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Probing new physics through lepton polarization asymmetry of $B_s^* \to \mu^+\mu^-$ decay

5.1 Introduction

In chapter 4, a model independent analysis of $B_s^* \to \mu^+\mu^-$ decay was performed to identify the new physics operators which can lead to a large enhancement of its branching ratio. It was found that such an enhancement is not possible due to the constraints from the present $b \to s\mu^+\mu^-$ data. Further, it was seen that the branching ratio of $B_s^* \to \mu^+\mu^-$ cannot discriminate between the existing new physics solutions. Therefore, it would be desirable to construct a new observable related to this decay mode to see whether such an observable has the potential to discriminate between the existing new physics solutions in $b \to s\mu^+\mu^-$ transition. In particular, we focus on the two distinct solutions, one with the operator of the form $(\bar{s}\gamma^\alpha P_L b)(\bar{\mu}\gamma_\alpha \mu)$ and the other whose operator is a linear combination of $(\bar{s}\gamma^\alpha P_L b)(\bar{\mu}\gamma_\alpha \mu)$ and $(\bar{s}\gamma^\alpha P_L b)(\bar{\mu}\gamma_\alpha \gamma_5 \mu)$. It was pointed out in Ref. [69, 70] that the third solution involving chirality flipped operator $(\bar{s}\gamma^\alpha P_R b)(\bar{\mu}\gamma_\alpha \mu)$ is disfavoured both experimentally and theoretically.

In this chapter, we consider the longitudinal polarization asymmetry of muon in $B_s^* \to \mu^+\mu^-$ decay ($\mathcal{A}_{LP}(\mu)$) to explore such a possibility. This asymmetry is theoretically clean because it has a very mild dependence on the decay constants unlike the branching ratio. We first calculate the SM prediction of $\mathcal{A}_{LP}(\mu)$ and then study its sensitivity to the two new physics solutions.

This chapter is organized as follows. In Sec. 5.2, we obtain the theoretical expressions for the longitudinal polarization asymmetry of the final state leptons in $B_s^* \to l^+ l^-$ decays, where $l = e, \mu$ or τ. This is done for the SM and for the case of new physics V and A operators. In Sec. 5.3, we obtain predictions of $\mathcal{A}_{LP}(\mu)$ in both the SM and the two new physics solutions which explain all $b \to s\mu^+\mu^-$ anomalies. The conclusions are presented in Sec. 5.4.

5.2 Calculation of Longitudinal Polarization Asymmetry for $B_s^* \to l^+l^-$ decay

5.2.1 Longitudinal Polarization Asymmetry in the SM

The purely leptonic decay $B_s^* \to l^+ l^-$ is induced by the quark level transition $b \to sl^+l^-$. In the SM, the corresponding effective Hamiltonian is given by Eq. 4.1. The decay rate for $B_s^* \to \mu^+\mu^-$ is obtained to be

$$\Gamma_{SM} = \frac{\alpha_{em}^2 G_F^2 f_{B_s^*}^2 m_{B_s^*}^3}{96\pi^3}|V_{ts}V_{tb}^*|^2\sqrt{1-\frac{4m_l^2}{m_{B_s^*}^2}} \times \left[\left(1+\frac{2m_l^2}{m_{B_s^*}^2}\right)\left|C_9^{eff}+\frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}}C_7^{eff}\right|^2 + \left(1-\frac{4m_l^2}{m_{B_s^*}^2}\right)|C_{10}|^2\right]. \tag{5.1}$$

We define the longitudinal polarization asymmetry for the final state leptons in $B_s^* \to l^+l^-$ decay. The unit longitudinal polarization four-vector in the rest frame of the lepton (l^+ or l^-) is defined as

$$\overline{s}_{l^\pm}^\alpha = \left(0, \pm\frac{\overrightarrow{p_l}}{|\overrightarrow{p_l}|}\right). \tag{5.2}$$

In the dilepton rest frame (which is also the rest frame of B_s^* meson), these unit polarization vectors become

$$s_{l^\pm}^\alpha = \left(\frac{|\overrightarrow{p_l}|}{m_l}, \pm\frac{E_l}{m_l}\frac{\overrightarrow{p_l}}{|\overrightarrow{p_l}|}\right), \tag{5.3}$$

where E_l, $\overrightarrow{p_l}$ and m_l are the energy, momentum and mass of the lepton (l^+ or l^-) respectively. We can define two longitudinal polarization asymmetries, $\mathcal{A}_{LP}^+$ for l^+ and $\mathcal{A}_{LP}^-$ for l^-, in the decay $B_s^* \to l^+ l^-$ as [197, 207, 208]

$$\mathcal{A}_{LP}^\pm = \frac{[\Gamma(s_{l^-}, s_{l^+}) + \Gamma(\mp s_{l^-}, \pm s_{l^+})] - [\Gamma(\pm s_{l^-}, \mp s_{l^+}) + \Gamma(-s_{l^-}, -s_{l^+})]}{[\Gamma(s_{l^-}, s_{l^+}) + \Gamma(\mp s_{l^-}, \pm s_{l^+})] + [\Gamma(\pm s_{l^-}, \mp s_{l^+}) + \Gamma(-s_{l^-}, -s_{l^+})]}. \tag{5.4}$$

