\phi_3\rangle|^2 = v^2 = -2\mu^2/\lambda$ breaks the symmetry. After the symmetry breaking, the Higgs field can be redefined using only physics components by making use of gauge freedom. This is known as unitary gauge and the Higgs doublet becomes

$$\Phi = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}. \tag{1.28}$$

Here Φ^+ and $\mathrm{Im}(\Phi^0)$, known as Goldstone boson, are "rotated away" by choice of a particular minima mentioned above. To study the spectrum, we consider fluctuations around the minima. So $\mathrm{Re}(\Phi^0) = \phi_4$ is shifted from the global minima where $h(x)$ represents the oscillation parameter or fluctuations around the minima. Now if we look at the kinetic part of the Higgs Lagrangian, it will give the mass terms associated with the gauge bosons. The kinetic term becomes

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) \sim \frac{1}{4}g^2v^2W^{+\mu}W^-_\mu + \frac{1}{8}v^2(gW^3_\mu - g^{'}B_\mu)^2 + ... \tag{1.29}$$

From Eq. (1.29), the mass term for charged gauge boson $W^\pm$ is $M_W = (1/2)g\,v$. The second term in Eq. (1.29) can be written as

$$\frac{1}{8}v^2(gW^3_\mu - g^{'}B_\mu)(gW^{3\mu} - g^{'}B^\mu) = \frac{v^2}{8}\begin{pmatrix}W^3_\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2, & -gg^{'} \\ -gg^{'} & g^{'2}\end{pmatrix}\begin{pmatrix}W_{3\mu} \\ B^\mu.\end{pmatrix} \tag{1.30}$$

From this Eq. (1.30), we can see that mass matrix is not diagonal so W^3_μ and B_μ cannot be mass eigenstates. The off-diagonal elements allow an admixture of the fields W^3_μ and B_μ . The physical gauge bosons correspond to a basis in which mass matrix is diagonal and the eigenvalues of the non-diagonal mass matrix will be the masses of physical gauge bosons. The physical gauge boson masses can be read as

$$M_A = 0 \quad \text{and} \quad M_Z = \tfrac{1}{2}v\sqrt{g^2 + g'^2}.$$

The massless neutral gauge boson A can be identified as photon and the massive neutral gauge boson can be identified as Z^0 . These physical field can be defined as

$$A_\mu = \frac{g^{'}W^3_\mu + gB_\mu}{\sqrt{g^2 + g^{'2}}},$$
$$Z_\mu = \frac{gW^3_\mu - g^{'}B_\mu}{\sqrt{g^2 + g^{'2}}} \tag{1.31}$$

Now we can see that all the massive gauge have got the mass and photon is massless. It should be noted that the Higgs mechanism is responsible for the mass generation of fermions and gauge bosons, but not for its own mass.

On 4 July 2012, the ATLAS [17] and CMS [18] experiments at CERN's Large Hadron Collider (LHC) announced they had each observed a new particle in the mass region around 125 GeV. This particle is consistent with the Higgs boson. On 8 October 2013 the Nobel prize in physics was awarded jointly to François Englert and Peter Higgs "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider."

1.4 The CKM paradigm

The Yukawa interactions of Higgs field with quarks in the SM are

$$\mathcal{L}_{\text{Yukawa}} = -Y^d_{ij}\overline{Q}_{i,L}\Phi d_{j,R} - Y^u_{ij}\overline{Q}_{i,L}\tilde{\Phi}u_{j,R} + h.c.. \tag{1.32}$$

