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Introduction

This thesis is based on a study of various facets of quantum correlations in particle physics systems such as neutrinos and neutral mesons. The work highlights the interplay between various measures of quantum correlations and the underlying dynamics, and also brings out the usefulness of such investigations by connecting them with some of the open problems in neutrino physics such as the mass ordering and CP (charge conjugation-parity) violation.

Quantum Mechanics has proved to be an incredibly successful theory. Not only have its predictions been verified with great accuracy, but it has also laid the foundation of new realms of technology, ready to revolutionize the information and communication sectors. Surprisingly, despite all of its success, the question of when does a system behave quantum mechanically rather than classically, still waits for a clear and unambiguous answer. This question becomes important while dealing with the nature of correlations between different subsystems of a composite system. These correlations can be spatial as well as temporal. Some of the widely studied spatial quantum correlations are entanglement [1], steering [2], non-locality [3] and quantum discord [4]. The temporal correlations include Leggett-Garg (LG) [5] and LG type inequalities [6].

Quantum correlations have been studied mainly in the optical and electronic systems [7–10]. Recently such studies have been extended to the systems in high energy physics owing to the advancement in various experimental facilities, see for example [11–25]. Also, the transmission of information using NuMI beam and MINERνA detector at Fermilab has been demonstrated for the very first time [26]. The concept of single particle entanglement has been introduced in previous studies [27–29] which have also been demonstrated experimentally with single photon systems [30–32]. Later, the experimental schemes to probe nonlocality were generalized to include massive particles [33]. In [15], an experimental scheme was discussed for transferring this form of entanglement to spatially separated modes of stable leptonic particles. This allows us to put mode entanglement in neutrino oscillations on equal footing with that in atomic and optical systems. Therefore, different flavour modes of neutrinos can be expressed as legitimate individual entities and entanglement in these flavour modes, i.e. mode-entanglement, can be studied. Quantum correlations in the context of two and three flavour neutrino oscillations were studied in [14–16, 18, 19]. However, in these works, matter and CP violating effects were not taken into account.

In this thesis, we study various facets of nonclassicality, quantified by spatial quantum correlations such as flavour entropy, geometric entanglement, Mermin and Svetlichny inequalities, in the context of three flavour neutrino oscillations, by taking into account the matter effects and CP violation. This helps to investigate these quantum correlations in various neutrinos experiments. Specifically, we analyze these quantum correlations taking into consideration the experimental setups of on going experiments such as NOνA and T2K, and also the upcoming experiment DUNE. This, in turn, helps to probe some of the open problems in neutrino sector, viz., the mass-hierarchy problem and CP violation.

The quest for understanding the diverse nature of quantum correlations has led in recent times to a rethinking in terms of the most pristine adjective associated with quantumness, viz., quantum coherence. Quantum coherence has been the subject of a lot of interest as a resource, having implications to implementation of technology [34–36] as well as in studies related to non-classicality under the action of general open system effects [37]. The resource-driven viewpoint is aimed at achieving a better understanding of the classical-quantum boundary, in a mathematically rigorous fashion. Some of the well known measures of quantum coherence are those based on relative entropy [35, 38] and skew-information [39]. Further, it has been recently shown that measures of entanglement can be put to use to understand quantum coherence [40].

Coherence is the characteristic feature of quantum mechanics which enfolds the defining features of the theory such as superposition principle and quantum correlations. In quantum information protocols, one aims to utilize these advantageous key features of quantum states for information transmission and quantum computation. Hence, it becomes very important to discern when a system is behaving classically and when it shows quantum behaviour and to what extent it can preserve its quantumness.

The quantum mechanical phenomena of neutrino oscillation is a consequence of superposition principle which makes the quantum coherence an indispensable part of the system. Due to weakly interacting nature, the system of oscillating neutrinos can maintain quantum coherence over a long distance, which can be detected in long baseline experiments [41]. Hence, neutrinos can prove to be promising candidates for various tasks related to quantum information [1].

