A study of Quantum Mechanical aspects in Neutrino Oscillations

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Conclusions

In this thesis, different facets of nonclassicality have been investigated for the neutrino system by considering the three flavour scenario of neutrino oscillation. The matter effects are included in order to carry out the analysis in the context of the ongoing neutrino experiments NOνA and T2K and also for the future experiment DUNE. The analysis is carried out by considering both neutrino and antineutrino beams for the experiments. Quantum correlations show sensitivity to the neutrino mass ordering, i.e. the sign of Δm^2_{31}. It is a general feature displayed by all the correlations that the sensitivity to the mass ordering becomes more prominent for the high energy and long baseline experiment like DUNE compared to NOνA and T2K experiments. Obtained results also suggest that in order to probe various measures of nonclassicality in neutrino sector, one must use neutrino beam for the positive sign of Δm^2_{31} and an antineutrino beam otherwise.

Since these results favour the efficiency of neutrinos for quantum information tasks, a more detailed analysis about quantum correlations in neutrino-system is demanded under different circumstances. Therefore, the effects of quantum decoherence on the quantumness inherent in the neutrino-system is investigated. Specifically, coherence and mixedness is studied in the context of a decoherence model explicitly suggested to explain the anomalous events observed in LSND experiment (however, this model also fits with the global data coming from other oscillation experiments). In this model, the decoherence parameter γ is a function of the energy of neutrino E_ν (has exponential dependence on E_ν). For this model, the neutrino-system, within three-flavour oscillation scenario is found to follow the complementarity relation between coherence and mixedness. Also, it is seen that the coherence embedded in the neutrino system decreases with the increase in γ and attains its minimum value at $E_\nu \approx 30$ MeV. Further, the coherence for the neutrino system is found to depend on the CP violating phase δ.

Following the same motivation, next, the impact of new physics on coherence embedded in the system of oscillating neutrinos is included in terms of non-standard neutrino-matter interaction. These NSI effects on coherence are incorporated in a model-independent way within the framework of effective Lagrangian where higher dimensional operators are added to the SM Lagrangian, by restricting our analysis to dimension-6 operators. Recently, a global analysis of all relevant neutrino oscillation data in the presence of non-standard interaction was performed which included constraints from ν_e and $\bar{\nu}_e$ appearance data from T2K and NOνA due to which the allowed non-standard interaction parameter space is now different for normal and inverted mass orderings. Further, it was observed that two scenarios, LMA-Light solution with normal ordering and LMA-Dark solution with inverted ordering, provide a good fit to all data. In the DUNE experimental set-up, we find that the first solution marginally decreases the value of coherence parameter in comparison to the SM. For the LMA-Dark solution with inverted ordering, the coherence in the system is enhanced around $E \approx 4$ GeV, the energy corresponding to maximum neutrino flux at DUNE, for almost all values of CP violating phase. It is found that neutrinos can maintain large coherence over macroscopic distances in case of both SM and NSI. However, the oscillating system of three flavours of neutrinos would never reach its maximum value $\chi = 2$ in the context of DUNE experiment. Further, it is also noticeable that a small change in probabilities due to NSI effects can trigger relatively large alteration in the coherence inherent in the neutrino-system.

Further, we analyze the non-cyclic geometric phase which would be, in principle, easier to observe than its cyclic counterpart, in the context of three flavour neutrino oscillations in the presence of matter and CP violating effects at various man-made facilities, such as the reactor and accelerator neutrino experimental set-ups. The geometric phase is seen to be sensitive to the sign ambiguity in Δm^2_{31}. We studied GP in the context of two reactor neutrino experimental set-ups, Daya-Bay & RENO and two accelerator experiments, T2K & NOνA. It is found that for experimental facilities where the geometric phase can complete at least one cycle, the geometric phase corresponding to different values of CP violating phase, converge to a single point, called the *cluster point*. These cluster points are distinct for positive and negative signs of Δm^2_{31}. Thus experimental set-ups, such as T2K, Daya-Bay and RENO where at least one complete cycle of GP is possible, could help in resolving the neutrino mass hierarchy problem. The normal and inverted types of mass ordering are distinguishable for all values of CP-violating phase $\delta \in [0, 2\pi]$ in the suggested energy range in these three experiments. Moreover, we have also found that matter effects will disturb the formation of cluster points and hence, it will be difficult to pin-down the position of distinct cluster points corresponding to the normal and inverted mass ordering when neutrinos experience matter density potential during their travel. This implies that matter effect does not provide a suitable environment in favour of resolving the neutrino mass hierarchy problem through the measurement of GP.

