5
Effect of Quantum Decoherence on Coherence and Mixedness in Neutrino-system for LSND setup

This chapter is based on Ref. [95]. Neutrino experiments so far have resulted in favour of the standard oscillation picture as a dominant mechanism to explain the experimental observations, excluding some exceptional cases, such as the observations found in *liquid scintillator neutrino detector* (LSND) experiment. However, owing to a more precise new generation of neutrino oscillation experiments, it is worth studying the secondary contributions coming from some exotic physics viz. quantum decoherence [169]. Moreover, for significant utilization of neutrinos to perform quantum information tasks, it is also important to study the nature of correlations present in the system under different circumstances. In this chapter we examine the agreement between coherence and mixedness present in the neutrino-system under the influence of quantum decoherence based on a model suggested to explain the LSND anomaly [74].

The fact is well known that the presence of decoherence effects causes the loss of information present in a quantum mechanical state that can be characterized in terms of the diminishing purity of the state. In other words, it represents the loss of coherence. Therefore, it becomes very important to appropriately quantify the purity or its reciprocal property, the mixedness. Also, it is reasonable to discuss about the complementarity between mixedness present in the system with the coherence it restrains. The quantification of mixedness as well as its trade-off relation with coherence has been discussed in [214] where the aim was to investigate the upper bound imposed by mixedness of a quantum system on the maximum amount of quantum coherence present in the system and to develop a mathematical formulation for the same. Here, in this chapter, we focus specifically on neutrino oscillation scenario to investigate such complimentarity relation between mixedness and coherence embedded in the neutrino system.

It is to be noted that a quantum mechanical system can exhibit maximum coherence when it is characterized by a state in a specific eigenbasis, however, it is possible to have zero coherence in a different eigenbasis for the same quantum system. Basically, coherence is a basis dependent quantity. In case of neutrinos, we have two options for the choice of basis, viz. mass eigenstate and flavor state basis. In this work, we are interested in investigating the trade-off relation between coherence and mixedness for neutrino system in the presence of decoherence. Effects of such dissipation are introduced in the mass eigenstates of neutrinos that can be described in terms of open quantum system formalism. Hence, it is meaningful to consider the coherence parameter in mass eigenstate basis. In section 5.1 we provide a detailed calculation following the results and discussions for such analysis.

5.1 Effect of decoherence on coherence and mixedness in oscillating neutrino system

Recently a trade-off between mixedness and quantum coherence, from the perspective of *resource theory* [215], inherent in the system was proposed in the form of a complementarity relation [109]. These studies have mostly been focused on quantum optical systems from the perspective of quantum information. Here the interplay between quantum coherence and mixedness inherent in the state (Eq. 5.4) of neutrinos has been studied. The definition of coherence $\chi(\rho)$ is given in [18], in terms of the off-diagonal elements of the density matrix. This has been discussed in detail in 2.2. The mixedness, which represents the disorder in the system, can be quantified in terms of entropic functionals, viz., linear and von Neumann entropy of the quantum state. For an arbitrary d-dimensional state, the mixedness, based on normalized linear entropy [214] can be given as

$$\eta(\rho) = \frac{n}{n-1}(1 - \mathrm{Tr}\,\rho^2), \tag{5.1}$$

while the coherence can be quantified as

$$\chi(\rho) = \sum_{i \neq j} |\rho_{ij}|, \tag{5.2}$$

Here ρ_{ij} are the off-diagonal elements of the density matrix ρ and n is the dimension of the system. The balance between coherence and mixedness was recently expressed in terms of a complementarity relation [109] which is as follows

$$\beta(\rho) = \frac{\chi^2(\rho)}{(n-1)^2} + \eta(\rho) \leq 1. \tag{5.3}$$

Here, $n = 3$ in case of neutrino system with three flavours. It would be $n = 4$ for the system of neutral K and B-mesons which is discussed with complete analysis in chapter 10.