For future applications of neutrino oscillation phenomena in the quantum information sector, it is important to quantify the quantumness of the oscillating neutrino-system. The experimentally observed neutrino oscillations can violate the classical bounds of measures such as Bell-type inequalities. However, the degree of violation of these correlation measures cannot be considered as a measure to quantify the quantummess of the system because in certain cases their maximum value depends on the channel parameters [42–44]. Therefore, in the context of quantum information, coherence becomes a fundamental concept which can signalize the quantum behaviour of the system and it shows the departure from its classical behaviour. Coherence can be rigorously characterized in the context of quantum resource theory. Further, quantum coherence is closely related to various measures of quantum correlations, such as entanglement and quantum discord [45]. Recently, quantum coherence has been quantified in terms of experimentally observed neutrino survival and transition probabilities [44].

Quantum correlations in neutrino system have been studied so far by assuming that the dynamics is governed by the Standard Model (SM). The fact that the SM cannot be considered as the quintessential theory of fundamental interactions of nature, one needs to explore the effects of beyond SM physics. The experiments such as BaBar, Belle and LHCb have already provided several tantalizing evidence of such new physics. A hint of lepton flavour non-universality, in disagreement with the SM, has been observed in the decays induced by the quark level transitions $b \to cl\nu$ $(l = e, \mu, \tau)$ [46–48] and $b \to s\, l^+ l^-$ $(l = e, \mu)$ [49, 50]. Several model independent analyses identified the Lorentz structure of possible new physics in these decay modes. These Lorentz structures can be generated in new physics models, such as Z' and leptoquark models, and hence can account for the observed anomalies in semi-leptonic B decays. The existence of these new particles can also affect the pattern of neutrino oscillations [51–55]. Also, the fact that the experimental facilities in neutrino physics are now tending towards higher precision and have potential to probe sub leading effects like nonstandard neutrino-matter interaction; it is worth considering new physics effects on various measures of quantum correlations in the context of

[1] Although the dynamics of the mesonic system is also driven by weak interactions, owing to its short lifetime, this system would be more suitable for understanding foundational issues rather than having any applicational implications.

neutrino systems. Hence, in this thesis, we also studied the effects of beyond SM physics on coherence embedded in the neutrino system quantified in terms of the coherence parameter[2]

A convenient way to describe these new physics effects in neutrino interactions in the electroweak broken phase are the so called non-standard interaction (NSI) parameters [56–71]. The SM Lagrangian contains only renormalizable interactions with canonical dimensions $D \leq 4$. Assuming that new physics exists at some high energy Λ, the effects of these new physics interactions at the energy scale much below Λ, can be described in a model independent way by including higher dimensional operators constructed out of the SM fields. In this work, we restrict our analyses to dimension-6 operators which are expected to give observable contributions for energy $<< \Lambda$.

A global analysis of oscillation data with NSI interactions in three flavour scenario was performed in [68]. In [68] observables sensitive to the CP-violating phase (such as ν_e and $\bar{\nu}_e$ appearance at long baseline experiments) were excluded from the fit and hence the constraints were obtained on the CP-conserving part only. Further, as some approximations ($\Delta m^2_{21} = m^2_2 - m^2_1 \to 0$ in atmospheric and long baseline CP-conserving experiments) were used to simplify the calculations, the effect of mass ordering was greatly reduced. This analysis was performed for both scenarios: first (LMA-Light) and second (LMA-Dark) octant solution of solar mixing angle θ_{12}. Recently this global analysis has been extended to include complex NSI neutral current interaction with quarks for observables sensitive to the leptonic CP-violating phase and mass ordering [69]. In [69] they analyzed all the four combinations, i.e., Light and Dark sector with normal ordering (NO) and inverted ordering (IO) of mass states, in which two solutions, Light octant with NO and Dark octant with IO, are favoured in global analysis of oscillation data. In view of these updates, the effects of NSI on the quantum coherence of the neutrino system within the context of Deep Underground Neutrino Experiment (DUNE) [72] are investigated in this thesis.

Further, the thesis is devoted to a study of the trade-off between mixedness and quantum coherence in neutrino and neutral meson systems. This is done by incorporating the decoherence effects (possibly) due to quantum gravity like fluctuations in neutrino system, using the formalism of [73, 74]. This model was developed to explain the LSND (a short baseline accelerator neutrino experiment) data which could not be described by usual three flavour neutrino oscillation phenomena.

