3
Basics of Neutrino Oscillations

3.1 Brief history

It all started in 1896, when Becquerel discovered the radioactivity of uranium which led to Rutherford's discovery of two of its by-products α and β after three years. γ rays were also detected latter. Then, in 1914 James Chadwick illustrated the continuous spectrum of β particles (while a discrete spectra was predicted for a two-body β-decay) in contrast with the discrete spectrum of α and γ. This continuous feature was confirmed by Elis and Wooster in 1927. Also the spin statistics could not be explained by Rutherford model. This observation made many physicists quite surprised and led them to think that the conservation laws of energy and spin are misconceptions for the real world, these are nothing but statistical concepts[1]. However, many people could not digest this ambiguity. Respecting the law of charge conservation, it was suggested that the missing energy in β-spectrum could be ascribed to a neutral particle. Later, Meitner demonstrated that the neutral γ rays cannot be implemented to solve this issue and here came the postulate of the existence of a new particle. W. Pauli, on 4 December 1930, took the courageous step to propose this new particle in a letter addressed to a conference at Tubingen. To solve this problem, along with the spin statistics, he suggested that a weakly interacting fermion, he named it *neutron*, can be the solution. E. Fermi concluded that this new particle could be massless and hence, after the discovery of neutron (which was quite heavy to be implemented in β spectrum) by J. Chadwick in 1932, Fermi renamed the particle postulated by Pauli, the *neutrino*, which means a tiny neutral particle[2]. Later, Fermi also proposed the theory for weak interaction in 1934, specifically for β-decay, which is now famous as Fermi-theory. This was also known as effective low-energy theory of the weak interaction involved in beta-decay.

Using the Fermi-theory of beta-decay, H. Bethe and R. Peierls predicted the strength of the weak interaction. The decay rate for this process was very low and the resulting extremely small value of cross-section created a disappointment. A big question mark was at the observation of this neutrino particle which remained unanswered for almost 26 years. However, Pontecorvo raised a hope in this direction by suggesting the observation of inverse beta-decay process, *i.e.,* $\nu_e + \mathrm{Cl}^- \to \mathrm{Ar}^- + e^-$. Using this idea, F. Reines and C.L. Cowan set up an experiment using a huge anti-neutrino ($\bar{\nu}_e$) flux burst out from a nuclear explosion in a reactor at Savannah River. This was the first reactor-neutrino experiment. They used another possibility of observing this anti-neutrino in the interaction $\bar{\nu}_e + p \to n + e^+$. Finally, on 14 June 1956, they sent a telegraph to Pauli informing him about the discovery of his postulated particle for the very first time. Later, in 1958, the polarization of a neutrino was also measured by Goldhaber, Grodzins and Sunyar in an electron capture e^- + $^{152}\mathrm{Eu} \to {}^{152}\mathrm{Sm}^*$ + ν_e and the direction of polarization was always found to be opposite to the direction of its propagation.

Meanwhile, the other lepton, muon (μ) was discovered in 1937, by J.C. Street and E.C.

[1] This is reflected in a Nuclear Physics text book by G. Gamow, 1931: "...This would mean that the idea of energy and its conservation fails in dealing with processes involving the emission and capture of nuclear electrons."

[2] In Italian, "ino" means small.

Stevenson [115] and S.H. Neddermeyer and C.D. Anderson [116] which initiated the concept of generation or family of particles. It was found that μ decays in electron and some other unknown particle. This emitted electron was also found to have energy distribution in the range 0 - 54 MeV which indicated that there should exist at least two more particles along with e^- because two body decay does not allow a continuous energy spectrum of e^-. First, it was thought that those two particles are a neutrino and an anti-neutrino but in that condition, they were supposed to annihilate with each other and produce a γ ray. But this process could never be observed. Therefore, it was concluded that these two neutrinos are unrelated with each other and the second type of neutrino, *i.e.,* ν_μ was introduced. Also a new series of lepton number L_e and L_μ came into picture to solve the problem of muon decay. In 1962, L.M. Lederman, M. Schwartz, J. Steinberger successfully discovered the second neutrino ν_μ at Brookhaven National Laboratory (BNL).

After the discovery of third charged lepton τ^- by M. Perl in 1975, the third type of neutrino ν_τ could also be considered in the picture and latter on discovered by DONUT collaboration in 2000. Finally, we have three types or *flavours* of neutrinos ν_e, ν_μ, ν_τ associated with three charged leptons e^-, μ^-, τ^- that matches the upper bound on the number of neutrino species, 2.984 $\pm$ 0.008 [117], *i.e.,* approximately 3, obtained form the LEP data.

3.1.1 History of neutrino oscillations

The phenomena of particle oscillation was first introduced by M. Gell-Mann and A. Pais [118] in 1955 by proposing the oscillation process $K^0 \leftrightarrows \bar{K}^0$ to explain the observed CP-violation in neutral K-meson decay. A deficit was observed in Raymond Davis's Cl-experiment (later known as Homestake experiment) in the solar neutrino flux calculated by John Bahcall in his *standard solar model* (SSM). This deficit of neutrino flux was known as *solar neutrino problem* where the observed neutrino flux was about 1/3 of the expected flux. Bruno Pontecorvo, in the late 1950s, proposed the phenomena of neutrino-antineutrino oscillation in analogy with the $K^0 \leftrightarrows \bar{K}^0$ to explain this deficiency. This analogy was in favor of the inaccurate announcement of successful observation of $\bar{\nu}$ + ^{37}Cl $\rightarrow$ ^{37}Ar + e^- in Davis's Cl-experiment. However, it was denied after the experimental verification on the fixed helicity (within the ultrarelativistic limit) of neutrinos (left-handed) and antineutrinos (right-handed). At that instant, since ν_μ had been discovered already, Z. Maki, M. Nakagawa and S. Sakata in 1962 could think of ν_e and ν_μ to be the mixed states of two mass eigenstates [119]. This was led by Pontecorvo's intuitive theory of two-neutrino mixing and oscillations [120] in 1967. According to this oscillation theory, neutrinos are *massive* and the neutrino produced in any flavour is not a mass eigenstate but a superposition of two or more mass eigenstates. During its propagation up to a distant place it can oscillate partly into some other flavour. Since Cl-experiment was incapable to detect ν_μ interaction, it seemed as some of the solar neutrinos ν_e had disappeared. Further, the solar neutrino deficit could be explained by taking into account the matter effect and the so called *MSW-resonance* within the two-flavour oscillation scenario. The matter effect in neutrino oscillation is discussed in detail in section 3.2.2. A similar kind of deficiency was also observed in atmospheric neutrino flux which could be interpreted via ν_μ-ν_τ oscillation theory. This was the point when three flavour neutrino-oscillation scheme was imported.