If the two spin projections, s_{l^-} and s_{l^+} are the same, the decay rate is given by

$$\begin{aligned}\Gamma(\pm s_{l^-}, \pm s_{l^+}) =&\mathcal{N}\left[\frac{4m_l^2}{3}|\mathcal{C}|^2 + \frac{\mathcal{C}C_{10}^*}{6m_l m_{B_s^*}}\left\{i\sqrt{m_{B_s^*}^2-4m_l^2}\left(\varepsilon_{\alpha\beta\gamma\nu}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu + \varepsilon_{\alpha\beta\gamma\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^+}^\sigma\right)\right.\right.\\ &\left.+im_l m_{B_s^*}\left(\varepsilon_{\alpha\beta\nu\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta s_{l^-}^\nu s_{l^+}^\sigma - \varepsilon_{\beta\gamma\nu\sigma}p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu s_{l^+}^\sigma\right)\right\} + \frac{\mathcal{C}^*C_{10}}{6m_l m_{B_s^*}}\left\{-i\sqrt{m_{B_s^*}^2-4m_l^2}\right.\\ &\left(\varepsilon_{\alpha\beta\gamma\nu}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu + \varepsilon_{\alpha\beta\gamma\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^+}^\sigma\right)\\ &\left.\left.-im_l m_{B_s^*}\left(\varepsilon_{\alpha\beta\nu\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta s_{l^-}^\nu s_{l^+}^\sigma - \varepsilon_{\beta\gamma\nu\sigma}p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu s_{l^+}^\sigma\right)\right\}\right],\end{aligned} \tag{5.5}$$

For opposite spin projections of s_{l^-} and s_{l^+}, we have

$$\begin{aligned}\Gamma(\mp s_{l^-}, \pm s_{l^+}) = \mathcal{N}&\left[\frac{2m_{B_s^*}^2}{3}|\mathcal{C}|^2 + \frac{\mathcal{C}C_{10}^*}{6m_l m_{B_s^*}}\left\{m_l m_{B_s^*}\left(-i\varepsilon_{\alpha\beta\nu\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta s_{l^-}^\nu s_{l^+}^\sigma + i\varepsilon_{\beta\gamma\nu\sigma}p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu s_{l^+}^\sigma\right.\right.\right.\\ &\left.\mp 4m_{B_s^*}\sqrt{m_{B_s^*}^2-4m_l^2}\right) - i\sqrt{m_{B_s^*}^2-4m_l^2}\left.\left(\varepsilon_{\alpha\beta\gamma\nu}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu + \varepsilon_{\alpha\beta\gamma\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^+}^\sigma\right)\right\}\\ &+\frac{\mathcal{C}^*C_{10}}{6m_l m_{B_s^*}}\left\{i\sqrt{m_{B_s^*}^2-4m_l^2}\left(\varepsilon_{\alpha\beta\gamma\nu}p_{l^-}^\alpha p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu + \varepsilon_{\alpha\beta\gamma\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta s_{l^-}^\nu s_{l^+}^\sigma\right) + m_l m_{B_s^*}\right.\\ &\left.\left(\mp 4m_{B_s^*}\sqrt{m_{B_s^*}^2-4m_l^2} + i\varepsilon_{\alpha\beta\nu\sigma}p_{l^-}^\alpha p_{B_s^*}^\beta s_{l^-}^\nu s_{l^+}^\sigma - i\varepsilon_{\beta\gamma\nu\sigma}p_{B_s^*}^\beta p_{l^+}^\gamma s_{l^-}^\nu s_{l^+}^\sigma\right)\right\}\\ &\left.+\frac{2}{3}\left(m_{B_s^*}^2-4m_l^2\right)|C_{10}|^2\right].\end{aligned} \tag{5.6}$$

