6
A model independent analysis of New Physics effects on Quantum Coherence

This chapter is based on Refs. [96, 97]. Several measures of quantum correlations such as Leggett-Garg and Bell-type inequalities have been extensively studied in the context of neutrino oscillations. However these analyses are performed under the assumption of standard model (SM) interactions of neutrinos. In this chapter we study new physics effects on l_1-norm based measure of quantum coherence, discussed in chapter 2, which quantifies the quantumness embedded in the system and is also intrinsically related to various measures of quantum correlations. Moreover, it is considered to be a resource theoretical tool which can be utilized in quantum algorithms and quantum channel discrimination. The new physics effects are incorporated in a model independent way by using the effective Lagrangian for the neutral current non-standard neutrino interactions (NSI), details can be found in 3.4. Bounds on the NSI parameters are extracted from a recent global analysis of oscillation experiments including COHERENT (coherent neutrino-nucleus scattering experiment) data [68, 69], values of these parameters are given in the next section.

In this chapter we show that in the context of upcoming DUNE experimental setup, the most favourable combination of LMA-Light sector of θ_{12} (*i.e.*, $\theta_{12} < 45^o$) with normal mass ordering decreases the coherence in the system in comparison to the SM prediction for all values of neutrino energy E and CP violating phase δ (except in the narrow region around $E \sim 2$ GeV). On the other hand, a large enhancement in the value of coherence parameter in the entire $(E - \delta)$ plane is possible for the dark octant of θ_{12} ($\theta_{12} > 45^o$) with inverted ordering. For almost all values of CP violating phase, the enhancement is more protuberant in the region around $E \sim 4$ GeV where maximum neutrino flux is expected in the DUNE experiment. Therefore for the normal mass ordering, the SM interaction provides favourable conditions for quantum information tasks while the NSI favours inverted ordering scenario for such tasks.

The aim of this work is to develop a formalism to incorporate the effects of NSI on quantum correlations present in neutrino oscillations. For such study, we have opted the well defined coherence measure based on the l_1 norm of the state, as coherence can be considered a keystone and core of the heart for the existence of all forms of quantum correlations for a system. In this chapter we aim to develop the mechanism in such a way that one can get a prior idea of possible NSI effects on observables such as entanglement in the neutrino sector. Since, the only observable quantities in neutrino oscillations are the so called transition probabilities which restrict us to be in the flavor state basis, it is appropriate to consider the flavor state basis for present study as well.

6.1 Effect of nonstandard neutrino-matter interaction on coherence in neutrino system

The parameter $\chi = |\rho_{ij}|, i \neq j$, also defined in Eq. (2.8), is probably the most convenient coherence-measure for neutrino experiments. $\rho = |\psi\rangle\langle\psi|$ can be calculated using the neutrino state represented by $|\psi(t)\rangle \equiv |\nu_\alpha(t)\rangle = \sum_j \mathcal{U}_{fij}(t)\,|\nu_\beta\rangle$, with $j = 1, 2, 3$ for respective values of $\beta = e, \mu, \tau$ and i is fixed according to the initial flavour (at $t = 0$) of neutrino, i.e., i will be 1, 2 or 3 if $\alpha = e, \mu$ or τ, respectively. As a result, a neutrino state at time t, initially produced in ν_μ-flavour, can be represented as

$$|\nu_\alpha(t)\rangle = \mathcal{U}_{f21}(t)\,|\nu_e(0)\rangle + \mathcal{U}_{f22}(t)\,|\nu_\mu(0)\rangle + \mathcal{U}_{f23}(t)\,|\nu_\tau(0)\rangle\,.$$

Here $\mathcal{U}_{fij}$ are elements of the evolution operator defined as $U_f(t) = UU_m(t)U^{-1}$, with U being the PMNS neutrino mixing-matrix and

$$U_m(L) = \phi\sum_{a=1}^{3}\frac{e^{-iL\lambda_a}}{3\lambda_a^2 + c_1}[(\lambda_a^2 + c_1)I + \lambda_a T + T^2],$$

with $\phi = e^{\frac{-i}{3}LTr\mathcal{H}_m}$, $T = \mathcal{H}_m - Tr[\mathcal{H}_m]/3$, $c_1 = \det T\; Tr(T^{-1})$ and λ_a are eigenvalues of T. The Hamiltonian including matter density potential terms (for standard and nonstandard neutrino-matter interactions), also given in Eq. (3.28), can be expressed as

$$\mathcal{H}_m = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} + UA\begin{pmatrix} 1+\epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon^*_{e\mu}(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon^*_{e\tau}(x) & \epsilon^*_{\mu\tau}(x) & \epsilon_{\tau\tau}(x) \end{pmatrix}U^{-1}$$