3.2 Neutrino oscillation phenomena

After the discovery of muon neutrino it became understandable that the quantum mechanical phenomenon of oscillations may occur between different neutrino flavours if neutrinos are massive and mixed. Neutrino oscillations are generated by the interference of different massive neutrinos that are produced and detected coherently because of their small mass differences.

In the quantum mechanical theory, a two level system is characterized by two eigenstates along with the associated eigenvalues of the Hamiltonian. A particle sitting in one of these eigenstates cannot make a transition to the other state in the absence of coupling between these two states. To establish the coupling an external stimulus (usually an electric of magnetic field in the atomic physics) is required. For example an e^-, resting on the ground state of a two level system can approach the higher level in the presence of a coherent beam of photons where the absorption and re-emission of photons will happen in a cyclic pattern by stimulated emission. This phenomena is also known as *Rabi oscillations*. However, the phenomena of neutrino oscillation is different from this usual two-level system in the sense that there is no any such external stimulus to drive the oscillations between neutrino mass eigenstates. Nature itself has established a coupling between these two states. One can notice here that such transitions should be forbidden by the energy conservation law. However, if the energy difference between two states is much shorter than the uncertainties in E_1 and E_2 (ΔE_1, ΔE_2) *i.e.,* if $(E_2 - E_1) \ll \Delta E_1, \Delta E_2$, then the spontaneous transition between such states is allowed, leading to the oscillations between them.

3.2.1 Neutrino oscillations in vacuum

When neutrinos are produced in some interaction, they are in the form of one of the three flavours ν_e, ν_μ, ν_τ. Also they can be detected by observing their interaction with matter which will again cause them to be in certain flavour. Since neutrinos can participate in weak interactions only, hence, these flavour states are also called weak eigenstates or interaction eigenstates. One the other hand, they propagate over a distance in the form of mass eigenstates ν_1, ν_2, ν_3 and hence these are also termed as propagation eigenstates.

To show mixing, it is necessary for neutrinos to have some mass. A massless particle will travel with the speed of light and, in terms of relativity, it will never experience time. Similar is the case with neutrinos. If neutrinos are massless then the flavour states are themselves the mass or energy eigenstates, *i.e.,* the eigenstates of the Hamiltonian. Eigenstates of Hamiltonian are stationary in nature implying no oscillation within each other. Hence, to explain the mixing phenomena it is necessary for neutrino flavour states to be not exactly equal to the propagation states and to be there superposition.

In this section, a brief description of the neutrino oscillation in vacuum is given. To this aim, consider an arbitrary neutrino state $|\Psi(t)\rangle$ at time t, which can be represented either in the flavour basis $\{|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle\}$ or in the mass-basis $\{|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle\}$ as:

$$|\Psi(t)\rangle = \sum_{\alpha=e,\mu,\tau} \nu_\alpha(t) |\nu_\alpha\rangle = \sum_{i=1,2,3} \nu_i(t) |\nu_i\rangle . \tag{3.1}$$

The coefficients in the two representations are connected by a *unitary* matrix

$$\nu_\alpha(t) = \sum_{i=1,2,3} U_{\alpha i} \nu_i(t). \tag{3.2}$$

A convenient parametrization for U in terms of mixing angles θ_{ij} and CP violating phase δ, also known as *PMNS* matrix, is given in Eq. (3.3).

$$U(\theta_{12}, \theta_{13}, \theta_{23}, \delta) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{23}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{13}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.3}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$.

The time evolution of massive states is given by $\nu_i(t) = e^{-iE_it}\ \nu_i(0)$, which, along-with Eq. (3.2), gives

$$\nu_\alpha(t) = U_f\ \nu_\alpha(0). \tag{3.4}$$

Here, U_f is the flavour evolution matrix, taking a flavour state from time $t = 0$ to some later time t. In matrix form

$$\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\\ \nu_\tau(t)\end{pmatrix} = \begin{pmatrix}a(t) & d(t) & g(t)\\ b(t) & e(t) & h(t)\\ c(t) & f(t) & k(t)\end{pmatrix}\begin{pmatrix}\nu_e(0)\\ \nu_\mu(0)\\ \nu_\tau(0)\end{pmatrix}. \tag{3.5}$$

If the state at time $t = 0$ is $|\nu_e\rangle$, then $\nu_\alpha(0) = \delta_{\alpha e}$ ($\alpha = e, \mu, \tau$). Therefore after time t, we have $\nu_e(t) = a(t)$, $\nu_\mu(t) = b(t)$ and $\nu_\tau(t) = c(t)$. Hence, the wave function can be written as

$$|\Psi_e(t)\rangle = a(t)\,|\nu_e\rangle + b(t)\,|\nu_\mu\rangle + c(t)\,|\nu_\tau\rangle\,. \tag{3.6}$$

The survival probability is then given by $|\langle\nu_e|\Psi_e(t)\rangle|^2 = |a(t)|^2$. Similarly, $|b(t)|^2$ and $|c(t)|^2$ are the transition probabilities to μ and τ flavour, respectively. The survival and transition probabilities are functions of energy difference $\Delta E_{ij} = E_i - E_j$ $(j, k = 1, 2, 3)$. Also, in the ultra-relativistic limit, following standard approximations are adopted:

$$\Delta E_{ij} \approxeq \frac{\Delta m^2_{ij}}{2E}; \quad E \equiv |\vec{P}|; \quad t \equiv L. \tag{3.7}$$

These approximations are quite reasonable in the context of the experiments considered here (mainly reactor and accelerator experiments), since the neutrinos are ultra relativistic with neutrino-masses of the order of a few electron-volts (eV) and the energy higher than 10^6 eV. A general form of probability $P_{\alpha\to\beta}$ can be expressed as (in the units $\hbar = 1$, $c = 1$)

$$\begin{aligned} P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) \\ + 2\sum_{i>j}\mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right), \end{aligned} \tag{3.8}$$