Quantum correlations in the context of gravitationally induced neutrino-antineutrino oscillations are also studied. For single flavour neutrino-antineutrino oscillations, entanglement is found to be maximum for the case when neutrino and antineutrino states are equally probable. Gravity, however, suppresses the entanglement between neutrino and antineutrino states, which is implied by a decrease in the von-Neumann entropy $\mathcal{S}$ with the increase in the gravitation potential B^g_0. Further, in case of 2-flavour neutrino-antineutrino oscillations, $\mathcal{S}$ is non-zero which indicates the absolute entanglement present in the system. Mermin inequality is violated while Svetlichny is not, implying that the system is having absolute nonlocal correlations (nonlocality shared by at least two parties) but genuine nonlocal correlations (nonlocality shared among all parties) are absent, a consequence of the degeneracy in the levels $\nu_1 - \nu_4$ and $\nu_2 - \nu_3$.

Summarizing overall discussions of this thesis, results can be categorized in two parts as follows:

Results Important for Quantum Information Sector
• Single neutrino-state exhibits large entanglement between its flavour modes and show prominent violation of Bell-type inequalities. This signifies that neutrino is a potential candidate for tasks related to quantum computation and information transmission. • Neutrino oscillations, under the influence of quantum decoherence effects, favour the trade-off relation between coherence and mixedness. • It is found that for inverted ordering, NSI effects can facilitate quantum information tasks owing to increase in coherence in comparison with SM interactions, whereas for normal ordering, new physics effects can decrease the coherence demeaning the neutrino efficiency for QIP tasks.

- In the presence of gravity, neutrino-antineutrino oscillations are induced and the two-flavor neutrino oscillation scenario can be mapped as a 4-qubit system. Genuine nonlocal features are observed to be absent for such scenario. However, absolute nonlocal correlations still exist.

Results Important for Neutrino Oscillation Sector

- Various entanglement measures, nonlocality witnesses, coherence and mixedness parameters as well as geometric phase are found to be sensitive to the CP-violating phase and neutrino mass ordering.
- Coherence parameter shows more deviation from its SM value due to NSI effects in comparison to the probabilities, both in case of normal and inverted mass ordering. Hence, measurement of coherence and other correlation features can also be used to probe new physics in neutrino sector.
- Measurement of quantum correlations can provide information related to mass ordering. This information can be more prominent in case of experimental setups with large baseline and higher neutrino-energy (*i.e.,* large matter effects) such as DUNE.
- The knowledge about the neutrino mass ordering can also be obtained by the observation of geometric phase in the neutrino-system in the context of certain oscillation experimental setups viz. T2K, Daya-Bay and RENO (where matter effect is almost negligible). This discrimination of mass orderings is possible due to the presence of two distinct cluster points (the point in $GP - E$ plane where the value of GP does not depend on the CP-phase) corresponding to normal and inverted neutrino mass orderings.
- We also expressed GP in terms of observable quantities, neutrino survival and oscillation probabilities, which provides a platform for an indirect measurement of GP in neutrino sector.

Future directions:

- The correlation measures included in this thesis are based on the nonclassical features represented in terms of coherence, entanglement and nonlocality that can be witnessed in terms of violations of Bell-type inequalities. However, there are several other aspects **related to** quantum information processing. For example, disturbance induced by a local measurement and persistence of coherence in all possible local bases are some of the crucial features that can enhance the performance of a number of quantum protocols over the classical ones experiencing noisy environments [242]. Hence, the platform is open for the study of several other nonclassicality measures in the neutrino oscillation dynamics.