Here, $A = \pm\sqrt{2}\,G_F\,n_e$, represents the matter density potential where G_F is the Fermi constant, n_e is the electron number density. The $\epsilon_{\alpha\beta}$ parameters represent the strength of neutral current NSI of neutrinos with matter during its propagation through material medium.

In case of three-flavour neutrino oscillations, the dimension of the system $d = 3$, and hence, the maximal value the neutrino-system can achieve will be $\chi = d - 1 = 2$. For SM interactions, coherence can then be obtained as

$$\chi^{SM} = \lim_{\epsilon_{\alpha\beta}\to 0}\chi^{NSI}.$$

The l_1-norm based coherence quantifier can be expressed in terms of observable neutrino survival and transition probabilities [44]. For example, if neutrino is produced initially in $|\nu_\mu\rangle$ state, then

$$\chi = 2\left[\sqrt{P_{\mu e}(t)P_{\mu\mu}(t)} + \sqrt{P_{\mu e}(t)P_{\mu\tau}(t)} + \sqrt{P_{\mu\mu}(t)P_{\mu\tau}(t)}\right] \tag{6.1}$$

with normalization constraint on probabilities, *i.e.*, $\sum_\alpha P_{\alpha\beta} = 1 = \sum_\beta P_{\alpha\beta}$ with $\alpha, \beta = e, \mu, \tau$. From above Eq. (6.1), it can be seen that χ achieves its maximal value 2 only when all the flavours of neutrinos are equally probable to appear, *i.e.*, when $P_{\mu\mu} = P_{\mu e} = P_{\mu\tau} = 1/3$. If neutrino is found to have unit probability of being in a specific flavour, in that case χ becomes minimum or zero which means that the coherence will be lost completely. Therefore, in case of three-flavour neutrino oscillations the χ parameter is bounded as $0 \leq \chi \leq 2$.

In the following, we present our results for χ^{SM} and χ^{NSI} in the context of experimental set-up for upcoming long-baseline accelerator experiment DUNE (baseline $L = 1300$ Km). Hence we have $\alpha = \mu$ for accelerator ν_μ beam and matter density potential is taken to be $A = 1.01\,\times$

10^{-13} eV ($\sim 2.8\,\text{g/cm}^3$)[1]. Further, oscillation parameters are as $\theta_{12} = 33.82^o$ (in case of SM interaction as well as for LMA-Light solution), $\theta_{23} = 49.6^o$, $\theta_{13} = 8.61^o$, $\Delta m^2_{21} = 7.39 \times 10^{-5}\text{eV}^2$ and $|\Delta m^2_{32}| = 2.525 \times 10^{-3}\text{eV}^2$ [218]. Due to CPT-transformation given in Eq. (3.33), the mixing-angle θ_{12} obtains the value 56.18^o for LMA-Dark solution. For a detailed study we have presented our results taking into account both the old and newer global analysis of neutrino experiments given in references [68] and [69], respectively.

Values of NSI parameters $\epsilon_{\alpha\beta}$ are taken from Refs.[68, 69] one can find these values in the next paragraph. In Ref. [68], bounds on NSI parameters were obtained mainly by using constraints from observables such as the disappearance data from solar and KamLAND experiments, atmospheric neutrino data from Super-K, DeepCore and IceCube experiments along with the long-baseline (LBL) experimental data such as ν_μ and $\bar{\nu}_\mu$ disappearance as well as ν_e and $\bar{\nu}_e$ appearance data from MINOS, ν_μ and $\bar{\nu}_\mu$ disappearance data from T2K, ν_μ disappearance data from NOνA experiment and also the data from COHERENT experiment. These observables are not sensitive to δ-value and the sign of mass squared difference $\Delta m^2_{31} (= m^2_3 - m^2_1)$ and hence the NSI parameters were the same for both signs of Δm^2_{31}. This analysis was updated in Ref. [69] by including all relevant data in the neutrino sector which includes observables having functional dependence on the CP-violating phase as well as the sign of the Δm^2_{31}, *i.e.*, ν_e and $\bar{\nu}_e$ appearance data from T2K and NOνA. In both of these works, the allowed parameter space for NSI couplings were obtained for the light ($\theta_{12} < 45^o$) and dark ($\theta_{12} > 45^o$) octant.