3.2.2 Matter effect on neutrino oscillations

In 1978, L. Wolfenstein [121] pointed out that during the neutrino-propagation through a material medium neutrinos have to face a potential induced due to neutrino-matter interaction and the neutrino gets scattered. It was considered that it exhibits coherent forward (elastic) scattering, *i.e.,* the particles participating in the scattering process retain their identities. This can change the pattern of neutrino oscillations. The only interaction that neutrinos can experience is weak interaction mediated by $W^\pm$ (charged current (CC) interaction) and Z boson (neutral current (NC) interaction). We can have a look over the neutrino interaction channels below which show elastic scattering, such as

$$\nu_e + p(n) \to \nu_e + p(n), \tag{3.9}$$

$$\nu_{\mu(\tau)} + p(n) \to \nu_{\mu(\tau)} + p(n), \tag{3.10}$$

$$\nu_e + e^- \to \nu_e + e^-. \tag{3.11}$$

The above Eqs. (3.9) and (3.10) show (NC) interactions while Eq. (3.11) represents both the NC and CC interactions. The CC interaction of ν_μ or ν_τ do not maintain the identity of the participant particles and hence, are not considered here. It can be given as

$$\nu_{\mu(\tau)} + e^- \to \mu(\tau) + \nu_e. \tag{3.12}$$

From Eqs. (3.9), (3.10) and (3.11) it is clear that the amplitudes corresponding to NC interactions of each neutrino-flavour with matter are identical. As a consequence, just a common phase factor is produced due to NC interactions during the calculation of oscillation or survival amplitudes which gets disappeared from the probability expressions. Hence, the amplitude due to charged current interaction of ν_e is the only one which is responsible to change the oscillation patterns. Since, only $\nu_e - \nu_e$ term is affected due to CC interaction, a potential A induced due to interaction of ν_e with matter through the CC channel given in Eq. (3.11) should be added to the first term of the Hamiltonian in flavour basis. This potential A can be calculated using Feynman rules and it turns out be $\sqrt{2}G_F N_e$, with G_F and N_e as Fermi coupling constant and electron number density, respectively. Analysis of matter effect in two-flavour scenario (in which one flavour is surely ν_e) is a bit easier than the case of three-flavour oscillations. In two-flavour mixing, the survival and oscillation probabilities are equivalent to those obtained for vacuum oscillation with vacuum mixing angle θ and mass squared difference Δ replaced by effective mixing angle θ_{eff} and mass squared difference Δ_{eff} in matter, such as

$$P_{ee}(L) = P_{\mu\mu}(L) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta_{\text{eff}} L}{4E} \tag{3.13}$$

and

$$P_{e\mu}(L) = P_{\mu e}(L) = \sin^2 2\theta \sin^2 \frac{\Delta_{\text{eff}} L}{4E} \tag{3.14}$$

with

$$\Delta_{\text{eff}} = \sqrt{(\Delta \cos 2\theta - 2EA)^2 + \Delta^2 \sin^2 2\theta}$$

and

$$\tan 2\theta_{\text{eff}} = \frac{\tan 2\theta}{1 - \frac{2EA}{\Delta \cos 2\theta}}.$$

The effects of earth's matter density on neutrino oscillations has been studied using various models for matter densities [122–126]. To incorporate the matter effect in the case of three-flavour oscillation, a convenient formalism developed by [127, 128] has been used. In vacuum, the Hamiltonian H_m is given by $H_m = diag(E_1, E_2, E_3)$, where $E_a = \sqrt{m_a^2 + p^2}$, $a = 1, 2, 3$ are the energies of the neutrino mass eigenstates $|\nu_a\rangle$, with masses m_a and momentum p. As discussed earlier, when neutrinos propagate through ordinary matter, the Hamiltonian picks up an additional term as a consequence of the CC weak interaction with the electrons in the matter. This additional potential term is diagonal in the flavour basis and is given by

$$V_f = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.15}$$

The sign of the matter density potential (A) is positive for neutrinos and negative for antineutrinos. It is assumed that the electron density N_e is constant throughout the matter in which the neutrinos are propagating. In the mass basis, the additional potential term becomes $V_m = U^{-1} V_f U$, where U is given in Eq. (3.3). Thus the Hamiltonian in mass basis is given by $\mathcal{H}_m = H_m + U^{-1} V_f U$. After some algebra, one finally obtains the matter counterpart of the flavour evolution matrix defined in Eq. (3.4):

$$U_f(L) = e^{-i\mathcal{H}_f L} = \phi \sum_{n=1}^{3} e^{-i\lambda_n L} \frac{1}{3\lambda_n^2 + c_1} \left[(\lambda_n^2 + c_1)\mathbf{I} + \lambda_n \tilde{\mathbf{T}} + \tilde{\mathbf{T}}^2 \right]. \tag{3.16}$$

Here $\phi \equiv e^{iLtrH_m/3}$, λ_n $(n = 1, 2, 3)$ are the eigenvalues of $\mathbf{T}$ matrix defined further in Eq. (3.17), $\tilde{\mathbf{T}} = U\mathbf{T}U^{-1}$ and $c_1 = det\mathbf{T} \times Tr\mathbf{T}^{-1}$.

$$\mathbf{T} = \begin{pmatrix} AU_{e1}^2 - \frac{1}{3}A + \frac{1}{3}(E_{12} + E_{13}) & AU_{e1}U_{e2} & AU_{e1}U_{e3} \\ AU_{e1}U_{e2} & AU_{e2}^2 - \frac{1}{3}A + \frac{1}{3}(E_{21} + E_{23}) & AU_{e2}U_{e3} \\ AU_{e1}U_{e3} & AU_{e2}U_{e3} & AU_{e3}^2 - \frac{1}{3}A + \frac{1}{3}(E_{31} + E_{32}) \end{pmatrix}. \tag{3.17}$$

For a multilayer model potential with density parameters $A_1, A_2, A_3 \ldots A_m$, and lengths $L_1, L_2, L_3 \ldots L_m$, the net flavour evolution operator will be the product of the operators corresponding to each density, that is, $U_f|_{Net} = U_f(L_1).U_f(L_2).U_f(L_3) \ldots U_f(L_m)$.