Here $Y^{u,d}$ are 3×3 complex Yukawa matrices, Φ is the Higgs doublet, $\tilde{\Phi} = i\sigma^2\Phi$ (σ^2 = Pauli matrix), i,j are generation labels. The left handed quark doublets $Q_{i,L}$ and the right handed quark singlets $u_{j,R}$, $d_{j,R}$ are written in the flavour eigenbasis. When the neutral Higgs field acquires a *vev* $\langle\Phi\rangle = (0, v/\sqrt{2})^T$, fermion mass arises. The quark mass terms are given by

$$\mathcal{L}_{\text{Yukawa}} \supset -(\overline{u},\, \overline{c},\, \overline{t})_L\,\mathcal{M}^u\begin{pmatrix}u \\ c \\ t\end{pmatrix}_R - (\overline{d},\, \overline{s},\, \overline{b})_L\,\mathcal{M}^d\begin{pmatrix}d \\ s \\ b\end{pmatrix}_R + h.c. \tag{1.33}$$

Here

$$\mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}} Y^u_{ij}, \quad \mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}} Y^d_{ij} \tag{1.34}$$

are the quark mass matrices in flavour space.

In order to find out the quark mass eigenstate, we need to diagonalize the mass matrices by a proper unitary transformation. The unitary matrices which diagonalize the mass matrices can be defined as

$$U_R^{-1} \mathcal{M}^u U_L = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad D_R^{-1} \mathcal{M}^d D_L = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \tag{1.35}$$

The quark mass eigenstates and flavour eigenstates are related through the following relations

$$d'_L = D_L\, d_L, \quad d'_R = D_R\, d_R, \quad u'_L = U_L\, u_L, \quad u'_R = U_R\, u_R. \tag{1.36}$$

The charged current interaction term in the mass basis is then given by

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{u}_{L,i} \gamma_\mu d_{L,j} W_\mu = \frac{g}{\sqrt{2}} \overline{u}'_{L,i} \gamma_\mu (U_L D_L^\dagger) d'_{L,j} W_\mu = \frac{g}{\sqrt{2}} \overline{u}'_{L,i} \gamma_\mu V^{\rm CKM}_{ij} d'_{L,j} W_\mu. \tag{1.37}$$

Here $V^{\rm CKM}$ is the Cabibo-Kobayashi-Maskawa (CKM) matrix which is defined as

$$V^{\rm CKM} = U_L\, D_L^\dagger. \tag{1.38}$$

It should be noted that the CKM matrix is a unitary matrix, $VV^\dagger = I$. Further, the rotation of right handed quark fields has no physical consequence in the SM as u_R and d_R do not appear in charged current interactions. Hence, instead of the physical quarks d, s and b, the states to be used in the weak interactions are $d^{'}$, $s^{'}$ and $b^{'}$. The weak interaction states $d^{'}$, $s^{'}$ and $b^{'}$ are "rotated" in the plane of physical quark states. These states are related by the CKM matrix.

$$\begin{pmatrix} d^{'} \\ s^{'} \\ b^{'} \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1.39}$$

where

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{1.40}$$

The unitarity of CKM matrix implies that the Flavour changing neutral current (FCNC) interactions cannot occur at tree level in the SM. These interactions can only occur at the loop diagrams and hence they are suppressed. Thus FCNC transitions can serve as an important tool to probe new physics. A $n \times n$ quark mixing matrix can be parametrized by $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases. For two generations of quarks, the CKM matrix is basically an orthogonal matrix which contains only one angle. Hence it is deprived of a CP violating phase. Therefore at least three generations of quarks are required to have CP violation.

The 3×3 CKM matrix can be parametrized by three angles and a single complex phase. The single phase of the CKM matrix is the only source of CP violation in the SM. This phase manifests itself in several observables in K and B decays. However, unlike parity violation, which is maximal, the observed CP violation is small. For e.g., the observed CP violation in the decays of neutral kaons is one part in a thousand [19]. We will see in the next section 1.5.2 that that the CKM paradigm is unable to account for the observed matter-antimatter asymmetry of the universe. The amount of baryons in the Universe predicted using the CKM mechanism falls several orders of magnitude short of the observed experimental value.