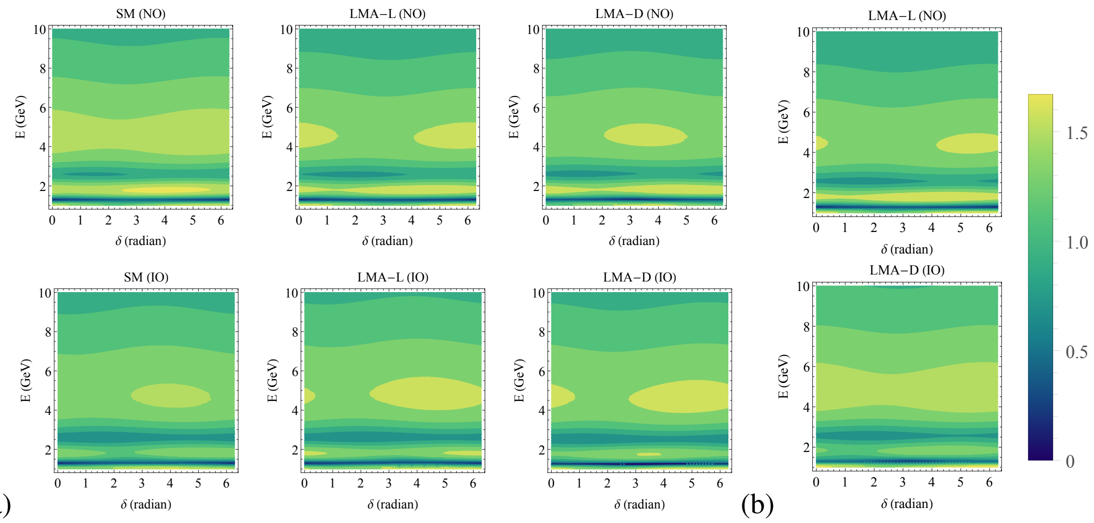
Here are the values of NSI parameters taken from the Ref. [68] corresponding to both LMA-Light and LMA-Dark sectors: $\epsilon_{ee} \approx 1.754796$, $\epsilon_{\mu\mu} \approx 0.828342$, $\epsilon_{\tau\tau} \approx 0.834372$, $\epsilon_{e\mu} \approx -0.030174$, $\epsilon_{e\tau} \approx -0.343944$ and $\epsilon_{\mu\tau} \approx -7.5 \times 10^{-3}$. We have extracted these values from 1σ allowed range of each parameter. These values were updated in [69] by including observables sensitive to the leptonic CP-violating phase and mass ordering in the global fit where the following NSI parameters were obtained for two favoured solutions:

- LMA-Light sector with normal ordering: $\tilde{\epsilon}_{ee} = \epsilon_{ee} - \epsilon_{\mu\mu} \approx -0.1$, $\tilde{\epsilon}_{\tau\tau} = \epsilon_{\tau\tau} - \epsilon_{\mu\mu} \approx 0.01$, $\epsilon_{e\mu} \approx -0.06$, $\epsilon_{e\tau} \approx -0.1$, $\epsilon_{\mu\tau} \approx -0.01$.
- LMA-Dark sector with inverted ordering: $\tilde{\epsilon}_{ee} \approx -1.8$, $\tilde{\epsilon}_{\tau\tau} \approx -0.01$, $\epsilon_{e\mu} \approx 0.06$, $\epsilon_{e\tau} \approx -0.07$, $\epsilon_{\mu\tau} \approx -0.01$.

We first study the impact of NSI on coherence parameter using results from first analysis of ref. [68]. We then see how these results change in view of updated results obtained in [69]. The results of our analysis are presented in Fig. 6.1. In this figure the observable quantifying coherence, χ, is shown in the plane of the neutrino-energy E (in GeV) and the CP-violating phase δ (in radian) for both positive (upper panel) and negative (lower panel) signs of Δm^2_{31}. The range of E ($1 \leq E \leq 10$ GeV) along with the baseline length $L = 1300$ km correspond to the DUNE experimental set-up.