3.3 Effects of quantum decoherence on neutrino-oscillations

The neutrino oscillation phenomenon is a conventional method to explain the deficit of neutrino flux coming from several natural and artificial sources. However, there also exit some non-oscillation phenomena (such as quantum decoherence, neutrino decay, flavor changing neutral current interactions with matter and so on) which could be considered as possible solutions of this issue. After several experimental verifications it turned out that the mass-induced neutrino oscillations have a dominating contribution in explaining the neutrino flux deficiency when it is traveled to a distant place. It can be called the conventional method while the other non-oscillatory solutions, which were excluded by $\approx 4\sigma$ significance level turned out to be non conventional solutions and are expected to add sub-leading effects on the standard neutrino oscillation patterns. As we are in precision era of measurements, currently running as well as planned experiments have potential to measure such effects in the neutrino oscillations. In this thesis, we include the effects of one such non oscillation method, quantum decoherence on various measures of quantum correlations in chapter 5.

In this section we discuss in detail the effects of quantum decoherence on neutrino oscillations. In the standard quantum mechanical framework, a system is considered fully isolated which represents an ideal case. Indeed these systems should be treated more likely as open systems, *i.e.,* the system should be considered to have interactions with its surroundings. We have a large variety of open systems which can be modeled as being subsystems in interaction with large environment. In this case, the total system (the subsystem plus environment) follows the unitary time evolution. However, the evolution of the system alone (obtained by eliminating the degrees of freedom introduced by the environment) will no longer remain unitary due to the development of dissipation and irreversibility [129, 130].

When there are no initial correlations between subsystem and environment and the mutual interaction shared can be considered weak, then the dynamics of subsystem can be described by *quantum dynamical semigroups*. These semigroups are linear time evolution maps $f_t : \rho(0) \to \rho(t)$, ρ represents the state of the system. To be physically acceptable, these maps should follow some general physical requirements which are essential for the correct physical interpretation of the subdynamics. First of all, the system-state, say neutrino states, should transform into neutrino states under the action of these maps, *i.e.,* $f_t[\rho(0)] = \rho(t)$. Further, physical requirements of (i) increasing entropy $S = -Tr[\rho \ln \rho]$ (*i.e.,* irreversibility), (ii) semigroup property of forward in-time composition law *i.e.,* $f_t[\rho(t')] = \rho(t + t')$, for $t, t' \geq 0$ and (iii) *complete positivity* should be satisfied. The density matrix defining the state of the system should be a positive operator, *i.e.,* its eigenvalues, representing probabilities, should be positive to make the formalism consistent. The condition of complete positivity assures that this positivity condition is maintained even for the density matrix describing a larger system in which the system is trivially coupled with another arbitrary finite-dimensional system. Hence, complete positivity sets down a stronger condition to develop a compatible formalism.

Such description of open quantum systems has been originally developed for quantum optical systems . Nevertheless, this description is quite general and can be considered to model several kinds of phenomena. This phenomenological treatment of open quantum systems can be supported by physical considerations such as it has been implemented to analyze the effects of dissipation and irreversibility in a number of particle physics phenomena [131–137]. The original motivation for the investigation of such dissipation effects was based on quantum gravity effects which are caused due to foamlike spacetime originated by quantum fluctuations of gravitational field and appearance of virtual black holes [138, 139]. In such situations, the spaetime looses its continuum aspect at distances of the order of Planck's scale (10^{19} GeV or 10^{-35} m) leading to a loss of quantum coherence [140]. Therefore, in general, these effects are seen to emerge naturally for the systems in weak interactions with suitable environments. In case of particle physics systems, one of the adequate examples of these so called environments could be the dynamics of fundamental extended objects (stings and branes) generating a weakly coupled environment at low energies [141]. In fact, the time evolution expressed in terms of quantum dynamical semi-groups can be the result of interaction with the gas like environment of quanta obeying infinite statistics (e.g., gas of D0 branes) [142]. These effects are expected to be suppressed by at least one inverse power of Planck mass according to a rough dimensional estimate and hence are very small. However, in some particular situations based on the interference phenomena, these effects can be possibly experienced in some present and future experiments. Indeed, there are a number of experiments looking for the interference effects exhibited by neutral K [131, 132] and B-meson systems [135, 137] as well as neutron interferometry [136] for which the analysis have been performed using quantum dynamical semigroups. In these studies, order of magnitude limits on some dissipation parameters have been obtained using experimental data [133, 136, 143]. Moreover, neutrino oscillation, being a pure quantum interference phenomenon over macroscopic distances also provides a good stage to probe quantum decoherence effects.

In case of neutrino system, more specifically, the total Hamiltonian can be decomposed as:

$$H_{tot} = H_S \otimes \mathbf{1} + \mathbf{1} \otimes H_E + gH', \tag{3.18}$$

where H_S represents the standard Hamiltonian exhibiting the unitary evolution of neutrino state, H_E describes the internal dynamics of the environment and H' characterizes the interaction of the system with environment with small coupling constant g. Since, the mechanism of neutrino production has no resemblance with that causing dissipation effects, the condition of the absence of any initial correlation between the system and environment can be safely assumed to be true in case of neutrinos. Hence the initial state of the total system can be articulated as $\rho_{tot} = \rho \otimes \rho_E$. The evolution of the neutrino state obtained by eliminating (or tracing over) the environmental degrees of freedom, is given by

$$\rho \rightarrow \rho(t) = Tr_E \Big[e^{-iH_{tot}t} (\rho \otimes \rho_E) e^{iH_{tot}t} \Big], \tag{3.19}$$

which is generally very complicated with lack of explicit description. However, it is possible to describe such evolution in terms of the Linblad-Kossakowski master equation when the system has weak interaction with the environment. The form of this master equation is given in Eq. (3.20). Furthermore, there are two essential ways of implementing the condition of weak interaction in terms of making the ratio τ/τ_E large [130, 144, 145], where τ is the evolution-time taken by $\rho(t)$ and τ_E stands for the typical decay time of correlations in the environment. The large τ/τ_E ratio implies that the memory effects encoded in Eq. (3.19) can be neglected and it validates the local in time evolution of the state. There are two ways of achieving large value of this ratio in terms of applying certain limiting conditions: (i) "singular coupling limit", when τ_E becomes small, while τ being finite (i.e., the time-correlations of the environment approach a δ-function.), and (ii) "weak coupling limit", when τ_E remains finite and τ becomes large. The later condition is realized by rescaling the time parameter as $t \to t/g^2$ with the coupling constant $g \to 0$. This is called van Hove limit.