- In chapter 6, it is seen that coherence embedded in neutrino-system gets significantly affected by the effects of nonstandard neutrino-matter interaction. Hence, it will be interesting to reanalyze previously studied correlation measures under the effects of NSI. The methodology provided in chapter 6 is the general one which can be implemented for such investigation of nonclassical features.

- A very important and open issue is about the experimental verification of various correlation measures in the neutrino sector. It is accessible to measure correlations quantities such as entanglement in meson-sectors as these particles are copiously produced in pairs (e.g. $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$) in K-meson and B-meson factories. However, it is challenging in general, to measure single-particle mode entanglement as is exhibited by the state describing the three flavour neutrino oscillation. Therefore, experimental proposals which can realize the observation of mode entanglement in the neutrino sector will have immense implications not only for performing various QI tasks but would also help in resolving some of the open questions related to neutrino physics.

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Other works done: Coherence and Mixedness in Mesonic systems

This chapter is based on Ref. [95]. So far, the results obtained by analyzing the nature of correlations have indicated the prominence of neutrinos to perform various tasks related to quantum information and computation. These results can be attributed to the weakly interacting nature of neutrinos. Furthermore, it is also interesting to inspect the correlations in the oscillating neutral mesonic systems, since these systems also participate in weak interactions. However, due to their decaying nature, mesons cannot be considered as promising candidates for information transmission. Nonetheless, foundational issues can be studied in these systems.

In this chapter, we first elaborate the meson-system dynamics in section 10.1. Then we present results of our analysis of coherence and mixedness in this system in section 10.2.

10.1 System dynamics

We briefly discuss the dynamics of the meson system. The meson being a decaying system, is treated as an open quantum system, leading to a generic description of decoherence in this system[1].

For the B-meson system, imagine the decay $\Upsilon \to b\bar{b}$ followed by hadronization into a $B\bar{B}$ pair. In the Υ rest frame, the mesons fly off in opposite directions (left and right, say); since the Υ is a spin-1 particle, they are in an antisymmetric spatial state. The same considerations apply to the K system, with the Υ replaced by a ϕ meson.

The flavour-space wave function of the correlated $M\bar{M}$ meson systems ($M = K,\ B_d,\ B_s$) at the initial time $t = 0$ is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|M\bar{M}\rangle - |\bar{M}M\rangle\right], \tag{10.1}$$

where the first (second) particle in each ket is the one flying off in the left (right) direction and $|M\rangle$ and $|\bar{M}\rangle$ are flavour eigenstates. As can be seen from Eq. (10.1), the initial state of the neutral meson system is a singlet (maximally entangled) state.

The Hilbert space of a system of two correlated neutral mesons, as in Eq. (10.1), is

$$\mathcal{H} = (\mathcal{H}_L \oplus \mathcal{H}_0) \otimes (\mathcal{H}_R \oplus \mathcal{H}_0), \tag{10.2}$$

[1]Neutrino dynamics was studied in chapter 5 in the context of three flavour oscillations within the framework of a decoherence model recently used in the context of LSND experiment.

where $\mathcal{H}_{L,R}$ are the Hilbert spaces of the left-moving and right-moving decay products, each of which can be either a meson or an anti-meson, and $\mathcal{H}_0$ is that of the zero-particle (vacuum) state. Thus, the total Hilbert space is the tensor sum of a two-particle space, two one-particle spaces, and one zero-particle state. The initial density matrix of the full system is

$$\rho_{\mathcal{H}}(0) = |\psi(0)\rangle \langle\psi(0)| . \tag{10.3}$$

The system, initially in the two-particle subspace, evolves in time into the full Hilbert space, eventually (after the decay of both particles) finding itself in the vacuum state. As can be appreciated from basic notions of quantum correlations such as entanglement, we need to project from the full Hilbert space $\mathcal{H}$ down to the two-particle sector $\mathcal{H}_L \otimes \mathcal{H}_R$. This is easily done once $\rho_{\mathcal{H}}(t)$ is written in the operator-sum representation [20, 243, 244]. The result is

$$\rho(t) = A \begin{pmatrix} |r|^4 a_- & 0 & 0 & -r^2 a_- \\ 0 & a_+ & -a_+ & 0 \\ 0 & -a_+ & a_+ & 0 \\ -r^{*2} a_- & 0 & 0 & a_- \end{pmatrix}, \tag{10.4}$$