Three flavour states ($|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$) of neutrino mix via a 3×3 unitary (PMNS) matrix $U(\theta_{ij}, \delta)$, $i, j = 1, 2, 3$; $i < j$, where θ_{ij} are the mixing angles and δ is CP-violating phase, to form three mass eigenstates ($|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$). The mass eigenstates evolve as plane waves, i.e. $\nu_a(t) = e^{-iE_a t}\, \nu_a(0)$, $a = 1, 2, 3$. A neutrino state $|\Psi_\alpha(0)\rangle$ at time $t = 0$ evolves unitarily to time t and can be written in flavour basis as

$$|\Psi_\alpha(t)\rangle = \sum_{\beta = e,\mu,\tau} \zeta_{\alpha,\beta}(t)\, |\nu_\beta\rangle\,. \tag{5.4}$$

Here $\zeta_{\alpha,\beta} = (UEU^{-1})_{\alpha,\beta}$, with $E = diag[e^{-iE_1 t}, e^{-iE_2 t}, e^{-iE_3 t}]$. Under the system environment interaction, a pure state is almost inevitable transformed into a mixture of pure states often called as *mixed state*. Such states are no longer represented by rays in Hilbert space, rather they are described by *density matrix* or *density operator*, an element of a set of trace class operators with unit trace. The density matrix formalism is in fact a general description for both *pure* and *mixed* states, characterized by $Tr[\rho^2] = 1$ and $Tr[\rho^2] < 1$, respectively. Hence we use this formalism here too. In this work we use the initial state as $\rho_\mu(0) = |\nu_\mu\rangle\,\langle\nu_\mu|$, relevant to the LSND experiment and in the context of the decoherence model discussed in section 3.3. The time evolution of the corresponding state can be expressed as

$$\rho(t) = \begin{pmatrix} \rho_{11}(0) & \rho_{12}(0)e^{-(\gamma_{12} - i\Delta_{12})t} & \rho_{13}(0)e^{-(\gamma_{13} - i\Delta_{13})t} \\ \rho_{21}(0)e^{-(\gamma_{21} - i\Delta_{21})t} & \rho_{22}(0) & \rho_{23}(0)e^{-(\gamma_{23} - i\Delta_{23})t} \\ \rho_{31}(0)e^{-(\gamma_{31} - i\Delta_{31})t} & \rho_{32}(0)e^{-(\gamma_{32} - i\Delta_{32})t} & \rho_{33}(0) \end{pmatrix}, \tag{5.5}$$

where, γ_{ij} are the decoherence parameters and $\Delta_{ij} \approx \frac{\Delta m_{ij}^2}{2E_\nu}$, with Δm_{ij}^2 and E_ν being the mass square difference and the energy of the neutrino, respectively. Here the elements of the density matrix at time $t = 0$ are given by $\rho_{ij}(0) = U_{\mu i}^* U_{\mu j}$ (for initial state ν_μ), $U_{\mu i}$ are the elements of the second row of the U (PMNS) matrix given in Eq. (3.3). The matrix defining the evolution operator (UEU^{-1}) contains, apart from the mixing angles θ_{ij}, the mass square differences $\Delta m_{ij}^2 = m_j^2 - m_i^2$. The numerical values of these quantities used in this work are $\theta_{12} = 33.48^o$, $\theta_{23} = 42.3^o$, $\theta_{13} = 8.5^o$, $\Delta m_{21}^2 = 2.457 \times 10^{-3}\text{eV}^2$, $\Delta m_{31}^2 \approx \Delta m_{32}^2 = 2.457 \times 10^{-3}\text{eV}^2$ [216].

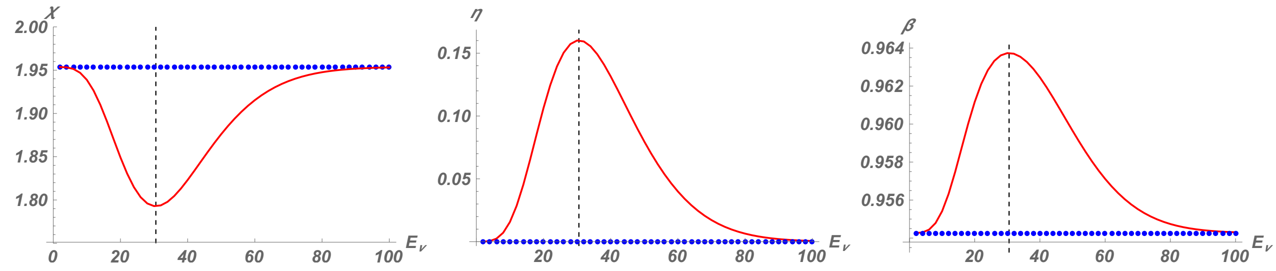

Figure 5.1: *Neutrino system* (LSND decoherence model): Coherence parameter $\chi(\rho)$ (left), mixedness parameter $\eta(\rho)$ (middle) and complementarity parameter $\beta(\rho)$ (right), plotted as a function of neutrino energy E_ν (MeV) with CP violating phase $\delta = 0$. The maximum value of decoherence parameter, defined in Eq. (3.24), corresponds approximately to 30 MeV (vertical dashed line). At this energy the coherence and the mixedness parameter attain their minimum and maximum values, respectively.