The standard parametrization of CKM is done by making use of three angles (θ_{12}, θ_{23} and θ_{13}) and one phase (δ) [20].

$$\begin{aligned} V_{\mathrm{CKM}} &= R_1(\theta_{23})\Gamma(\delta)R_2(\theta_{13})\Gamma(-\delta)R_3(\theta_{12}) \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \end{aligned} \tag{1.41}$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, $R(\theta_{ij})$ is the rotational matrix in $i-j$ plane ($i, j = 1, 2, 3$) and $\Gamma(\delta) = \mathrm{diag}(1, 1, \mathrm{e}^{\mathrm{i}\delta})$. From experiments the well known hierarchy between the mixing angles is $s_{13} << s_{23} << s_{12} << 1$. This is evident from the measurements of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$. The element $|V_{us}|$ is determined from the semi-leptonic kaon decays $K \to \pi\ell\nu$ ($K_{\ell 3}$), where ℓ is either an electron or muon, using the form factor at zero momentum transfer, $f_+(0)$. Using the average $f_+(0) = 0.9706 \pm 0.0027$ [21] obtained from $N_f = 2+1+1$ lattice QCD calculations, $|V_{us}|$ is obtained to be 0.2231 ± 0.0007. The matrix element $|V_{cb}|$ can be determined from exclusive and inclusive semi-leptonic decays of B mesons to charm. The current value determined from inclusive (exclusive) decay mode is $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$ ($|V_{cb}| = (41.0 \pm 1.4) \times 10^{-3}$) [20, 22]. The CKM element $|V_{ub}|$ is determined from inclusive $B \to X_u l\bar{\nu}$ decay as well as the exclusive mode $B \to \pi l\bar{\nu}$. The average of these determinations gives $|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}$ [20, 22].

An alternate parametrization, the Wolfenstein parametrization [23] was introduced in 1983. This parametrization depends on four parameters A, λ, ρ and η and each matrix element is expanded as a power series of the λ parameter:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{1.42}$$

Here the matrix elements are written up to the order of λ^3, with $\lambda \sim 0.23$ the Cabibbo angle. The four parameters are defined as

$$\lambda \equiv s_{12}, \quad A \equiv \frac{s_{23}}{\lambda^2}, \quad \rho = \frac{s_{13}\cos\delta}{s_{12}s_{23}}, \quad \eta = \frac{s_{13}\sin\delta}{s_{12}s_{23}}. \tag{1.43}$$

In addition one often uses $\bar{\rho}$ and $\bar{\eta}$ instead of ρ and η which are defined as

$$\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}. \tag{1.44}$$

and in terms of Wolfenstein parameters, the relation is

$$\rho + i\eta \simeq \left(1 + \frac{\lambda^2}{2}\right)(\bar{\rho} + i\bar{\eta}) + \mathcal{O}(\lambda^4). \tag{1.45}$$

The values of Wolfenstein parameters, taken form Unitarity Triangle (UT) fit [24], are

$$\rho = 0.152 \pm 0.014, \quad \eta = 0.357 \pm 0.010, \quad A = 0.826 \pm 0.012, \quad \lambda = 0.225 \pm 0.001 \tag{1.46}$$

and

$$\bar{\rho} = 0.148 \pm 0.013, \quad \bar{\eta} = 0.348 \pm 0.010. \tag{1.47}$$

Hence the CKM matrix elements can be written as [24]

$$V_{\mathrm{CKM}} = \begin{pmatrix} (0.97431 \pm 0.00012) & (0.22514 \pm 0.00055) & (0.00365 \pm 0.00010)e^{i(-66.8\pm 2.0)^\circ} \\ (-0.22500 \pm 0.00054)e^{i(0.0351\pm 0.0010)^\circ} & (0.97344 \pm 0.00012)e^{i(-0.001880\pm 0.000052)^\circ} & (0.04241 \pm 0.00065) \\ (0.00869 \pm 0.00014)e^{i(-22.23\pm 0.63)^\circ} & (-0.04124 \pm 0.00056)e^{i(1.056\pm 0.032)^\circ} & (0.999112 \pm 0.000024) \end{pmatrix}.$$