where $a_\pm = (e^{2\lambda t} \pm 1)(1 \pm \delta_L)$, $A = (1 - \delta_L)/4(e^{2\lambda t} - \delta_L^2)$, $\delta_L = 2\mathrm{Re}(\epsilon)/(1 + |\epsilon|^2)$ and $r = (1+\epsilon)/(1-\epsilon)$. $\rho(t)$, which is written in the basis $\{|MM\rangle, |M\bar{M}\rangle, |\bar{M}M\rangle, |\bar{M}\bar{M}\rangle\}$, is trace-preserving. Here ϵ is a small CP-violating parameter [245]. It is of order $\sim 10^{-3}$ for K mesons and 10^{-5} for $B_{d,s}$ mesons. λ is the decoherence parameter, representing interaction between the one-particle system and its environment which could be ascribed to quantum gravity effects [140, 141, 246–253]. Some quantum gravity models are characterized by quantum fluctuations of space-time geometry, such as microscopic black holes, giving rise to quantum space-time foam backgrounds and hence may lead to decoherence. A stochastic fluctuation of point-like solitonic structure, known as D-particles, can constitute an environment for matter propagation [248, 249]. Further, if the ground state of quantum gravity consists of stochastically-fluctuating metrics, it can lead to decoherence [251, 252]. The decoherence in the mesonic systems can also be due to the detector background itself. Irrespective of the microscopic origin of the environment, its effect on the neutral meson systems, in our formalism, is modelled by a phenomenological parameter λ.

Diverse experimental techniques are used for testing decoherence, ranging from laboratory experiments to astrophysical observations [23, 74, 165, 169, 254–261]. In the case of the K meson system, its value has been obtained by the KLOE collaboration by studying the interference between the initially entangled kaons and the decay product in the channel $\phi \to K_S K_L \to \pi^+\pi^-\pi^+\pi^-$ [256]. The value of λ is at most $2.0 \times 10^8\,\mathrm{s}^{-1}$ at 3σ. Using existing Belle data on the time-dependent flavour asymmetry of semi-leptonic B_d decays as given in Ref. [262], an estimate on λ for the B_d system was obtained in [244]. At 3σ, the value of λ is restricted to $0.012 \times 10^{12}\,\mathrm{s}^{-1}$ [244]. For B_s mesons, to the best of our knowledge, there is no experimental information about λ. We consider it to be same as is in case of B_d mesons. The mean life time for K, B_d and B_s mesons are $\tau_k = 1.7889 \times 10^{-10}$ sec, $\tau_{Bd} = 1.518 \times 10^{-12}$ sec and $\tau_{Bs} = 1.509 \times 10^{-12}$ sec, respectively. In case of K-mesons the CP-violating parameter is $|\epsilon| = 2.228 \times 10^{-3}$, $Re(\epsilon) = 1.596 \times 10^{-3}$. The dynamics of B-mesons is similar to that of K-meson with ϵ replaced by $\frac{p-q}{p+q}$. The decay widths are $\Gamma_k = 5.59 \times 10^9\mathrm{sec}^{-1}$, $\Gamma_{Bd} = 6.58 \times 10^{11}\mathrm{sec}^{-1}$ and $\Gamma_{Bs} = 6.645 \times 10^{11}\mathrm{sec}^{-1}$ [245, 263].

10.2 Measures of coherence and mixedness in meson systems

Here the study of decaying systems is made using their evolution. Another approach would be to study, instead, the construction of effective operators and study their evolution, instead of studying the state evolution, i.e., by invoking the Heisenberg picture. This approach was used to construct Bell inequality violations in neutral Kaon systems [264].