In Eqs. (5.5) and (5.6), we have used the abbreviations $\mathcal{N} = \frac{\alpha_{em}^2 G_F^2}{128\pi^3}|V_{tb}V_{ts}^*|^2 f_{B_s^*}^2\sqrt{m_{B_s^*}^2-4m_l^2}$, $\mathcal{C} = \left(C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}}C_7^{eff}\right)$. Using Eqs. (5.4),(5.5) and (5.6), we get the lepton polarization asymmetry to be

$$\mathcal{A}_{LP}^{\pm}|_{SM} = \mp \frac{2\sqrt{1-\frac{4m_l^2}{m_{B_s^*}^2}}\; Re\left[\left(C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}} C_7^{eff}\right) C_{10}^*\right]}{\left(1+2m_l^2/m_{B_s^*}^2\right)\left|C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}} C_7^{eff}\right|^2 + \left(1-4m_l^2/m_{B_s^*}^2\right)|C_{10}|^2}. \tag{5.7}$$

5.2.2 Longitudinal polarization asymmetry in presence of new physics

We now investigate the lepton polarization asymmetry in the presence of new physics. As the new physics solutions to the $b \to s l^+ l^-$ anomalies are in the form of V and A operators, we consider the addition of these operators to the SM effective Hamiltonian of $b \to s l^+ l^-$.

The effective Hamiltonian now takes the form

$$\mathcal{H}_{eff}(b \to s l^+ l^-) = \mathcal{H}_{SM} + \mathcal{H}_{VA}, \tag{5.8}$$

where $\mathcal{H}_{VA}$ is

$$\mathcal{H}_{VA} = \frac{\alpha_{em} G_F}{\sqrt{2}\pi} V_{ts}^* V_{tb}\left[C_9^{NP}(\bar{s}\gamma^\mu P_L b)(\bar{l}\gamma_\mu l) + C_{10}^{NP}(\bar{s}\gamma^\mu P_L b)(\bar{l}\gamma_\mu\gamma_5 l)\right].$$

Here $C_{9(10)}^{NP}$ are the new physics Wilson coefficients. Within this framework, the branching ratio and $\mathcal{A}_{LP}$ are obtained to be

$$\mathcal{B}(B_s^* \to l^+ l^-) = \frac{\alpha_{em}^2 G_F^2 f_{B_s^*}^2 m_{B_s^*}^3 \tau_{B_s^*}}{96\pi^3}|V_{ts}V_{tb}^*|^2\sqrt{1-\frac{4m_l^2}{m_{B_s^*}^2}}\left[\left(1+\frac{2m_l^2}{m_{B_s^*}^2}\right)\left|C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}} C_7^{eff} + C_9^{NP}\right|^2 + \left(1-\frac{4m_l^2}{m_{B_s^*}^2}\right)|C_{10}+C_{10}^{NP}|^2\right], \tag{5.9}$$

$$\mathcal{A}_{LP}^{\pm}|_{NP} = \mp \frac{2\sqrt{1-4m_l^2/m_{B_s^*}^2}\; Re\left[\left(C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}} C_7^{eff} + C_9^{NP}\right)\left(C_{10}+C_{10}^{NP}\right)^*\right]}{\left(1+2m_l^2/m_{B_s^*}^2\right)\left|C_9^{eff} + \frac{2m_b f_{B_s^*}^T}{m_{B_s^*} f_{B_s^*}} C_7^{eff} + C_9^{NP}\right|^2 + \left(1-4m_l^2/m_{B_s^*}^2\right)|C_{10}+C_{10}^{NP}|^2}. \tag{5.10}$$

5.3 $\mathcal{A}_{LP}(\mu)$ with new physics solutions

In this section we first calculate $\mathcal{A}_{LP}(\mu)$ for the $B_s^* \to \mu^+\mu^-$ decay. The numerical inputs used for this calculation are listed in table 7.1. The SM prediction is

$$\mathcal{A}_{LP}^{+}(\mu)|_{SM} = -\mathcal{A}_{LP}^{-}(\mu)|_{SM} = 0.9955 \pm 0.0003. \tag{5.11}$$

The uncertainty in this prediction (about 0.03%) is much smaller than the uncertainty in the decay constants (about 2%), making it theoretically clean.

There are basically two new physics solutions which can account for all the $b \to s\mu^+\mu^-$ anomalies [69, 70]. The $\mathcal{A}_{LP}^{\pm}(\mu)$ predictions for these solutions are listed in table 7.2. From this table it is obvious that the prediction of $\mathcal{A}_{LP}(\mu)$ for the first solution deviates from the SM at the level of 3.4σ whereas, for the second solution, it is the same as that of the SM. Hence any large deviation in this asymmetry can only be

Parameter	Value
m_b	4.18 ± 0.03 GeV [209]
$m_{B_s^*}$	$5415.4^{+1.8}_{-1.5}$ MeV [210]
$f_{B_s^*}/f_{B_s}$	0.953 ± 0.023 [211]
$f^T_{B_s^*}/f_{B_s}$	0.95 [199]

Table 5.1: Numerical inputs used in our calculations.