1.5 Limitations of SM

Although SM successfully accounts for the phenomena within its domain, still it cannot be considered as the quintessential theory of fundamental interactions of nature. There are strong experimental evidences as well as theoretical motivations that SM is an effective field theory of a more general theory at high energy scales. It is glaringly evident from the gauge structure of the SM that it cannot be considered as a theory of everything as it fails to account for the gravitational interactions. Therefore one needs to extend SM by adding quantized version of gravity. Some of the other experimental evidences and theoretical motivations for the beyond SM physics are discussed below.

1.5.1 Flavour Structure

The flavour structure of the SM depends upon following free parameters in the SM Lagrangian:

- three gauge couplings (mentioned in Table 1.2) are related to the gauge sector,
- there are two parameters (μ and λ appearing in Eq. (1.18)) in the Higgs sector,
- fourteen parameters are related to the masses and mixing of quarks and leptons. These are six quark masses, three masses of the charged lepton, three angles in the quark mixing matrix, one weak CP violating phase and one strong CP violating term.

The three flavours of neutrinos were assumed to massless in the SM. This implies that neutrinos don't mix and hence flavour change is not allowed in the SM. The phenomenon of neutrino oscillation was first predicted by Bruno Pontecorvo in 1957 [25] and is now a well established fact owing to a series of experiments measuring fluxes of neutrinos produced in the Sun, in the atmosphere, in accelerators and in nuclear reactors. The Ray Davis's Homestake experiment was the first to detect a deficit in the solar neutrinos flux in comparison to the Standard Solar Model predictions [26]. However, the first conclusive evidence of this deficit was provided by the Sudbury Neutrino Observatory (SNO) collaboration [27] in 2001. The first experimental evidence for atmospheric neutrino oscillations was provided by Super-Kamiokande experiment in 1998 [28]. The discovery of neutrino oscillations implies non zero neutrino masses and mixing angles. After including neutrino oscillation, there are nine additional parameters in form of three neutrino masses, three mixing angles and three CP violating phases. Due to this freedom, the SM fails to provide any insight into the hierarchies among fermion masses and mixing angles. The hierarchy between the fermion mass ranges from 0.511 MeV (mass of electron) to about 173 GeV (mass of top quark). This becomes even worse, if one includes neutrino masses which are less than 1 eV. Although we can measure the numerical values of these parameters, there is no explanation or understanding as to why this flavour structure exists. Further there is no mechanism within SM that can explain and predict them. Therefore one hopes that beyond SM models can provide answers to the flavour puzzle.

1.5.2 Matter Antimatter asymmetry

SM fails to account for the observed matter-antimatter asymmetry of the Universe. The particle spectrum of SM includes both baryonic matter and anti-matter, and hence we expect that the number density of both types of matter must be equal. However, the universe is matter dominated. As known from different types of astronomical data, the universe is only populated with particles, while antiparticles are practically absent. A small number of the observed anti-protons or positrons in cosmic rays can be explained by their secondary origin through particle collisions. Further, there is no evidence of large antimatter objects such as anti-stars, anti-planets or gaseous clouds of antimatter in our universe. The asymmetry between matter and antimatter is quantified as

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma}, \tag{1.48}$$

where n_B and $n_{\bar{B}}$ are the number density of baryons and anti-baryons respectively and n_γ is the number density of photons in the universe. The Wilkinson Microwave Anisotropy Probe (WMAP) has measured η_B to be $(6.19 \pm 0.15) \times 10^{-10}$ [29].

This weird asymmetry can be generated if following Sakharov's conditions [30] are satisfied:

1. Baryon number violation,

2. Breaking of symmetry between particles and antiparticles, both C and CP,

3. Deviation from thermal equilibrium.

Although there are sources of CP violation within the SM due to a phase in the quark mixing matrix, it is not sufficient to generate an asymmetry of the observed magnitude. Therefore one needs to go beyond the CKM paradigm of the SM. Thus various CP violating observables are particularly important in probing new physics.