In order to take into account the effect of decay in the system under study, the measures given in Eqs. (5.2) and (5.1) must be multiplied by the probability of survival of the pair of particles up to that time, $P_S(t)$, which can be shown to be

$$P_S(t) = e^{-2\Gamma t}\frac{(1-\delta_L^2 e^{-2\lambda t})}{1-\delta_L^2}, \tag{10.5}$$

where Γ is the meson decay width. For the K meson system, $\Gamma = (\Gamma_S + \Gamma_L)/2$ where Γ_S and Γ_L are the decay widths of the short and the long neutral Kaon states, respectively. For the state given in Eq. (10.4), we have

$$\chi(\rho) = 2P_S(t)A\left[a_+ + a_-\left|\left(\frac{\epsilon+1}{\epsilon-1}\right)^2\right|\right]. \tag{10.6}$$

Also, the mixedness parameter $\eta(\rho)$ is given by

$$\eta(\rho) = \frac{4}{3}P_S(t)\left[1 - a_-^2 A^2\left\{1 + 4\left(\frac{a_+}{a_-}\right)^2 + 2\delta_\epsilon^2 + \delta_\epsilon^4\right\}\right], \tag{10.7}$$

where

$$\delta_\epsilon = \frac{1+2\mathrm{Re}(\epsilon)+|\epsilon|^2}{1-2\mathrm{Re}(\epsilon)+|\epsilon|^2}. \tag{10.8}$$

The corresponding simplified expressions, neglecting the small CP violation, can be obtained from these expressions by setting ϵ equal to zero. For meson-systems, coherence is a function of λ, ϵ, t and Γ. For $\lambda = 0$, $\chi = e^{-2\,t\,\Gamma}$. Thus we see that, in the absence of decoherence, coherence depends only on Γ and t and not on CP violation. In the absence of CP violation in mixing, $\chi = e^{-2\,t\,\Gamma}$. Hence in the limit of neglecting CP violation in mixing, coherence is independent of decoherence parameter.

Like coherence, mixedness is also function of λ, ϵ, t and Γ. $\eta(\rho) = 0$ only if $\lambda = 0$, i.e., the state (10.4) will become mixed only in the presence of quantum gravity like background fluctuations. Also the maximum value of $\eta(\rho)$ cannot approach 1, as can be seen from Fig. 10.1. Hence the concept of maximally coherent mixed state, which is valid for states satisfying the equality in the complementarity equation will never happen in these decaying systems, as can be seen from the right most panels of Fig. 10.1. Apart from the violation of the well-known relation between non-locality and teleportation fidelity, as observed in [20], this brings out another difference between the stable and decaying system. The effect of CP violating parameter ϵ in mixedness is negligible. It is obvious from the middle panel of Fig. 10.1 that $\eta(\rho)$ increases with t till about one life time of the mesons. After one life time, the average mixedness is seen to decrease with time. This is an artifact of the decaying nature of the system and can be attributed to the modulation by $P_S(t)$, Eq. (10.5).

The interplay between coherence and concurrence, $C(\rho)$, can be represented as

$$C(\rho) = \chi(\rho)\, e^{-2t\lambda}. \tag{10.9}$$

It can be seen from the above relation that coherence and concurrence are not synonyms with each other, a fact that is highlighted by the interest in coherence resource theory [35]. Further, the interplay between coherence, mixedness and nonlocality [2], $M(\rho)$, is given by

$$\eta(\rho) = \frac{4}{3}\left(\chi(\rho) - \frac{M(\rho)}{2}\right). \tag{10.10}$$

[2]Here we do not study an experimental test of Bell's inequality from local realism. Instead, we make use of an important result obtained in [265] which enables us to make quantitative statements about Bell inequality violations, indicative of nonlocality, just by making use of the parameters of the density operator describing the system.