SM interaction: Results shown in the left most column of Fig. 6.1(a) correspond to the variation of coherence parameter for the SM interaction. It is observed that for the SM interaction, the range of χ is $\sim (0.1, 1.67)$ for positive (upper left figure) and $\sim (0.1, 1.5)$ for negative (lower left figure) sign of Δm^2_{31}. $\chi \approx 1.67$ for energy ~ 2 GeV in case of SM + NO. However, in the energy range 4 - 6 GeV (the maximum neutrino flux is expected around 4 GeV for DUNE experiment) for positive Δm^2_{31}, $\chi^{SM} \geq 1.5$, *i.e.*, the system has large coherence for all values of CP-violating phase in this energy range. For the case of negative Δm^2_{31}, the value of χ parameter

[1]We converted the unit of matter density potential from g/cm^3 to eV using $A = 7.6 \times Y_e\,\rho \times 10^{-14}$, where $Y_e = N_e/(N_e + N_n)$. Here N_e and N_n are the number densities of electrons and neutrons in the Earth and ρ is the matter density in g/cm^3. We then have $Y_e \approx 0.48$ and $\rho = 2.8\,\text{g/cm}^3$ which are convenient for Earth's matter as shown by Dziewonski et al. [217]. Hence, the value of A turns out to be $\approx 1.01 \times 10^{-13}$ eV.



(a) (b)

Figure 6.1: The parameter χ plotted with respect to E and δ in the context of DUNE ($L = 1300$km & $E = 1 - 10$GeV) experiment. (a) The upper and lower panels correspond to normal and inverted mass ordering, respectively. Further, the figures in the left most panel represent the effect of SM interaction whereas the middle and right panel of figures (a) show the effects of NSI for LMA-Light and LMA-Dark solutions, respectively. The $\epsilon_{\alpha\beta}$ parameters associated with LMA-Light and LMA-Dark solutions are taken from the first analysis given ref. [68]. (b) The upper and lower panels correspond to the LMA-Light+NO and LMA-Dark+IO combinations, respectively. The $\epsilon_{\alpha\beta}$ parameters are taken from the second analysis given in ref. [69]. Minimum value (zero) of χ parameter represents the complete loss of coherence whereas the maximally coherent state is represented by $\chi = 2$.

is relatively reduced. Thus we see that within the SM, the quantumness of system quantified in terms of coherence, is sensitive to the sign of Δm^2_{31} as well as the CP violating phase.

NSI effect (first analysis): The second column of Fig. 6.1(a) shows χ^{NSI} for LMA-Light solution corresponding to NSI inputs calculated in the first analysis. The effect of NSI is to reduce the coherence for positive Δm^2_{31} and enhance for the negative sign in comparison to the SM case. For positive Δm^2_{31} (upper figure in second column), it can be seen that the NSI effect results in an overall decrease in coherence in the entire $E - \delta$ plane barring few small regions such as $\delta \in (0 - 1.5)$ and $(4 - 6)$ for $E \in (4 - 6)$ GeV, where we see a marginal increase in comparison to the SM scenario. For negative Δm^2_{31} (lower figure in second column), there is an overall emplacement in the coherence of the system due to NSI effects in comparison with SM + IO. This enhancement is more prominent in the energy range 4 - 6 GeV for all values of δ phase along with energy around 2 GeV for $\delta \in (0 - 0.8)$ and $(5 - 6)$.

The results for LMA-Dark solution are depicted in the third column of Fig. 6.1(a). It can be seen from the figure that for normal mass ordering (upper figure in third column), the NSI effects provide almost similar pattern of coherence χ as that is for LMA-Light + NO, except for the case at around 4 GeV where large coherence can be seen in $\delta \in (2 - 5)$ region for $E \in (4 - 6)$ GeV. For inverted mass ordering, a large value of χ is observable in $\delta \in (0 - 0.8)$ and $(3 - 6.23)$.