1.5.3 Dark Matter and Dark Energy

There is overwhelming evidence that the matter content of our Universe cannot be explained by the particle content of the SM. The existence of the dark matter has been confirmed by various astrophysical and cosmological evidences at different scales [31–35]. It turns out that dark matter constitutes 26.8% of the universe, whereas 68.3% of the universe is filled by exotic matter known as dark energy. The expansion of the universe has not been slowing due to gravity, as seemed logical, but it has been accelerating. The unknown so called "anti-gravity" force at work is termed "dark energy". Its density remains same even after expansion. All normal matter adds only up to 4.9% of the universe [36].

These evidences include measurements of galactic rotation curves, precision measurements of the Cosmic Microwave Background (CMB) and observation of the abundances of heavy isotopes produced by primeval nucleosynthesis. These observations indicate that a substantial fraction of the universe's energy density is in the form of matter which interacts very weakly with the SM particles. This fact is corroborated by the numerical simulations of large-scale structure which indicates that the dark matter should be non-relativistic, known as cold dark matter. One of the favoured candidates for the cold dark matter particle is weakly interactive massive particle (WIMP), with masses in the range of GeV to TeV. Another possibility of finding dark matter is in the postulated primordial black holes which were formed in the early universe, much before the stars were formed. Within the SM, there are no candidates with such properties. Thus one needs new physics to account for non-baryonic cold dark matter.

1.5.4 Flavour Anomalies

Flavour physics is one of the most elegant ways to probe beyond SM physics. It is a rich laboratory that probes physics beyond the SM by precision measurements that look for the virtual production of new particles in quantum loops. The promising feature lies in the discovery potential of rare processes in various flavour systems, in particular B-meson systems. The currently running LHC experiments, ATLAS [17], CMS [18] and LHCb [37], at CERN along with BaBar experiment [38] at Stanford Linear Accelerator Center (SLAC) and Belle experiment [39] at KEKB have already provided several tantalizing hints of beyond SM physics [1]. These hints are in the rare decays of B mesons, in particular the decays induced by the quark level transitions $b \to s\, l^+\, l^-$ ($l = e,\, \mu$) [40–50] and $b \to c\, l\, \bar{\nu}$ ($l = e,\, \mu,\, \tau$) [51–61] which are discussed in detail in Sec.3.3 and 3.4, respectively.

The piling up of anomalies in these sectors in a coherent fashion can be considered as signatures of new physics. In order to identify the Lorentz structure of this possible new physics, several groups across the

[1] A brief description of relevant experiments at the B-factories and at the LHC are provided in Sec. 3.2.

globe performed model independent analyses within the framework of effective field theories, in $b \to s\mu\mu$ [62–86] as well as in $b \to c\tau\nu$ sector [87–95].

Motivated by these model independent analyses, several new physics models were proposed, see for e.g. [84, 87, 96–139]. The Belle-II experiment [140] at Super-KEKB and high luminosity LHCb experiment have immense potential to convert some of these signatures into possible discoveries of new physics and hence reveal the hidden secrets of the Universe.

1.6 Motivation and Plan of the thesis

Precision measurements in flavour sector have lead to a number of discoveries in particle physics. For e.g., construction of $(V - A)$ theory due to the smallness of $\pi^- \to e^-\bar{\nu}$[2], prediction of second neutrino due to non-occurrence of $\mu \to e\gamma$, prediction of third generation due to the discovery of CP violation. Therefore, flavour decays play an important role in probing beyond SM physics. In this regard, study of decays of B mesons are particularly interesting. This is because heavier the particle, there are large number of decay channels. Further, we should look for new physics in the decays suppressed in SM. Many B decays are suppressed and experimentally accessible, e.g. decays induced by quark level transition $b \to s\,\mu^+\mu^-$. Also, many observables in the B meson sector are sensitive to CP violation. The fact that no new particles have been observed at the LHC so far has put B-physics at the forefront of the hunt for beyond SM physics. The importance of B-physics has increased considerably over last few years due to the fact that several apparent deviations from the SM have been observed in semileptonic decays of B mesons induced by the quark level transitions $b \to s\,l^+\,l^-$ $(l = e,\, \mu)$ and $b \to c\,l\,\bar{\nu}$ $(l = e,\, \mu,\, \tau)$.

