4
Quantum Correlations in Neutrino Oscillations

This chapter is based on the analysis made in Ref. [94]. Quantum correlations have been studied previously in the neutrino-system in case of both two-flavour and three-flavour oscillation scenarios [14–16, 18, 19, 21]. However, these analyses have taken into account the lack of knowledge of the CP violating phase and the ambiguity in the sign of the third mass squared difference Δm^2_{31}. Further, the new physics effects beyond the standard model remain unprobed. Therefore, in this work, the sensitivity of various spatial nonclassicality measures to the neutrino mass ordering and CP-violation has been investigated. In order to attempt such investigation, neutrinos are treated as qubits in a way that we assume the three flavor neutrino oscillation picture equivalent to a three-qubit system, and correlations such as entanglement have been studied between three flavour-modes of single neutrino particle. For this study, the standard neutrino-matter interaction has been incorporated. In section 4.1, we start with expressing the mode entanglement hidden in neutrino dynamics by assigning occupation numbers to the neutrino states. Then, in section 4.2, variety of correlation measures have been analyzed for the system of three flavour neutrino oscillations.

This analysis is carried out for parameters relevant to two ongoing experiments NOνA and T2K, and also for the upcoming experiment DUNE. Various quantum correlations turn out to be sensitive to the mass-hierarchy problem in neutrinos, elaborated in 4.2. This sensitivity is found to be more prominent in DUNE experiment as compared to NOνA and T2K experiments. This can be attributed to the large baseline and high energy of the DUNE experiment. Further, we find that to probe these correlations, the neutrino (antineutrino) beam should be preferred if the sign of mass square difference Δ_{31} turns out to be positive (negative).

4.1 Neutrino dynamics with mode entanglement

–Occupation number representation: In continuation with the dynamics of neutrino system discussed in chapter 3, one can introduce the occupation number associated with a given flavour or mass mode and a correspondence with three-qubit states can be established, such as [15, 19, 206]

$$\left.\begin{aligned} |\nu_e\rangle &\equiv |1\rangle_e |0\rangle_\mu |0\rangle_\tau \\ |\nu_\mu\rangle &\equiv |0\rangle_e |1\rangle_\mu |0\rangle_\tau \\ |\nu_\tau\rangle &\equiv |0\rangle_e |0\rangle_\mu |1\rangle_\tau \end{aligned}\right\} \text{flavour modes} \tag{4.1}$$

$$\left.\begin{aligned}|\nu_1\rangle &\equiv |1\rangle_1\,|0\rangle_2\,|0\rangle_3\\|\nu_2\rangle &\equiv |0\rangle_1\,|1\rangle_2\,|0\rangle_3\\|\nu_3\rangle &\equiv |0\rangle_1\,|0\rangle_2\,|1\rangle_3\end{aligned}\right\}\text{massive modes} \tag{4.2}$$

In Eq. (4.1), $|n\rangle_\alpha$ represents the n-th occupation number state of a neutrino in mode α *i.e.,* states $|1\rangle_\alpha$ and $|0\rangle_\alpha$ represent the presence and the absence, respectively, of neutrino in flavor mode α. Hence, one can write the time evolved electron-neutrino state

$$|\Psi_e(t)\rangle = a(t)\,|\nu_e\rangle + b(t)\,|\nu_\mu\rangle + c(t)\,|\nu_\tau\rangle \tag{4.3}$$

in the occupation number representation as

$$|\Psi_e(t)\rangle = a(t)\,|100\rangle + b(t)\,|010\rangle + c(t)\,|001\rangle\,. \tag{4.4}$$

It is clear that entanglement will be established among flavor modes in the single-particle state. The state of three-flavour neutrino can be considered equivalent to a three-qubit state where three qubits represent three different flavours of neutrinos. This prescription is in fact very general and can be applied to any n level system. The textbook example of a hydrogen atom in superposition of $|n_k, l_k, m_k\rangle$, $k = 1, 2, 3$, could be written as Eq. (4.4) with the logical states $|1, 0, 0\rangle$, $|0, 1, 0\rangle$ and $|0, 0, 1\rangle$ identified with $|n_k, l_k, m_k\rangle$ for $k = 1, 2$, and 3, respectively. It is important to mention here that such states are difficult to sustain in atomic systems due to deteriorating effects from system environment interactions. However, neutrino provides a self sustained example of such states because of its weakly interacting nature; making it easy to probe various quantum correlations among such states. In case of neutrinos, mode entanglement can be expressed in terms of flavour transition probabilities, and therefore that single-particle entangled states acquire a precise operational characterization in the context of particle mixing. **Moreover,** an experimental scheme to transfer the quantum information encoded in single-neutrino states to spatially delocalized two-flavor charged-lepton states has been discussed already in [15] where the single-particle entangled states of neutrino mixing are represented as legitimate physical resources for quantum information tasks.

