Appendix B: Mathematica programs for calculating the fixed point of the map $\mathscr{R}$ defined on feasible domain

P1: PROGRAM FOR PERIOD TRIPLING RENORMALIZATION

```
F[x_]:=1 − ((x−c)/(1−c))^2;
l = Solve[{s1 == Nest[F,0,3], s2 == Nest[F,0,4] − Nest[F,0,1], s3 == 1 − Nest[F,0,2]}, {s1,s2,s3}];
s1 = l[[1,1,2]];
Plot[s1, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S1(c)",18]}, FrameTicks → All];
s2 = l[[1,2,2]];
Plot[s2, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S2(c)",18]}, FrameTicks → All];
s3 = l[[1,3,2]];
Plot[s3, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S3(c)",18]}, FrameTicks → All];
s123 = s1 + s2 + s3;
Plot[{s123,1}, {c,0,0.5}, Frame → {{True,False},{True,False}}, PlotRange → Full,
FrameLabel → {Style[c,20], Style["S1(c)+S2(c)+S3(c)",18]}, FrameTicks → All,
 PlotLegends → {"S1(c)+S2(c)+S3(c)", "S1(c)+S2(c)+S3(c)=1"}];
fd = N[Reduce[{s1 > 0, s2 > 0, s3 > 0, Nest[F,0,2] − Nest[F,0,4] > 0, Nest[F,0,1] − Nest[F,0,3] > 0}, c]];
a1 = fd[[1,1]];
b1 = fd[[1,5]];
a2 = fd[[2,1]];
b2 = fd[[2,5]];
f4 = Nest[F,0,4];
f0 = Nest[F,0,1];
cnew = (f4 − c)/(f4 − f0);
Plot[{cnew,c}, {c,a1,b1}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
Style["R(c)",18]}, FrameTicks → All, PlotLegends → {Style["R(c)",17], Style["R(c) = c",17]},
PlotStyle → {Normal,Dashed}, AspectRatio → 1]
sols1 = N[Solve[{y==cnew, y == c, a1 < c < b1}, {y,c}, Reals]];
Plot[{cnew,c}, {c,a2,b2}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
Style["R(c)",18]}, FrameTicks → All, PlotLegends → {Style["R(c)",17], Style["R(c) = c",17]},
PlotStyle → {Normal,Dashed}, AspectRatio → 1]
sols2 = N[Solve[{y==cnew, y == c, a2 < c < b2}, {y,c}, Reals]];
cp = sols2[[1,2,2]]
D[cnew,c]/.c → cp;
```

P2: PROGRAM FOR PERIOD QUADRUPLING RENORMALIZATION

```
F[x_]:=1 − ((x−c)/(1−c))^2
l = Solve[{s1 == Nest[F,0,4], s2 == Nest[F,0,2] − Nest[F,0,6], s3 == Nest[F,0,5] − Nest[F,0,1],
s4 == 1 − Nest[F,0,3]}, {s1, s2, s3, s4}];
s1 = l[[1,1,2]];
Plot[s1, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style ["S1(c)", 18]} , FrameTicks → All] ;
s2 = l[[1,2,2]];
Plot[s2, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style ["S2(c)", 18]} , FrameTicks → All] ;
s3 = l[[1,3,2]];
Plot[s3, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style ["S3(c)", 18]} , FrameTicks → All] ;
s4 = l[[1,4,2]];
Plot[s4, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style ["S4(c)", 18]} , FrameTicks → All] ;
sum = s1 + s2 + s3 + s4;
Plot[{sum,1}, {c,0,0.5}, Frame → {{True,False},{True,False}}, PlotRange → Full,
FrameLabel → {Style[c,20], Style ["S1(c)+S2(c)+S3(c)+S4(c)", 18]} , FrameTicks → All,
 PlotLegends → {"S1(c)+S2(c)+S3(c)+S4(c)", "S1(c)+S2(c)+S3(c)+S4(c)=1"}] ;
g1 = Nest[F,0,6] − Nest[F,0,4];
g2 = Nest[F,0,1] − Nest[F,0,2];
g3 = Nest[F,0,3] − Nest[F,0,5];
fd = N[Reduce[{s1 > 0, s2 > 0, s3 > 0, s4 > 0, g1 > 0, g2 > 0, g3 > 0}, c]];
a1 = fd[[1,1]];
b1 = fd[[1,5]];
a2 = fd[[2,1]];
b2 = fd[[2,5]];
f6 = Nest[F,0,6];
cnew = (c − f6)/(s2);
Plot[{cnew,c}, {c,a1,b1}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
Style["R(c)", 18]}, FrameTicks → All, PlotLegends → {Style["R(c)", 17], Style["R(c) = c", 17]},
PlotStyle → {Normal, Dashed}, AspectRatio → 1]
sols1 = N[Solve[{y==cnew, y == c, a1 < c  < b1}, {y,c}, Reals]];
Plot[{cnew,c}, {c,a2,b2}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
Style["R(c)", 18]}, FrameTicks → All, PlotLegends → {Style["R(c)", 17], Style["R(c) = c", 17]},
PlotStyle → {Normal, Dashed}, AspectRatio → 1]
sols2 = N[Solve[{y==cnew, y == c, a2 < c  < b2}, {y,c}, Reals]];
cp = sols2[[1,2,2]]
D[cnew,c]/.c → cp;
```