Most of the neutrino experimental data could be described very nicely with the phenomena of neutrino oscillations. However, the observations in some short baseline experiments such as LSND (Liquid Scintillator Neutrino Detector) and MiniBooNE (Mini Booster Neutrino Experiment) require some other explanation along with the neutrino-oscillation solution. The LSND result gives an indication of the sterile neutrino oscillations which however, has difficulties in explaning the global data [146] along with the cosmological observations [147]. To explain the experimental data coming from these two experiments along with the global data analysis many ideas and scenarios have been proposed, some of them involving very exotic physics such as sterile neutrino decay [148–150], violation of the CPT [151–154] and/or Lorentz [155, 156] symmetries, mass-varying neutrinos [157, 158], short-cuts of sterile neutrinos in extra dimensions [159], a non-standard energy dependence of the sterile neutrino parameters [160], or sterile neutrinos interacting with a new gauge boson [161]. A possible source of the origin of the LSND signal might be quantum decoherence in neutrino oscillations [162, 163]. Furthermore, the possibility of neutrinos to probe decoherence effects has been investigated for atmospheric neutrinos [164], solar neutrinos [165], KamLAND [166] and long-baseline experiments [167, 168].

Possible sources of decoherence effect have been discussed before in previous paragraphs. A phenomenological approach in terms of quantum dynamical semigroups, without taking care of the possible source, can also be used to analyze the neutrino oscillation scenario under the effect of quantum decoherence [169]. A sophisticated model was proposed in [73, 74] whose aim was to explain the LSND signal via quantum-decoherence of the mass states leading to the damping of the interference terms in the oscillation probabilities. In case of neutrino-system, the neutrino state can be described by the density matrix ρ in the mass state basis, which is a Hermitian and positive operator, *i.e.,* with constant trace (Tr(ρ) = 1) and positive eigenvalues. The evolution of the density matrix under the effect of quantum decoherence is given as

$$\frac{d\rho}{dt} = -i[H, \rho] - \mathcal{D}[\rho]. \tag{3.20}$$

Here H is the Hamiltonian (driving the unitary evolution of the state) while $\mathcal{D}[\rho]$ is the dissipator parameterizing the decoherence effect. This term violates the conservation of Tr(ρ^2), consequently the pure state evolves into a mixed state. Physical requirements carried by the quantum dynamical semigroup formalism (discussed before) construct the form of $\mathcal{D}[\rho]$. The requirement of *complete positivity* leads to the Lindblad form for $\mathcal{D}[\rho]$ such as,

$$\mathcal{D}[\rho] = \sum_m [\{\rho, D_m D_m^\dagger\} - 2 D_m \rho D_m^\dagger], \tag{3.21}$$

D_m being general complex matrices. These general complex matrices can allow the violation of unitarity, *i.e.,* $dTr(\rho)/dt$ can be nonzero. Hence, to take care of the unitarity condition D_m are required to be Hermitian. This Hermiticity, along with the unitarity condition, also guarantees that the entropy $S(\rho) = -Tr(\rho \ln \rho)$ cannot decrease. The conservation of average energy leads to $[H, D_m] = 0$. This allows us to diagonalize these two matrices H and D_m simultaneously, *i.e.,*

$$H = \mathrm{diag}(E_1, E_2, E_3), \qquad D_m = \mathrm{diag}(d_{m,1}, d_{m,2}, d_{m,3})$$

with $E_i = (p^2 + m_i^2)^{1/2}$ and $d_{m,i}$ are unknown energy dependent real parameters having the dimensions of $(mass)^{1/2}$. Under these conditions Eq. (3.20) obtains the following form

$$\dot{\rho}(t) = -i(H\rho(t) - \rho(t)H) - \sum_m [(\rho(t)D_m^2 + D_m^2\rho(t)) - 2D_m\rho(t)D_m]$$

Decoherence effects when taken into account lead to the following form of the density matrix describing a neutrino state in mass basis [73]

$$\rho(t) = \begin{pmatrix} \rho_{11}(0) & \rho_{12}(0)e^{-(\gamma_{12}-i\Delta_{12})t} & \rho_{13}(0)e^{-(\gamma_{13}-i\Delta_{13})t} \\ \rho_{21}(0)e^{-(\gamma_{21}-i\Delta_{21})t} & \rho_{22}(0) & \rho_{23}(0)e^{-(\gamma_{23}-i\Delta_{23})t} \\ \rho_{31}(0)e^{-(\gamma_{31}-i\Delta_{31})t} & \rho_{32}(0)e^{-(\gamma_{32}-i\Delta_{32})t} & \rho_{33}(0) \end{pmatrix}. \quad (3.22)$$

Here $\gamma_{ij} = \sum_m (d_{m,i} - d_{m,j})^2$ are the decoherence parameters and $\Delta_{ij} = E_i - E_j \approx \frac{\Delta m_{ij}^2}{2E_\nu}$, with Δm_{ij}^2 and E_ν being the mass square difference and the energy of the neutrino, respectively. Elements of the density matrix at time $t = 0$ are given by $\rho_{ij}(0) = U_{\alpha i}^* U_{\alpha j}$, such that $U_{\alpha i}$ are the elements of the PMNS matrix. One can write the flavour transition probability given by

$$P_{\alpha\beta} = \langle \nu_\beta | \rho^\alpha(t) | \nu_\beta \rangle = \sum_{ij} U_{\beta i}^* U_{\beta j} \rho_{ij}^\alpha(t).$$

The most economic scenario that describes the LSND data and also consistent with the results of other experiments, leads to the consideration of $d_1 = d_2 \neq d_3$ which is also inspired by the pattern ($m_1 \approxeq m_2 \neq m_3$) of masses associated to the neutrino mass eigenstates. This leads to

$$\gamma_{12} = 0 \quad \text{and } \gamma \equiv \gamma_{13} = \gamma_{32}. \quad (3.23)$$