The thesis also brings out the behaviour of the geometric phase (GP) in neutrinos and its role to address the neutrino mass hierarchy problem. The concept of GP exists in the standard Pontecorvo formulation of neutrino oscillations [75]. The adiabaticity condition needs to be satisfied in order to have Berry phase. This condition is provided by varying the matter density. The Berry phase in the context of two flavour neutrino oscillations was calculated in Ref. [75]. The possibility of observation of Berry phase in long baseline neutrino oscillation experiments was discussed in [76]. In order to extract Berry phase, one needs to vary the oscillation distance L while maintaining the cyclicity condition [76]. Thus one has to measure this phase after the state has undergone a complete cycle with time period T. In neutrino oscillations, this period is relatively long so, one needs to place the detector at sufficiently large distance from the source to observe the Berry phase. The generalization of GP to noncyclic paths was made shortly after Berry's discovery involving closed cyclic paths, see for example [77]. The noncyclic phase could be easier to observe because the cyclicity constraint is removed as shown in Ref. [78], where the analysis has been done for vacuum case only. Recently, attempts have been made towards establishing the Dirac or Majorana nature of neutrinos using noncyclic GP [79, 80]. In [81] the topological phase in two-flavour neutrino oscillations was discussed by using the analogy between the two-flavour state and the

[2] The methodology of this work can serve as a guideline for implementing new physics effects in the study of various other measures of quantum correlations in neutrinos.

polarization states in optics. In [82] neutrino geometric phases in high-density environments were studied.

Implication of earth's matter effects and inputs from ongoing neutrino oscillation experimental setups takes this up further. The thesis investigates the non-cyclic GP in the context of three flavour neutrino oscillations in the presence of matter and CP violating effects. A discussion on how GP can be used to distinguish between the normal and inverted ordering of neutrino masses is made in this thesis. GP is analyzed in the context of two reactor neutrino experiments, Daya-Bay & RENO and two accelerator experiments, T2K & NOνA and it is found that the GP in T2K, Daya-Bay and RENO experimental set-ups can discriminate effects of normal and inverted mass ordering for all values of CP violating phase. Also, the possibility of measurement of geometric phase at the currently running neutrino physics experimental facilities is discussed.

The effect of gravitational field on neutrino oscillation was also studied [83]. It was shown that while a nonzero (Majorana) mass of neutrinos is required for the neutrino-antineutrino mixing, for oscillation between neutrino and antineutrino (or their mass eigenstates) to occur, their energies must be split. This splitting of energy due to gravitational effect, "gravitational Zeeman effect", also gives rise to the effective CPT violation, as the effective masses (pertaining to the mass eigenstates) are different [84]. Hence, along with the lepton number violation in the neutrino sector (due to Majorana mass), gravitational Zeeman effect leads to neutrino and antineutrino oscillation. In fact, gravitational Zeeman effect has many other consequences, e.g. neutrino asymmetry, baryogenesis, experimental tests of curvature couplings of spinors etc. [85–89]. Moreover, the geometric phase in neutrinos has been a part of various studies in this context [76, 78, 79, 81, 90, 91]. It is well-known that the neutrinos propagating in a varying magnetic field acquire a geometric phase [92]. However, the Zeeman like splitting was also shown to give rise to a geometric phase where the space-time curvature plays the role of the magnetic field [93].

Gravitational field also modifies the mixing. In absence of gravity, this mixing is *passive* in the sense that the value of mixing angle is always $\pi/4$. The effect of gravity leads to *active* mixing, which depends on the strength of CPT violation being determined by the strength of gravity itself. The gravity induced neutrino-antineutrino mixing also affects the flavour oscillations, thereby leading to modified neutrino oscillation even in the flavour sector. This naturally invites the investigation of different aspects of neutrino oscillations. In this direction, the present work is devoted to analyzing the neutrino-antineutrino mixing using various tools of quantum foundations.

The chapters in this thesis are arranged in such a manner so that reader can first get familiar with the concepts related to quantum information as well as neutrino oscillations**. A**fter this, we proceed with new findings obtained in Refs. [91, 94–98]. A brief outline of the thesis is as follows:

- In chapter 2, we first discuss the history of the origin of the concept of quantum correlations and its significance. We then discuss some correlation measures and nonclassicality witnesses in detail in 2.1. We also provide a general introduction of quantum coherence in 2.2. Then, in 2.3, we discuss the notion of geometric phase for a general quantum system. Specifically, we talk about the specialized case of geometric phase in 2.3.1, the so called Berry phase, which is supposed to appear for the quantum state undergoing cyclic and adiabatic evolution. Following this, in 2.3.2, we discuss the generalization of geometric phase dropping conditions of cyclicity and adiabaticity.