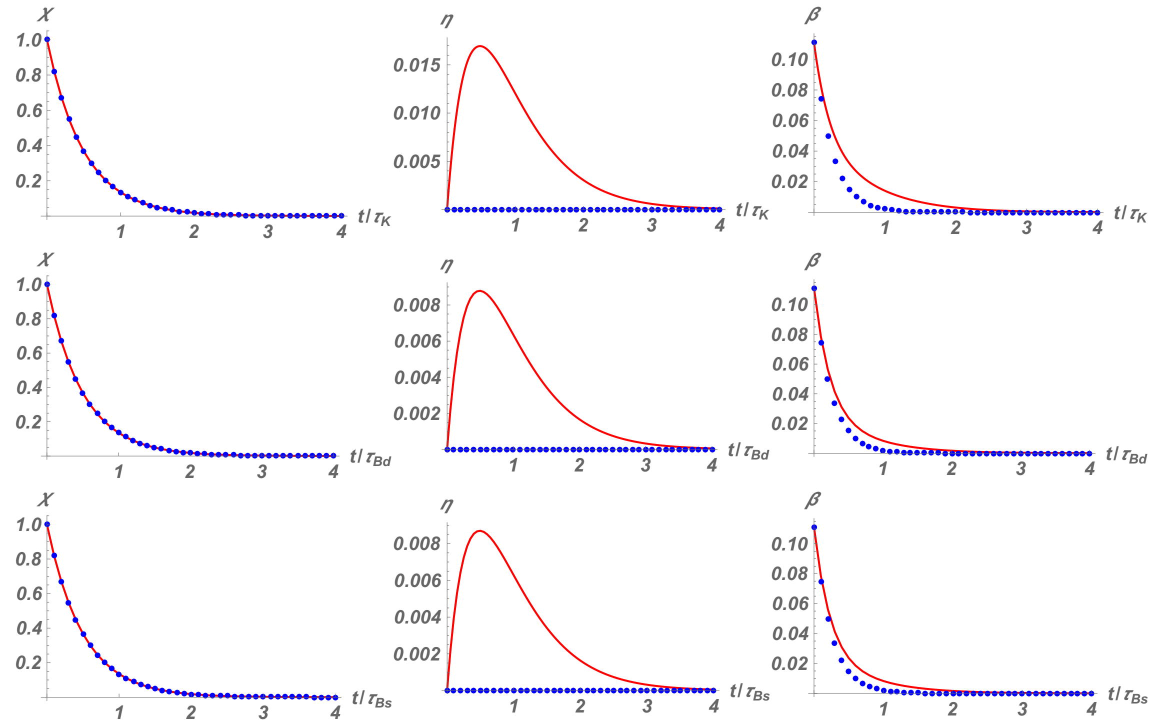

Figure 10.1: *Meson system*: Coherence parameter $\chi(\rho)$ (left), mixedness parameter $\eta(\rho)$ (middle) and complementarity parameter $\beta(\rho)$ (right) as function of the dimensionless quantity $t/\tau_{K(B_d,B_s)}$. Top, middle and bottom panels pertain to the case of K, B_d and B_s mesons, respectively. The solid (red) and dotted (blue) correspond to the case with and without decoherence, respectively. In all the three cases, the average mixedness increases for about one lifetime of the particles.

Here $M(\rho)$ denotes non-locality modulated by the probability of survival of the pair of particles up to the time of interest [20]. $M(\rho) > 1$ implies non-locality [265]. To keep the expression simple, here we have neglected the small CP violating parameter ϵ. From the above equation, we get $M(\rho) = 2\chi(\rho) - \frac{3}{2}\eta(\rho)$. Using the theoretical expressions for $\chi(\rho)$ and $\eta(\rho)$, given in Eqs. (10.6) and (10.7), respectively, along with the numerical inputs for Γ and λ, we can reproduce the $M(\rho)$ plots as obtained in [20]. This brings out the consistency of the present analysis.

In this chapter we have studied the interplay between coherence and mixedness for the systems of neutral mesons which are governed by weak interactions. The meson systems are interesting as well as challenging due to their both oscillatory and decaying nature. The impact of decoherence and CP violation is also included. In the limit of neglecting CP violation in mixing, it is observed that coherence is independent of the decoherence parameter. It is also shown that the concept of maximally coherent mixed state, which is valid for states satisfying the equality in the complementarity relation will never happen for the correlated neutral meson system. An interesting feature that comes out is that for about one life time of these particles, the average mixedenss increases with time in consonance with our usual notion of mixedness. However, after this, the mixedness decreases with time, a behaviour that can be attributed to the decaying nature of the system. Further, for these correlated meson systems, quantum coherence and entanglement are found not to be synonyms with each other, a fact that has been bolstered by the interest in coherence resource theory.