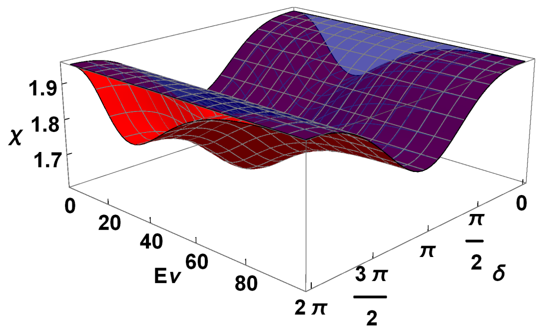

Figure 5.2: Coherence parameter χ as defined in Eq. (5.2), for the state in Eq. (5.5), is plotted with respect to the CP violating phase δ and the energy E_ν of the neutrino. The blue and red surfaces correspond to the cases with and without decoherence parameter, respectively. The minimum of the coherence parameter occurs for $E_\nu \approx 30$ MeV and $\delta = \pi$.

In case of neutrinos, since we are restricted to use a specific model to incorporate decoherence effects, we used the phenomenological approach of [73, 74] which was motivated to explain the LSND signal. The decoherence parameter γ (also discussed in section 3.3) has exponential dependence on neutrino-energy E_ν in the following way

$$\gamma = \gamma_0 \Bigg(\exp\left[-\left(\frac{E}{E_3}\right)^n \right] - \exp\left[-\left(\frac{E}{E_1}\right)^n \right] \Bigg)^2, \tag{5.6}$$

with best fit values of E_3 and E_1 as 55 MeV and 20 MeV, respectively, $n = 2$ and $\gamma = 0.01\ \text{m}^{-1}$ suitable to explain LSND data.

Using this formalism, we plotted coherence $\chi(\rho)$ (left), mixedness parameter $\eta(\rho)$ (middle) and the parameter $\beta(\rho)$ showing the trade-off relation (right) in Fig. 5.1. The baseline is considered to be $L = 30$ m corresponding to the LSND experiment, while we have kept the CP-violating parameter δ to be zero. The energy range is also considered to be associated to the LSND setup that is around 0 - 100 MeV. It can be seen that γ defined in Eq. (5.6) attains its maximum value at approximately 30 Mev. Consequently, the coherence parameter χ and the mixedness parameter η attain their minimum and maximum values, respectively, at this energy as is shown in Fig. 5.1. The complementarity relation is found to be satisfied in case of neutrinos as can be seen in right panel of Fig. 5.1.

In Fig. 5.2, we have also plotted the coherence parameter with respect to the neutrino-energy and CP phase δ. The coherence for the neutrino system is found to depend on the CP violating phase. It is clear that the coherence parameter decreases (increases) in the upper half plane $0 < \delta < \pi$ (lower half plane $\pi < \delta < 2\pi$), and attains its minimum value at $\delta = \pi$ and energy $E_\nu \approx 30$ MeV.

For neutrino system, coherence and mixedness is studied in the context of the decoherence model for LSND experiment. In this model, the decoherence parameter γ is a function of the energy of neutrino E_ν. The coherence parameter decreases with the increase in γ and attains its minimum value at $E_\nu \approx 30$ MeV. Further, the coherence for the neutrino system is found to depend on the CP violating phase δ.

One point is to note that the trade-off relation mentioned here provides and upper bound on coherence that can be acquired by a state having a fixed value of mixedness and that upper bound is defined by considering the equality sign in Eq. (5.3). Such class of states is named as *maximally coherent mixed states*. However, the three flavor neutrino state (in mass eigenstate basis) is not found to achieve that upper bound for the present bounds on oscillation parameters (*i.e.,* mixing angles and mass squared differences) for the LSND setup.