P3: PROGRAM FOR PERIOD QUINTUPLING RENORMALIZATION

```
F[x_]:=1 − ((x−c)/(1−c))^2
l = Solve[{s1 == Nest[F,0,5], s2 == Nest[F,0,8] − Nest[F,0,3], s3 == Nest[F,0,6] − Nest[F,0,1],
s4 == Nest[F,0,2] − Nest[F,0,7], s5 == 1 − Nest[F,0,4]}, {s1,s2,s3,s4,s5}];
s1 = l[[1,1,2]];
Plot[s1, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S1(c)",18]}, FrameTicks → All];
s2 = l[[1,2,2]];
Plot[s2, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S2(c)",18]}, FrameTicks → All];
s3 = l[[1,3,2]];
Plot[s3, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S3(c)",18]}, FrameTicks → All];
s4 = l[[1,4,2]];
Plot[s4, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S4(c)",18]}, FrameTicks → All];
s5 = l[[1,5,2]];
Plot[s5, {c,0,0.5}, Frame → {{True,False},{True,False}}, FrameLabel → {Style[c,20],
 Style["S5(c)",18]}, FrameTicks → All];
sum = s1 + s2 + s3 + s4 + s5;
Plot[{sum,1}, {c,0,0.5}, Frame → {{True,False},{True,False}}, PlotRange → Full,
FrameLabel → {Style[c,20], Style["S1(c)+S2(c)+S3(c)+S4(c)+S5(c)",18]}, FrameTicks → All,
 PlotLegends → {"S1(c)+S2(c)+S3(c)+S4(c)+S5(c)", "S1(c)+S2(c)+S3(c)+S4(c)+S5(c)=1"}];
g1 = Nest[F,0,3] − Nest[F,0,5];
g2 = Nest[F,0,1] − Nest[F,0,8];
g3 = Nest[F,0,7] − Nest[F,0,6];
g4 = Nest[F,0,4] − Nest[F,0,2];
fd = N[Reduce[{0 < s1 < 1, 0 < s2 < 1, 0 < s3 < 1, 0 < s4 < 1, 0 < s5 < 1, g1 > 0, g2 > 0, g3 > 0, g4 > 0,
0 < c < 0.5}, c]];
a1 = fd[[1,1]];
b1 = fd[[1,5]];
a2 = fd[[2,1]];
b2 = fd[[2,5]];
f8 = Nest[F,0,8];
cnew = (f8 − c)/(s2);
Plot[{cnew,c}, {c,a1,b2}]
sols = FindRoot[cnew == c, {c,0.387}]
cp = sols[[1,2]]
D[cnew,c]/.c → cp;
```