- In chapter 3 we provide description of various relevant aspects related to neutrino physics. A brief history of development in the neutrino sector and neutrino oscillations are discussed in 3.1, the detail of neutrino oscillation dynamics is given in 3.2. The vacuum oscillations are described in 3.2.1 and propagation under different environmental conditions such as standard matter interaction, quantum decoherence as well as nonstandard neutrino-matter interactions are described in 3.2.2, 3.3 and 3.4, respectively. To describe the phenomenon of quantum decoherence, the neutrino-system is treated as open quantum system, details of that are given in 3.3.

- Having an adequate knowledge from previous chapters of both the concepts of quantum correlations and neutrino oscillations, in chapter 4, several entanglement measures viz. von-Neumann entropy (named as flavor entropy in case of neutrino oscillations) & genuine tri-partite entanglement and Bell-type Mermin and Svetlichny inequalities are analyzed for the system of three-flavour neutrino oscillations incorporating the standard matter interaction of neutrinos in the context of some ongoing experimental facilities such as T2K and NOνA and upcoming DUNE experiment [94]. The concept of single particle mode-entanglement has been discussed for the neutrino state in 4.1. Then, various results of the study of correlation measures in the time evolution of three-flavour neutrino state have been discussed in 4.2.

- In further chapters, we aim to analyze the quantumness embedded in the neutrino system under different circumstances. The study of the trade-off determined between coherence and mixedness parameters is given in chapter 5 for the neutrino system. Nonetheless, when neutrino is propagating through vacuum or constant matter, the state describing neutrino flavour oscillations maintains its pureness through out the time evolution. However, under the influence of an environment significant to produce decoherence effect in the system, neutrino-state tends to loose its pureness achieving mixedness. Hence, the effect of quantum decoherence is examined on the quantum nature exhibited by neutrino-system, represented in terms of coherence and mixedness [95]. To incorporate the decoherence effect, we have considered a specific model introduced in [74] to account for the anomalous LSND data along with the rest of the oscillation data. Results of this analysis are given in 5.1.

- Next, in chapter 6, new physics effects in terms of nonstandard neutrino-matter interactions have been analyzed on quantumness present in the neutrino system in the environmental conditions suitable for DUNE experiment [96, 97]. Effects of nonstandard interactions are considered in a model-independent way through a 6-dimensional four fermion operator in the effective Lagrangian. Since, quantum coherence can be considered the keynote revealing quantum nature of a system, a well known coherence parameter is considered for such interpretation. Detailed discussions of results in this study are given in 6.1.

- In chapter 7 we have studied the concept of non-cyclic geometric phase in three flavour neutrino oscillation scenario for reactor experiments Daya-Bay and RENO and accelerator experiments NOνA and T2K [91]. In 7.1 we spell out the noncyclic geometric phase for the time evolution of a general quantum state. Then, in 7.2, this phase has been analyzed and discussed in detail for neutrino-system, firstly in vacuum in 7.2.1 and then in a more realistic case of neutrino-matter interaction in 7.2.2. The observation of such phase is quite difficult due to present limitations of experimental facilities in case of neutrinos, hence in 7.3, we have attempted to express the so called noncyclic GP in terms of observable neutrino oscillation probabilities so that an indirect measurement of this phase can be planned with the present resources in neutrino sector.

- Furthermore, in chapter 8, we explore various spatial quantum correlations in neutrinos propagating and oscillating in curved spacetimes [98]. A possible origin of CPT (charge conjugation-parity-time) violation in neutrino sector in terms of gravitationally induced neutrino-antineutrino oscillations has been suggested previously in [83, 84], we discuss this formalism in 8.1. In our investigation, we consider this model to study nonclassicality witnesses such as Mermin and Svetlichny inequalities. Results are presented for neutrino-antineutrino mixing both in case of single-flavour and two-flavour scenarios in 8.2.1 and 8.2.2, respectively.

Finally, collective conclusions are presented in chapter 9. Some details on the study of coherence and mixedness parameters in neutral meson systems based on the analysis presented in [95] is given in chapter 10, however, this work is not included in the main content of this thesis.