The time evolved flavour state given in Eq. (4.3) can be viewed as an entangled superposition of flavour modes as in Eq. (4.4) with the time dependent coefficients given by Eq. (3.5). Care should be taken in dealing with the above defined Fock representations in the flavour and mass basis as they are unitarily inequivalent in the quantum field theoretic description of neutrino oscillations [207]. Specifically, the unitary equivalence of the flavour and the mass state given in Eq. (3.2), is not valid under the infinite volume approximation as the flavour and mass eigenstates become orthogonal and the vacuum for definite flavour neutrinos can not be identified with the vacuum state for definite mass neutrinos. However, in this work, ultra relativist approximation has been taken into account, Eq. (3.7), under which the unitary equivalence holds and we can analyze the various nonclassical witnesses viz., entanglement existing among different flavour modes in a single particle setting. It would be interesting to investigate the behaviour of these witnesses by incorporating various non trivial effects arising from the quantum field theoretic treatment of neutrino oscillation viz, vacuum condensation [208].

In the next section, the behaviour of various quantum correlations has been analyzed in the context of T2K, NOνA and upcoming DUNE experiment with specific experimental conditions viz. baseline L and neutrino-energy E. These experimental facilities have been discussed before in section 3.5 of chapter 3.

4.2 Measures of Quantum Correlations in neutrino-system

The general form of Eq. (3.6), for initial state $|\nu_\alpha\rangle$, can be written as:

$$|\Psi_\alpha(t)\rangle = \xi_1(t)\,|\nu_e\rangle + \xi_2(t)\,|\nu_\mu\rangle + \xi_3(t)\,|\nu_\tau\rangle\,. \tag{4.5}$$

with

$$\begin{cases} \xi_1(t) = a(t),\ \xi_2(t) = b(t),\ \xi_3(t) = c(t), & \text{if } \alpha = e \\ \xi_1(t) = d(t),\ \xi_2(t) = e(t),\ \xi_3(t) = f(t), & \text{if } \alpha = \mu \\ \xi_1(t) = g(t),\ \xi_2(t) = h(t),\ \xi_3(t) = k(t), & \text{if } \alpha = \tau \end{cases} \tag{4.6}$$

where $a(t),\ b(t),\ c(t)\ \ldots\ k(t)$ are the elements of U_f matrix defined in Eq. (3.4) for vacuum. In case of matter, the corresponding elements of the flavour evolution given in Eq. (3.16), are used. Equivalently, Eq. (4.5) can be written in the occupation number representation as:

$$|\Psi_\alpha(t)\rangle = \xi_1(t)\,|100\rangle + \xi_2(t)\,|010\rangle + \xi_3(t)\,|001\rangle\,. \tag{4.7}$$

With this general setting, we now discuss various facets of quantum correlation

1. *Flavour Entropy*: For the pure states in Eq. (4.7), the standard measure of entanglement is given as [19]

$$\mathcal{S}(|\xi_i|^2) = -\sum_{i=1}^{3} |\xi_i|^2 \log_2(|\xi_i|^2) - \sum_{i=1}^{3}(1-|\xi_i|^2)\log_2(1-|\xi_i|^2). \tag{4.8}$$

   This measure serves as a tool to probe the nonclassicality of the system. In the context of neutrino oscillation, the flavour entropy parameter $\mathcal{S} = 0$ for an initially prepared neutrino state ν_α ($\alpha = e, \mu, \tau$), and reaches its upper bound $\mathcal{S} = 1$ for the maximally nonclassical state in the W class $\frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$[42].

2. *Tripartite geometric entanglement*: Tripartite geometric entanglement G for the pure states, given in Eq. (4.7), is defined as the cube of the geometric mean of Shannon entropy over every bipartite section.

$$G = H(\xi_1(t)^2)H(\xi_2(t)^2)H(\xi_3(t)^2), \tag{4.9}$$

   where $H(p) \equiv -p\, log_2(p) - (1-p)\, log_2(1-p)$ is the bipartite entropy. This is a weaker condition than genuine tripartite nonlocality discussed below. The genuine tripartite entanglement does not exist if $G = 0$.

