Certificate

This is to certify that the thesis titled *A study of Quantum Mechanical aspects in Neutrino Oscillations*, submitted by *Khushboo Dixit (P15PH003)* to the Indian Institute of Technology Jodhpur for the award of the degree of *Doctor of Philosophy*, is a bonafide record of the research work done by her under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. Ashutosh Kumar Alok
Ph.D.Thesis Supervisor

Declaration

I hereby declare that the work presented in this thesis entitled *A study of Quantum Mechanical aspects in Neutrino Oscillations* submitted to the Indian Institute of Technology Jodhpur in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy, is a bonafide record of the research work carried out under the supervision of Dr. Ashutosh Kumar Alok. The contents of this thesis in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.

Khushboo Dixit
P15PH003

List of articles

This thesis is based on the following articles:

1. **Khushboo Dixit**, Javid Naikoo, Subhashish Banerjee, Ashutosh Kumar Alok, "*Quantum correlations and the neutrino mass degeneracy problem*", Eur. Phys. J. C, **78** 914 (2018).

2. **Khushboo Dixit**, Javid Naikoo, Subhashish Banerjee, Ashutosh Kumar Alok, "*Study of coherence and mixedness in meson and neutrino systems*", Eur. Phys. J. C **79** 96 (2019).

3. **Khushboo Dixit**, Ashutosh Kumar Alok, "*New physics effects on quantum coherence in neutrino oscillations*", Eur. Phys. J. Plus **136** 334 (2021).

4. **Khushboo Dixit**, Ashutosh Kumar Alok, "*Effects of Nonstandard Interactions on Coherence in Neutrino Oscillations*", Springer Proc. Phys. **248** (2020), 343-347.

5. **Khushboo Dixit**, Ashutosh Kumar Alok, Subhashish Banerjee, Dinesh Kumar, "*Geometric phase and neutrino mass hierarchy problem*", J. Phys. G **45** (2018) no.8, 085002.

6. **Khushboo Dixit**, Javid Naikoo, Banibrata Mukhopadhyay, Subhashish Banerjee, "*Quantum correlations in neutrino oscillations in curved spacetime*", Phys. Rev. D **100**, 055021 (2019).

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Acronyms

CP	Charge conjugation - Parity
NuMI	Neutrinos at the Main Injector
MINERνA	Main Injector Experiment for V-A
LBL	Long Baseline
NOνA	NuMI Off-Axis ν_e Appearance
T2K	Tokai-to-Kamioka
DUNE	Deep Underground Neutrino Experiment
PMNS	Pontecorvo-Maki-Nakagawa-Sakata
SM	Standard Model
NSI	Nonstandard Interaction
NC	Neutral Current
CC	Charged Current
NO	Normal Ordering
IO	Inverted Ordering
LGI	Leggett-Garg Inequality
KamLAND	Kamioka Liquid Scintillator Antineutrino Detector
GP	Geometric Phase
AA	Aharonov and Anandan
RENO	Reactor Experiment for Neutrino Oscillation
CPT	Charge conjugation - Parity - Time reversal
EPR	Einstein - Podolsky - Rosen
CHSH	John Clauser, Michael Horne, Abner Shimony, Richard Holt

NIM	Noninvasive measurement
LOCC	Local Operations and Classical Communication
BNL	Brookhaven National Laboratory
DONUT	Direct observation of the nu tau
MSW	Mikheyev-Smirnov-Wolfenstein
SSM	Standard Solar Model
LSND	Liquid Scintillator Neutrino Detector
MiniBooNE	Mini Booster Neutrino Experiment

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