NSI effect (second analysis): We now present results using the NSI parameters obtained by including ν_e and $\bar{\nu}_e$ appearance data from T2K and NOνA in the global fit to neutrino oscillation data [69] which we call the second analysis. Based on goodness of fit, two solutions were obtained: LMA-Light solution with NO and LMA-Dark solution with IO. The results are presented in Fig. 6.1(b) corresponding to these results.

The upper plot in Fig. 6.1(b) depicts χ^{NSI} for LMA-Light solution with NO. One can compare this plot with the upper figure in the second column of Fig. 6.1(a) and it can be seen that the values of χ^{NSI} using the updated results are almost similar in these plots except for the case where large coherence value is shifted to $\delta \in (0-0.2)$ and $(4-6)$ for $E \in (4-6)$ GeV. However, as compare to SM + NO case, over all coherence is reduced for $E \in (4-6)$ GeV.

The lower panel of Fig. 6.1(b) represents the case of LMA-Dark + IO for second analysis. This can be seen by comparing this figure with the lower plot in the third column of Fig. 6.1(a) that the updated values of NSI parameters result in a large enhancement in the value of coherence parameter in the entire $(E-\delta)$ plane. This enhancement is more prominent in the region around $E \sim 4$ GeV where maximum neutrino flux is expected in the DUNE experiment. For LMA-Dark solution with IO, the effect of NSI is to increase the inherent coherence of the system in comparison to the SM (IO) case in the energy range corresponding to the maximum neutrino flux in the DUNE experiment.

Furthermore, to be specific, one should compare the results obtained for SM neutrino-interaction (Fig. 6.1(a)) with that obtained using NSI parameters updated in the second analysis including all the possible constraints (Fig. 6.1(b)). It is observed that for the SM + NO, the maximal coherence achieved is $\chi_{max} \approx 1.67$ at around $E \sim 2$ GeV for $2.5 \lesssim \delta \lesssim 5.5$ which is approximately 84% of the maximum allowed value of coherence, *i.e.,* 2. Further, in the energy range 4 - 6 GeV (where the maximum neutrino flux is expected for the DUNE experiment), χ^{SM} can achieve quite large value ($\approx$ 1.5) for all values of CP-violating phase. For the SM + IO scenario, the maximal value of coherence is 1.5 in the energy range 4 - 5 GeV for $3 \lesssim \delta \lesssim 5.5$. This is 75% of the maximum allowed value of the coherence parameter. Thus we see that within SM interaction, the quantumness of the system which we have quantified in terms of coherence, is sensitive to the sign of Δ_{31} as well as to the CP violating phase and the system will have relatively large coherence (or quantumness) in case of NO.

In the upper plot of Fig. 6.1(b) that depicts χ^{NSI} for LMA-Light solution with NO, it can be seen that the overall effect of this NSI solution is to reduce the coherence. The maximal coherence obtained for the LMA-L + NO solution is ≈ 1.58 (79% of the maximum allowed value) at around $E \approx 2$ GeV (for all δ) and $E = 4-5$ GeV (for $0 \leq \delta \leq 0.5$ and $4 \leq \delta \leq 6.28$). This means that χ_{max} for the LMA-L + NO scenario is reduced by 5% as compared to the SM + NO. Contrarily, for IO, the LMA-D solution (lower panel of Fig. 6.1(b)) provides slightly larger value of χ_{max} for all values of δ in the energy range 4 - 6 GeV in comparison to the SM. Moreover, in this energy range, the overall enhancement in the coherence of the system for the LMA-D + IO solution is around 6% as compared to the SM + IO for all values of δ except for $2.5 \lesssim \delta \lesssim 5.5$.

In the left panel of Fig. 6.2 we have plotted the survival probability $P_{\nu_\mu \to \nu_\mu}$ (blue) and transition probabilities $P_{\nu_\mu \to \nu_e}$ (red) and $P_{\nu_\mu \to \nu_\tau}$ (green) with respect to E with $L = 1800$ km for DUNE. The upper and lower panel of Fig. 6.2 represent the case of NO (with SM (solid lines) and LMA-L (dashed lines)) and IO (with SM (solid lines) and LMA-D (dashed lines)), respectively. The right panel in Fig. 6.2 depicts the variation of χ parameter with respect to the neutrino-energy E ($L = 1800$ km). We have taken the value of CP-violating phase δ to be $\pi/2$. It is evident from the figure that the deviation in the coherence parameter χ from its SM value due to NSI effects is larger in comparison to the probabilities. This is true for both NO and IO scenarios. For example, the maximum difference observed in $P_{\nu_\mu \to \nu_\mu}$ is around 2%, while this difference is higher (around 5%) in case of χ parameter for NO. Similarly, for IO, the difference observed in $P_{\nu_\mu \to \nu_\mu}$ is less then 2%, while the coherence is increased by $\sim 6\%$ for the LMA-D scenario as compared to the SM. Therefore a small deviation in probability due to NSI effects can trigger a larger deviation in the coherence parameter. Hence, it is worth re-examining the NP effects on various measurements of quantum correlations. In particular, the nature of correlations viz. entanglement and non-locality