In [74] an exponential dependence of the decoherence parameters on neutrino energy was conjectured to be

$$\gamma = \gamma_0 \left(\exp\left[-\left(\frac{E}{E_3}\right)^n \right] - \exp\left[-\left(\frac{E}{E_1}\right)^n \right] \right)^2. \quad (3.24)$$

Best fit values of E_3 and E_1 are 55 MeV and 20 MeV, respectively with $n = 2$ and $\gamma = 0.01\ \mathrm{m}^{-1}$. It turns out that γ attains its maximum value at around 30 MeV; consequently, one expects maximum decoherence at this energy. The exponential energy dependence of the γ parameter given in Eq. (3.24) along with the best fit values of corresponding parameters restricts deviations from the standard three flavour oscillations only in the range of 20-50 MeV (the energy range corresponding to the LSND experiment) leaving the other experimental observations unchanged with energy-range higher then 200 MeV or lower than few MeV. It makes this model consistent with the global neutrino oscillation data.

3.4 Effects of nonstandard neutrino-matter interactions (NSI) on neutrino oscillations

The present understanding of fundamental interactions of nature is encapsulated in a theory known as the Standard Model (SM) of the strong and electroweak interactions. The SM successfully survived stringent tests of their efficacy in several high precision experiments. Although SM successfully accounts for the phenomena within its domain, still it cannot be considered as the quintessential theory of fundamental interactions. This is because there are several phenomenon which SM simply cannot explain. These include the observed baryon asymmetry, gravitational interactions and the origin and nature of dark matter and dark energy. Therefore one needs to explore physics beyond SM.

The phenomena of neutrino oscillation implies physics beyond the SM as the neutrinos are assumed to be massless within the SM whereas the observation of neutrino oscillations implies that neutrinos have a non-zero mass. Few measurements in the muon sector, for instance, the anomalous magnetic moment [170, 171] and the charge radius of the proton extracted from muonic hydrogen [172] are indications of beyond SM physics. Besides this, in contrast with the SM, we have hints of lepton flavor universality violation in the decays induced by the quark level transitions $b \to cl\nu$ ($l = e, \mu, \tau$) [46–48] and $b \to s\, l^+ l^-$ ($l = e, \mu$) [49, 50]. There have been several model-independent investigations in search for identifying the Lorentz structure for the possible new physics involved in these decay modes [173–186]. These Lorentz structures can be generated in new physics models, such as Z' and leptoquark models that can account for the observed anomalies in semi-leptonic B decays. These new physics models can also affect the dynamics of neutrino oscillations significantly.

New physics effects in the neutrino sector can be realized through nonstandard neutrino-matter interaction (NSI). One can think of the existence of new physics at some higher energy level Λ the effects of what can be observed at lower energy scales in terms of sub-leading effects encountered by present experimental results. Such effects can be described by adding higher dimensional operators constructed out of SM fields to the SM Lagrangian (which has renormalizable interactions with canonical dimension ≤ 4). These NSIs can be classified as charged current (CC) as well as neutral current (NC) interactions. The charged-current NSI of neutrinos with matter (*i.e.,* e,u,d) can affect the production and detection of neutrinos in general, called zero distance effect and can become discernible in near detectors. On the other hand, the NSI-NC with two neutrinos can also affect the forward coherent scattering as the neutrino propagate through matter via so called Mikheev-Smirnov-Wolfenstein (MSW) mechanism [121, 187]. The effect of incoherent scattering is neglected in case of Earth matter density $\rho \sim 2.8 gm/cc$, as the mean free path for the process is much larger than Earth's diameter when the neutrino energy is lower than $\sim 10^5$ GeV [188]. Consequently, a significantly enhanced effect of NSI-NC can be seen in large baseline oscillation experiments where neutrinos have to travel through a large region of matter. While the scattering bounds on NSI-CC are rather stringent, these bounds are quite weaker for NSI-NC [189, 190]. Therefore, we consider the neutral-current interactions driven by NSI relevant to neutrino propagation in matter. The Lagrangian for neutral current NSI neutrino interactions can be written as

$$\mathcal{L}_{NSI} = -2\sqrt{2} G_F \sum_{f,P,\alpha,\beta} \epsilon^{f,P}_{\alpha,\beta} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta)(\bar{f} \gamma_\mu P f), \tag{3.25}$$

where α and β are flavour indices, P_L & P_R are the projection operators and f is the charged fermion. Here, $\epsilon^{f,P}_{\alpha,\beta} \sim \mathcal{O}(G_x/G_F)$ represents the strength of the new interaction with respect to the SM interaction quantified by G_F. If the flavour of neutrinos participating in the interaction is considered to be independent of the charged fermion type, one can write

$$\epsilon^{f,P}_{\alpha\beta} \equiv \epsilon^{\eta}_{\alpha\beta}\, \xi^{f,P}, \tag{3.26}$$

where matrix elements $\epsilon^{\eta}_{\alpha\beta}$ correspond to the coupling between neutrinos and the coefficients $\xi^{f,P}$ represent the coupling to the charged fermions. Hence the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{NSI} = &- 2\sqrt{2} G_F \sum_{f,P,\alpha,\beta} \epsilon^{\eta}_{\alpha,\beta} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta) \\ &\times \sum_{f,P} \xi^{f,P} (\bar{f} \gamma_\mu P f). \end{aligned} \tag{3.27}$$

The Hamiltonian for the evolution of neutrino-state, in mass eigenstate basis, including NSI effect can be written as $\mathcal{H}_m = H_m + U^{-1}V_f U$. The matter part V_f of the Hamiltonian including the operators corresponding to the NSI effect becomes now

$$V_f = A \begin{pmatrix} 1+\epsilon_{ee}(x) & \epsilon_{e\mu}(x) & \epsilon_{e\tau}(x) \\ \epsilon^*_{e\mu}(x) & \epsilon_{\mu\mu}(x) & \epsilon_{\mu\tau}(x) \\ \epsilon^*_{e\tau}(x) & \epsilon^*_{\mu\tau}(x) & \epsilon_{\tau\tau}(x) \end{pmatrix}, \tag{3.28}$$

Here, $+1$ in the 1×1 element of V_f corresponds to the standard matter interaction of neutrinos and

$$\epsilon_{\alpha\beta} = \sum_{f=e,u,d} \frac{N_f(x)}{N_e(x)} \epsilon^f_{\alpha\beta}, \tag{3.29}$$

represents the non-standard part. Here, $N_f(x)$ is the number density of fermion f as a function of the distance x traveled by neutrino. According to the quark-structure of protons (p) and neutrons (n), we can write

$$N_u(x) = 2N_p(x) + N_n(x), \qquad N_d(x) = N_p(x) + 2N_n(x). \tag{3.30}$$

Therefore, from Eq. (3.29) and (3.30) we can write

$$\epsilon_{\alpha\beta} = (2+Y_n)\epsilon^u_{\alpha\beta} + (1+2Y_n)\epsilon^d_{\alpha\beta}, \tag{3.31}$$

with $Y_n = N_n/N_e$, N_e is the number density of electrons and $N_p = N_e$.