New Physics type	New physics WCs	$\mathcal{B}(B_s^* \to \mu^+\mu^-)$	$\mathcal{A}^+_{LP}(\mu) = -\mathcal{A}^-_{LP}(\mu)$
SM	0	$(1.10 \pm 0.60) \times 10^{-11}$	0.9955 ± 0.0003
(I) $C_9^{NP}(\mu\mu)$	-1.25 ± 0.19	$(0.83 \pm 0.45) \times 10^{-11}$	0.8877 ± 0.0312
(II) $C_9^{NP}(\mu\mu) = -C_{10}^{NP}(\mu\mu)$	-0.68 ± 0.12	$(0.79 \pm 0.43) \times 10^{-11}$	0.9936 ± 0.0057

Table 5.2: New physics predictions of branching ratio and $\mathcal{A}_{LP}(\mu)$ for $B_s^* \to \mu^+\mu^-$ decay with real new physics WCs. The new physics WCs are taken from Ref. [78]

New physics Type	[Re(WC), Im(WC)]	$\mathcal{B}(B_s^* \to \mu^+\mu^-)$	$\mathcal{A}^+_{LP}(\mu) = -\mathcal{A}^-_{LP}(\mu)$
(I) $C_9^{NP}(\mu\mu)$	$[(-1.1 \pm 0.2), (0.0 \pm 0.9)]$	$(0.85 \pm 0.27) \times 10^{-11}$	0.91 ± 0.13
(II) $C_9^{NP}(\mu\mu) = -C_{10}^{NP}(\mu\mu)$	(A) $[(-0.8 \pm 0.3), (1.2 \pm 0.7)]$	$(0.80 \pm 0.27) \times 10^{-11}$	0.99 ± 0.02
	(B) $[(-0.8 \pm 0.3), (-1.2 \pm 0.8)]$	$(0.80 \pm 0.28) \times 10^{-11}$	0.99 ± 0.11

Table 5.3: New physics predictions of branching ratio and $\mathcal{A}_{LP}(\mu)$ for $B_s^* \to \mu^+\mu^-$ decay with complex new physics WCs. The new physics WCs are taken from Ref. [68]

due to the first new physics solution. We also provide the predictions for $\mathcal{B}(B_s^* \to \mu^+\mu^-)$ in table 7.2. It is clear that neither of the two solutions can be distinguished from each other or from the SM via the branching ratio.

In the discussion above, the new physics WCs are assumed to be real. If these WCs are complex, they can lead to various CP asymmetries in $B \to (K, K^*)\mu^+\mu^-$ decays [198]. These asymmetries can distinguish between the two new physics solutions. In Ref. [68], it was assumed that $C_9^{NP}(\mu\mu)$ and $C_{10}^{NP}(\mu\mu)$ are complex and a fit to all the $b \to s\mu^+\mu^-$ data was performed. The resulting values of new physics WCs from this fit are given in table 7.3. The predictions for $\mathcal{B}(B_s^* \to \mu^+\mu^-)$ and $\mathcal{A}_{LP}(\mu)$ are also given in this table. Because of the large uncertainties, neither of these two observables can distinguish between the two new physics solutions. However, it may be possible to make a distinction based on the CP asymmetries mentioned above [68].

5.4 Conclusions

There are several measurements in the decays induced by the quark level transition $b \to sl^+l^-$ which do not agree with their SM predictions. All these discrepancies can be explained by considering new physics only in $b \to s\mu^+\mu^-$ transition. These new physics operators are required to have V and/or A form to account for the fact that R_K and R_{K^*} are less than 1. A global analysis of all the measurements in $b \to sl^+l^-$ sector leads to only two new physics solutions. The first solution has $C_9^{NP}(\mu\mu) < 0$ and the second has $C_9^{NP}(\mu\mu) = -C_{10}^{NP}(\mu\mu) < 0$. In this chapter we consider the ability of the muon longitudinal polarization asymmetry in $B_s^* \to \mu^+\mu^-$ decay to distinguish between these two solutions. This observable is theoretically clean because it has only a very mild dependence on the decay constants. For the case of real new physics WCs, we show that this asymmetry has the same value as the SM case for the second solution but is smaller by 11% for the first solution. Hence, a measurement of this asymmetry to 10% accuracy can distinguish between these two solutions. But for the complex new physics WCs, the discrimination power is lost because of the large theoretical uncertainties.