3. *Absolute and genuine tripartite nonlocality (Mermin and Svetlichny inequalities)*: The violation of a Bell type inequality (viz., CHSH) for a two qubit state is said to imply nonlocality. A generalization to three party system is not straightforward. Mermin inequality is based on the assumptions that all the three qubits are locally and realistically correlated; hence a violation would be a signature of the tripartite nonlocality shared among the qubits. It was shown in [209, 210] that the biseparable states also violate the Mermin inequality. This motivated Svetlichny to formulate a hybrid nonlocal-local realism based inequality, the Svetlichny inequality. A three qubit system may be nonlocal if nonclassical correlations exist between

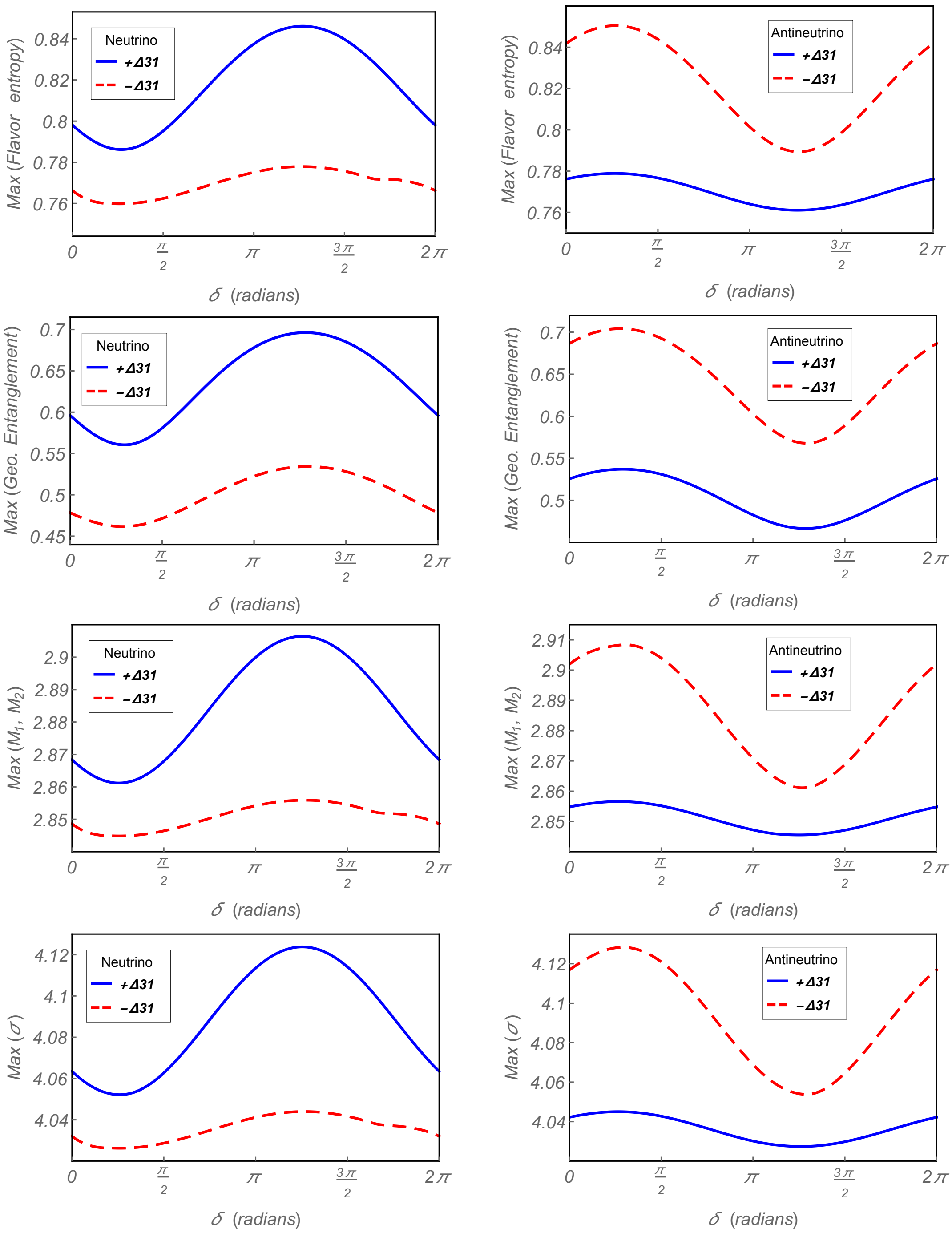

Figure 4.1: DUNE: The maximum of various quantum correlations such as Flavour entropy (First row), Geometric entanglement (second row), Mermin parameters (M_1, M_2) (third row) and Svetlichny parameter (σ) (fourth row) depicted with respect to the CP violating phase δ for DUNE experiment. The left and right panels pertain to the neutrino and antineutrino case, respectively. Solid(blue) and dashed(red) curves correspond to the positive and negative signs of Δm^2_{31}, respectively. The mixing angles and the squared mass differences used are $\theta_{12} = 33.48^o$, $\theta_{23} = 42.3^o$, $\theta_{13} = 8.5^o$, $\Delta m^2_{21} = 7.5 \times 10^{-5} eV^2$, $\Delta m^2_{32} \approx \Delta m^2_{31} = 2.457 \times 10^{-3} eV^2$. The energy range used is $E = 1 - 10$ GeV and the baseline used is 1300 km. The neutrinos pass through a matter density of $2.8 gm/cc$.