P4: PROGRAM FOR LEFT PERIOD TRIPLING RENORMALIZATION R^l

f[x_]:=1 − $\left(\frac{1}{8}\left(4+\frac{6}{1-2c}-\frac{2}{(1-2c)^3}\right)*(1-2x)+x+\frac{2}{(1-2c)^3}*\left(\frac{x}{2}-\frac{3x^2}{2}+x^3\right)\right)$;

```
pl = Solve [f[0] == 1/2, c] ;
ltl = pl[[1,1,2]];
l = Solve[{sl0==(Nest[f,0,1] − Nest[f,0,4])/Nest[f,0,1], sl1==(Nest[f,0,2] − Nest[f,0,5])/Nest[f,0,1],
sl2== Nest[f,0,3]/Nest[f,0,1]}, {sl0, sl1, sl2}];
sl0 = l[[1,1,2]];
Plot[sl0, {c,0,ltl}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c,20],
 Style ["S10(c)", 18]} , FrameTicks → All] ;
sl1 = l[[1,2,2]];
Plot[sl1, {c,0,ltl}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c,20],
 Style ["S11(c)", 18]} , FrameTicks → All] ;
sl2 = l[[1,3,2]];
Plot[sl2, {c,0,ltl}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c,20],
 Style ["S12(c)", 18]} , FrameTicks → All] ;
suml = sl0 + sl1 + sl2;
Plot[{suml, 1}, {c, 0, 0.5}, Frame → {{True, False}, {True, False}}, PlotRange → Full,
FrameLabel → {Style[c,20], Style ["S10(c)+S11(c)+S12(c)", 18]} , FrameTicks → All,
 PlotLegends → {"S10(c)+S11(c)+S12(c)", "S10(c)+S11(c)+S12(c)=1"}] ;
gl1 = (Nest[f,0,5] − Nest[f,0,3])/Nest[f,0,1];
gl2 = (Nest[f,0,4] − Nest[f,0,2])/Nest[f,0,1];
fdl = N[Reduce[{sl0 > 0, sl1 > 0, sl2 > 0, gl1 > 0, gl2 > 0, 0 < suml < 0.5}, c]];
al1 = fdl[[1,1]];
bl1 = fdl[[1,5]];
al2 = fdl[[2,1]];
bl2 = fdl[[2,5]];
cnewl = (Nest[f,0,2] − c)/sl1;
Plot[{cnewl, c}, {c, al1, bl1}]
Plot[{cnewl, c}, {c, al2, bl2}]
solsl = FindRoot[cnewl == c, {c, 0.1967}];
cpl = solsl[[1,2]]
D[cnewl, c]/.c → cpl;
```

P5: PROGRAM FOR RIGHT PERIOD TRIPLING RENORMALIZATION R^r

```
f[x_]:=1 − (1/8 (4 + 6/(2c−1) − 2/(2c−1)^3) ∗ (1 − 2x) + x + 2/(2c−1)^3 ∗ (x/2 − 3x^2/2 + x^3));
pr = Solve [f[1] == 1/2, c] ;
ltr = pr[[2, 1, 2]];
r = Solve[{sr0==(Nest[f, 1, 4] − Nest[f, 1, 1])/(1 − Nest[f, 1, 1]),
sr1==(Nest[f, 1, 5] − Nest[f, 1, 2])/(1 − Nest[f, 1, 1]),
sr2== (1 − Nest[f, 1, 3])/(1 − Nest[f, 1, 1])}, {sr0, sr1, sr2}];
sr0 = r[[1, 1, 2]];
Plot[sr0, {c, ltr, 1}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c, 20],
 Style ["Sr0(c)", 18]} , FrameTicks → All] ;
sr1 = r[[1, 2, 2]];
Plot[sr1, {c, ltr, 1}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c, 20],
 Style ["Sr1(c)", 18]} , FrameTicks → All] ;
sr2 = r[[1, 3, 2]];
Plot[sr2, {c, ltr, 1}, Frame → {{True, False}, {True, False}}, FrameLabel → {Style[c, 20],
 Style ["Sr2(c)", 18]} , FrameTicks → All] ;
sumr = sr0 + sr1 + sr2;
Plot[{sumr, 1}, {c, ltr, 1}, Frame → {{True, False}, {True, False}}, PlotRange → Full,
FrameLabel → {Style[c, 20], Style ["Sr0(c)+Sr1(c)+Sr2(c)", 18]} , FrameTicks → All,
 PlotLegends → {"Sr0(c)+Sr1(c)+Sr2(c)", "Sr0(c)+Sr1(c)+Sr2(c)=1"}] ;
gr1 = (Nest[f, 1, 4] − Nest[f, 1, 1])/(1 − Nest[f, 1, 1]);
gr2 = (Nest[f, 1, 5] − Nest[f, 1, 2])/(1 − Nest[f, 1, 1]);
fdr = N[Reduce[{sr0 > 0, sr1 > 0, sr2 > 0, gr1 > 0, gr2 > 0, 0 < sumr < 0.5}, c]];
ar1 = fdr[[1, 1]];
br1 = fdr[[1, 5]];
ar2 = fdr[[2, 1]];
br2 = fdr[[2, 5]];
cnewr = 1 − (c−Nest[f,1,2])/sr1;
Plot[{cnewr, c}, {c, ar1, br1}]
Plot[{cnewr, c}, {c, ar2, br2}]
solsr = FindRoot[cnewr == c, {c, 0.8033}];
cpr = solsr[[1, 2]]
D[cnewr, c]/.c → cpr;
```