Motivated by recent developments in the B-sector, in this thesis, we have studied the impact of anomalies in $b \to s\,l^+\,l^-$ transition. Assuming new physics only in the muon sector, we study the impact of new physics solutions in $b \to s\,l^+\,l^-$ on the branching ratio of $B_s^* \to \mu^+\,\mu^-$ decay as its sensitivity to new physics is quite complementary to that of $B_s \to \mu^+\mu^-$. This is because the branching ratio of $B_s^* \to \mu^+\mu^-$ decay is sensitive to different combinations of Wilson coefficients whereas $Br(B_s \to \mu^+\mu^-)$ is sensitive only to the axial-vector operator O_{10}. Further, as B_s^* meson is a vector meson, $Br(B_s^* \to \mu^+\mu^-)$ is not chirally suppressed. This helps to reduce the effect of short lifetime of B_s^* meson. Moreover, this decay is theoretically very clean as the amplitude depends only upon decay constants which are accurately determined in the lattice QCD and the invariant mass of the process, $q^2 = m^2_{B_s^*} = 28\ \text{GeV}^2$, is well above the charmonium states. We then introduce a new observable in $B_s^* \to \mu^+\,\mu^-$ decay, the lepton polarization asymmetry, and study its sensitivity to various types of new physics. Further, we also relate anomalies in $b \to c\,\tau\,\bar{\nu}$ sector to $B_s^* \to \tau^+\,\tau^-$ decay. Furthermore, we consider the case when new physics contributes only to $b \to s\,e^+\,e^-$ decay and provide first complete model independent analysis of new physics in this sector. Finally, we study the impact of measurements in B-sector on FCNC top quark decay $t \to c\,Z$.

The thesis is organized in following chapters as described below.

- In chapter 2, we first describe the effective field theory approach to four fermion processes. We then describe the application of this approach to $b \to s\,l^+\,l^-$ and $b \to c\,\tau\,\bar{\nu}$ transitions.

- In chapter 3, we describe the anomalies in semileptonic B decays. First we describe anomalous measurements related to $b \to s\,l^+\,l^-$ decays and then we describe anomalies $b \to c\,l\,\bar{\nu}$ sector.

- In chapter 4, we study the impact of current new physics solutions in $b \to s\mu^+\mu^-$ on the branching ratio of $B_s^* \to \mu^+\mu^-$ [141].

[2] The phase-space considerations demand larger decay of $\pi^- \to e^-\bar{\nu}$ in comparison to that of $\pi^- \to \mu^-\bar{\nu_\mu}$. However, the experimental value of the ratio $\frac{\Gamma(\pi^- \to e^-\bar{\nu_e})}{\Gamma(\pi^- \to \mu^-\bar{\nu_\mu})}$ is 1.283×10^{-4}. This is quite contrary to what one would expect from phase-space considerations. This is known as helicity suppression and is attributed to the left-handed or (V-A) structure of the charged current weak interaction.

- In chapter 5, we study new physics effects in lepton polarization asymmetry of muons in $B_s^* \to \mu^+\mu^-$ decay [142].
- In chapter 6, we establish correlations between branching ratio of $B_s^* \to \tau^+\tau^-$ and $R_{D,D^*,J/\psi}$ anomalies [142].
- In chapter 7, we provide a complete model independent analysis of new physics in $b \to s e^+ e^-$ decay [143].
- In chapter 8, we investigate the impact of measurements in B and K meson sectors on the effective anomalous tcZ couplings [144].