U is again the PMNS matrix but it differs from the usual one (Eq. (3.3)) by an overall phase matrix $P = \mathrm{diag}(\mathrm{e}^{\mathrm{i}\delta}, 1, 1)$ and represented as $U_v = PUP^*$, and can also be given by

$$U_v = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\delta} & s_{13} \\ -s_{12}c_{23}e^{-i\delta} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23}e^{-i\delta} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}. \tag{3.32}$$

This rephasing does not affect the probability expressions in the absence of NSI. The advantage of this convention of U-matrix is that one can easily perform the CPT-transformation, $H_{vac} \to -H^*_{vac}$, as just by doing simple replacements, such as

$$\begin{aligned} \Delta m^2_{31} &\to -\Delta m^2_{31} + \Delta m^2_{21} \to -\Delta m^2_{32}, \\ \theta_{12} &\to \pi/2 - \theta_{12}, \\ \delta &\to \pi - \delta. \end{aligned} \tag{3.33}$$

Moreover, we are considering here the complex NSI parameters. This means that one has to deal with extra phase factors $\phi_{\alpha\beta}$ which can affect the correct estimation of δ parameter. The form of mixing matrix given in Eq. (3.32) helps to overcome this difficulty [69]. The CPT-transformation of H_{vac}, in which neutrino evolution remains invariant, involves the change of the octant of θ_{12} (Dark octant with $\theta_{12} > 45^o$) and also the change in the sign of Δm^2_{31}. The octant selection of mixing-angle θ_{12} becomes important when neutrino is traveling through a dense material medium as the possibility of NSI increases. For example, the deficit of solar neutrinos at the detectors can be resolved by considering the vacuum mixing angle in the light-side ($0 \leq \theta_{12} \leq \frac{\pi}{4}$) with standard neutrino-matter interactions as well as the dark-side solution ($\frac{\pi}{4} \leq \theta_{12} \leq \frac{\pi}{2}$) with large enough values of NSI parameters [191]. This specific feature is called the *generalized mass ordering degeneracy* that was first noticed in [192, 193].

To include CPT-transformation in matter-part of Hamiltonian, the replacements, along with Eq. (3.33), are

$$[\epsilon_{ee} - \epsilon_{\mu\mu}] \to -[\epsilon_{ee} - \epsilon_{\mu\mu}] - 2,$$
$$[\epsilon_{\tau\tau} - \epsilon_{\mu\mu}] \to -[\epsilon_{\tau\tau} - \epsilon_{\mu\mu}],$$
$$\epsilon_{\alpha\beta} \to -\epsilon^*_{\alpha\beta}\ (\alpha \neq \beta).$$

The parameters $\epsilon_{\alpha\beta}$ can have complex values, however, here these are considered to be real. The evolution operator can again be obtained using the formalism given by [127, 128] whose functional form remains same as that given in Eq. (3.16) with $\mathrm{Tr}(\mathcal{H}_m) = E_\nu + A(1 + \epsilon_{ee} + \epsilon_{\mu\mu} + \epsilon_{\tau\tau})$ and $T = \mathcal{H}_m - \mathrm{Tr}(\mathcal{H}_m)\mathbf{I}/3$.

3.5 Experimental facilities

From the verification of non zero neutrino mass to the extraction of the oscillation parameter-values, the neutrino oscillation experimental facilities have been developed more significantly tending to achieve higher precisions. Various experiments have tested the oscillation properties using MeV energy solar and reactor neutrinos and the highly energetic atmospheric and accelerator neutrinos of GeV order as well in this direction. Solar neutrino experiments such as *Homestake* (1970), *Kamiokande* (late 1980s), *GALLEX/GNO* and *SAGE* (1990), the *Super-Kamiokande* and SNO (late 1990s), atmospheric neutrino experiments viz. Kolar Gold Field Mine in South India and East Rand Property Gold Mine in South Africa (1965) and Super-Kamiokande (1996) are some of the well known oscillation experimental setups using natural neutrino sources. However, in this thesis, we have included various experimental conditions (setups) corresponding to several man-made neutrino sources *i.e.,* accelerator (also known as LBL - long baseline experiments) and reactor facilities and have been discussed below:

–Accelerator experiments:

- *T2K (Tokai-to-Kamioka)* is an off-axis experiment [194] using a ν_μ– neutrino beam originating at J-PARC (Japan Proton Accelerator Complex) in $Tokai$ with neutrino energy-range of approximately 100 MeV to 1 GeV (maximum flux is observed at around 0.6 GeV) and the baseline of 295 km with the *Super-Kamiokande* (SK) far detector situated in $Gifu, Japan$ (located 1 km underground in the Mozumi Mine) and started taking data in 2010. SK was able to measure both the disappearance of muon neutrino flux as well as the electron neutrino appearance in the beam [195]. Properties and composition of the initial accelerator neutrino flux is observed using the near detector located 280 m from the beam production place. It was the first experiment which observed the appearance of electron neutrinos in muon neutrino beam [196] and it also provided the world's best measurement of oscillation parameter θ_{23} [197]. Recently, the first significant constraint on the δ_{CP} parameter, responsible for the matter-antimatter asymmetry in the neutrino sector has also been obtained by the T2K collaboration [198].