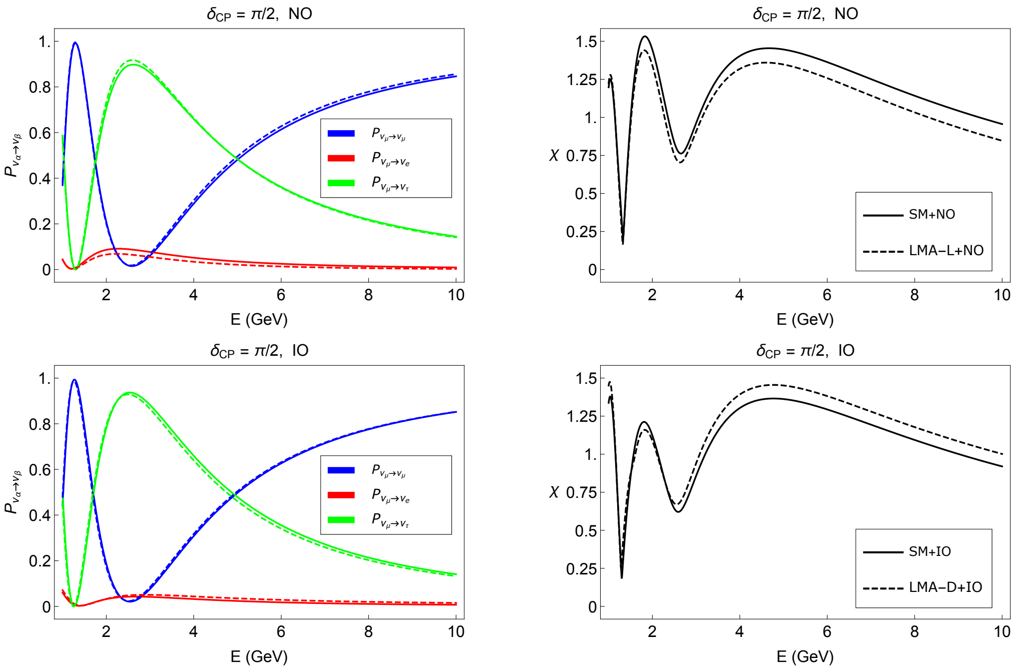


Figure 6.2: In the left panel probabilities $P_{\nu_\mu \to \nu_\mu}$ (blue), $P_{\nu_\mu \to \nu_e}$ (red) and $P_{\nu_\mu \to \nu_\tau}$ (green) are plotted with respect to E in the context of DUNE ($L = 1300$ km) experiment for $\delta = \pi/2$ where solid and dashed lines correspond to the SM and NSI interaction, respectively. The upper and lower panels correspond to normal mass hierarchy (with SM and LMA-L scenario) and inverted mass ordering (with SM and LMA-D scenario), respectively. The right panel shows the variations of χ parameter with E for $\delta = \pi/2$. The upper right and lower right panels correspond to the NO (with SM and LMA-L scenario) and IO (with SM and LMA-D scenario), respectively. The $\epsilon_{\alpha\beta}$ parameters are taken from recently updated analysis given in ref. [69].

in terms of Bell-type and Leggett-Garg inequalities in neutrino oscillations should be reanalyzed under NSI effects. To test these correlations, the long baseline (LBL) experiments can serve the purpose as it is possible to have good control over the neutrino-source and the detector. Among numerous LBL facilities, DUNE will be more sensitive to NSI effects since neutrinos will have to travel longer distance in Earth's material medium. Thus the NSI effects will be more notable in the correlation measures for the DUNE experimental setup.