Abstract

Renormalization theory plays a key role in describing the dynamics of a given system at a small spatial scale by an induced dynamical system in the same class. The concept of renormalization arises in many forms though Mathematics and Physics. Period doubling renormalization operator was introduced by M. Feigenbaum and by P. Coullet and C. Tresser, to study asymptotic small scale geometry of the attractor of one dimensional systems which are at the transition from simple to chaotic dynamics.

In this thesis, we discuss the renormalizations of unimodal maps and symmetric bimodal maps whose smoothness is C^{1+Lip}, just below C^2. In the first section of the thesis, we explore the period tripling and period quintupling renormalizations below C^2 class of unimodal maps. In the context of period tripling renormalization, there exists only one valid period tripling combinatorics. For a given proper tri-scaling data, we construct a nested sequence of affine pieces whose end-points lie on the unimodal map and shrinking down to the critical point. Consequently, we prove the existence of a renormalization fixed point, namely f_{s^*}, in the space of piece-wise affine maps which are period tripling infinitely renormalizable, corresponding to a proper tri-scaling data s^*. Furthermore, we show that the renormalization fixed point f_{s^*} is extended to a C^{1+Lip} unimodal map with a quadratic tip, by considering the period tripling combinatorics. Moreover, this leads to the fact that the period tripling renormalizations acting on the space of C^{1+Lip} unimodal maps has unbounded topological entropy. Finally, by considering a small variation on the scaling data we show the existence of an another fixed points of renormalization in the space of C^{1+Lip} unimodal maps. This shows the continuum of fixed points of period tripling renormalization operator. In the context of period quintupling renormalization, we have only three valid period quintupling combinatorics. For each combinatorics, there exists a piece-wise affine fixed point of renormalization operator, namely g_{s^*}, corresponding to a proper quint-scaling data s^*. We show that the geometry of the invariant Cantor set of the map g_s is more complex than the geometry of the invariant Cantor set of f_{s^*}. Next, we describe the extension of g_{s^*} to a C^{1+Lip} space, and the topological entropy of period quintupling renormalization operator. The continuum's result also holds for this case. Consequently, we observe that the increment of periodicity of renormalization increases the number of the possible combinatorics and each of them leads to such construction of renormalization fixed point.

The second section of the thesis studies the renormalization of symmetric bimodal maps with low smoothness. The renormalization operator R is a pair of period tripling renormalization operators R^l and R^r which are defined on piece-wise affine period tripling infinitely renormalizable maps corresponding to a proper scaling data s_l and s_r, respectively. We prove the existence of the renormalization fixed point in the space C^{1+Lip} symmetric bimodal maps. Moreover, we show that the topological entropy of the renormalization operator defined on the space of C^{1+Lip} symmetric bimodal maps is infinite. Further, we prove the existence of a continuum of fixed points of renormalization. The main result lead to the fact that the non-rigidity of the Cantor attractors of infinitely renormalizable symmetric bimodal maps, whose smoothness is below C^2.

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