- *NOνA (NuMI Off-Axis ν_e Appearance)*, the long baseline experiment, uses neutrinos from Fermilab's NuMI (Neutrinos at the Main Injector) beamline optimized to observe $\nu_\mu \to \nu_e$ oscillations. The near and far detectors, both located at 14 mrad off the axis of the NuMI beamline (*i.e.,* 12 km west from the beam's central axis) are positioned at 1 km (at Fermilab) and 810 km (in northern Minnesota) from the source, respectively. The flavour composition of the beam is 92.9% of ν_μ and 5.8% of $\bar{\nu}_\mu$ and 1.3% of ν_e and $\bar{\nu}_e$; the energy of the neutrino beam varies from 1 GeV to 4 GeV (maximum flux at $\approx$2 GeV). The spectra for NuMI beamline for various off-axis locations is given in [199–201]. Due to relatively higher energy of neutrinos, NOνA has the potential to resolve the issue of mass ordering. Besides this, it also aims to measure the precise value of θ_{23} mixing angle along with its octant information, the precise measurement of the large mass-squared difference Δm^2_{32} and the δ_{CP} phase. NOνA started taking data in 2010 while it was fully operational in the year 2014.

- *DUNE (Deep Underground Neutrino Experiment)* is an experimental facility planned for the future which is supposed to use NuMI neutrino beam with energy range of 1 - 10 GeV from Fermilab with a near detector placed in Fermilab itself and a far detector at the Sanford Underground Research Facility having a long baseline of 1300 km. It enables the L/E-value of about 10^3 km/GeV, to reach good sensitivity for CP measurement and determination of mass ordering [72]. DUNE is expected to be fully operational in the year 2027.

The matter density in all these experiments is approximately $2.8 gm/cc$, which corresponds to the density parameter $A \approx 1.01 \times 10^{-13} eV$.

–Reactor experiments:

- *Reactor Experiment for Neutrino Oscillation (RENO)* is a short baseline reactor neutrino oscillation experiment in South Korea. RENO has two identical detectors, placed at distances of 294 m and 1383 m (∼1.4 km), that observe electron antineutrinos produced by six reactors at the Hanbit Nuclear Power Plant in Korea with energy range of few MeV. In the year 2012, the RENO collaboration announced a 4.9σ observation of nonzero value of θ_{13} [202] which was updated in 2013 [203] on the 6.3σ level to be

$$\sin^2(2\theta_{13}) = 0.100 \pm 0.010(\text{stat.}) \pm 0.015(syst.)$$

- *Daya-Bay Reactor Neutrino Experiment* is situated at Daya Bay, approximately 52 kilometers northeast of Hong Kong, China designed to measure the mixing angle θ_{13} using antineutrinos of around 1 - 10 MeV energy produced by the reactors of the Daya Bay Nuclear Power Plant and the Ling Ao Nuclear Power Plant. Two antineutrino detectors (AD) are located in underground experimental halls EH1 and one in EH2 (also known as near halls). Three ADs are positioned near the oscillation maximum in the far hall (∼2 km from reactor pairs), EH3. The collaboration presented an analysis of their results in 2014 [204] providing the nonzero value of θ_{13} parameter and later on, at Moriond 2015, presented updated values of θ_{13} and effective mass squared difference Δm^2_{ee} (which is defined such that $\sin^2\Delta_{ee} = \cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}$, with $\Delta_{ee} = \frac{\Delta m^2_{ee}}{2E}$) [205] as

$$\sin^2(2\theta_{13}) = 0.084 \pm 0.005, \qquad |\Delta m^2_{ee}| = 2.44^{+0.10}_{-0.11} \times 10^{-3}\text{eV}^2$$

Reactor experiments can not provide significant information about the mass ordering due to very low Earth-matter effect owing to the relatively low energy (of the order of MeV) of antineutrinos produced in detectors.

3.6 Open questions

The phenomena of neutrino oscillations has been well established in various experimental setups, which are discussed in the previous section. However, many unanswered questions are still there associated to the neutrino oscillation parameters, such as

- *Mass hierarchy problem:* Owing to the existence of two different mass squared differences (of the order 10^{-5} and $10^{-3} eV^2$), the complete picture of three-flavour neutrino oscillations is described through the mixing between three flavour states (ν_e, ν_μ, ν_τ) using three mass eigenstates (ν_1, ν_2,ν_3). Though, the sign of the small mass squared difference Δm^2_{21} is well known from the MSW effect driven explanation of solar neutrino problem which is a consequence of neutrino-propagation through dense material medium inside the Sun's core. It was explained that in case of neutrino (antineutrino) the MSW effect appears for positive (negative) sign of Δm^2_{21}. The position of the third mass eigenstate is still not known. One can notice that the sign of Δm^2_{21} could be uncovered when neutrino oscillations took place under a significant matter effect due to Sun's interior. Same fact can be used to find out the sign of Δm^2_{31}. Experiments sensitive to Δm^2_{31}, such as atmospheric and LBL experiments, can experience the matter effect only due the earth's material. Hence, several currently running and planned experiments in neutrino sector also aim to resolve the mass hierarchy problem, *i.e.,* whether the mass appearing in ν_3-eigenstate of neutrino is the lightest (*normal ordering*) or the heaviest (*inverted ordering*) of the three masses.

- *Leptonic charge-parity (CP) violation:* One of the Sakharov conditions to explain the matter-antimatter asymmetry in the universe is to have CP-violation in the nature. The evidences of CP-violation have been found in the quark sector or more generalized meson-sector. However, the mesonic CP-violaton is quite small and is not sufficient to describe the amount of matter-antimatter asymmetry. The CP-violation in leptonic sector, appearing in the PMNS mixing matrix as nonzero complex phase δ_{CP}, leaves us some hope whose exact value is yet to be determined. The T2K collaboration has reported a measurement of δ_{CP} that favors large enhancement of the neutrino oscillation probability, excluding values of δ_{CP} which result in a large enhancement of the observed anti-neutrino oscillation probability. However, one should still consider the complete range of $\delta_{CP} \in [0, 2\pi]$ until the measurement of the exact value of this parameter.

- Due to very small mass of mass eigenstates and constraints on the experimental precisions, the *absolute mass* of these eigenstates could not be found yet.

- The *existence of sterile neutrinos* is also an interesting and open question in the neutrino sector since it can be considered as a possible dark matter candidate.

Works included in this thesis focus mainly on the problem of mass ordering and, up to certain extent, on the leptonic CP-violation.