two of the three qubits. Such a state would be absolute nonlocal and will violate Mermin inequality [105] for a particular set of detector setting (A,B,C) and (A',B',C'). The two

Mermin inequalities are:

$$M_1 \equiv \langle ABC' + AB'C + A'BC - A'B'C' \rangle \leq 2,$$
$$M_2 \equiv \langle ABC - A'B'C - A'BC' - AB'C' \rangle \leq 2. \tag{4.10}$$

However, a violation of Mermin inequality does not necessarily imply genuine tripartite nonlocality. A state violating a Mermin inequality may fail to violate a Svetlichny inequality, which provides a sufficient condition for genuine tripartite nonlocality [106] and is given by

$$\sigma \equiv M_1 + M_2 \leq 4. \tag{4.11}$$

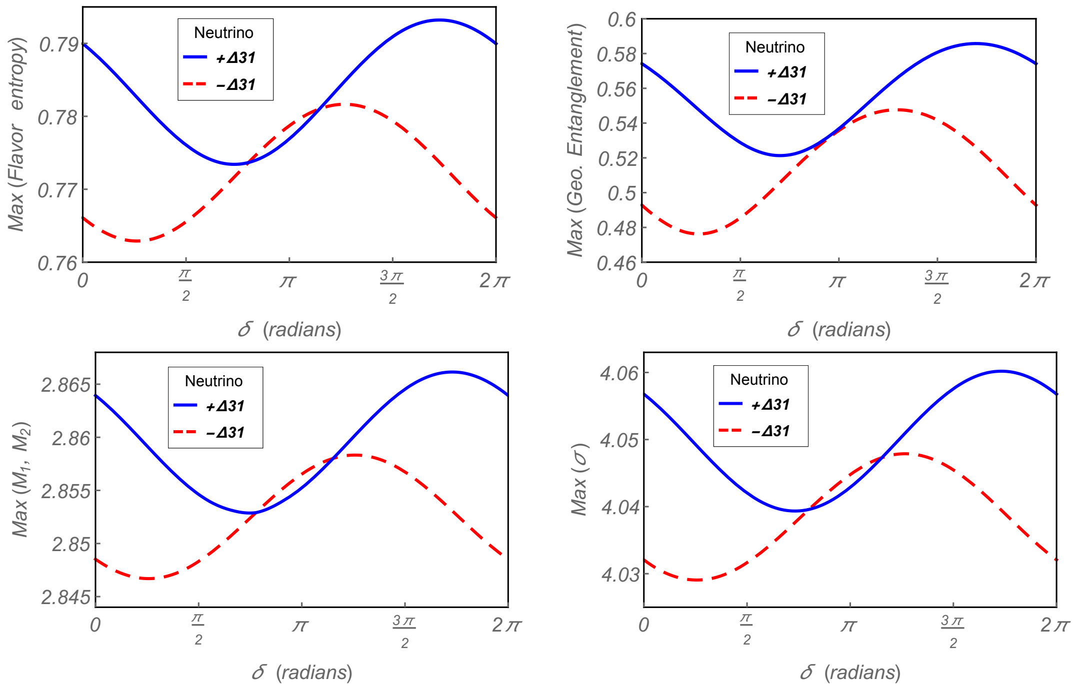

Figure 4.2: NOνA: Quantum correlations such as Flavour entropy (first), Geometric entanglement (second), Mermin parameter (M_1, M_2) (third) and Sevtlichny parameter (σ) (fourth) parameters, plotted with respect to the CP violating phase δ for NOνA experiment for the case of neutrinos. The energy is varied between $1.5 - 4$ GeV and the baseline is taken as 810 km. The various mixing angles and squared mass differences used are the same as for Fig. 4.1.

For DUNE experiment, Fig. 4.1 depicts the variation of the maximum of various quantum witnesses like flavour entropy, geometric entanglement, Mermin parameters (M_1, M_2) and Svetlichny parameter (σ) with respect to the CP violating phase δ, for the case of neutrino (first column) and antineutrino (second column), respectively. It can be seen that all the witnesses show different characteristics for the positive and negative signs of large mass square difference Δm^2_{31}. Figures 4.2 and 4.3 depict the same for ongoing NOνA and T2K experiments, for neutrino beam. The corresponding antineutrino plots show similar features, such as inversion of mass ordering, as in the DUNE plots and hence are not depicted here.

A general feature observed in these results is that the different measures of nonclassicality are sensitive to the sign of Δm^2_{31}. The distinction being more prominent in DUNE experiment compared to the NOνA and T2K experiments. This can be attributed to the high energy and long baseline of the DUNE experiment.

The quantum correlation measures studied in this work can attain their upper bounds for some specific values of L/E [19]. In the present study, however, by taking into account the matter

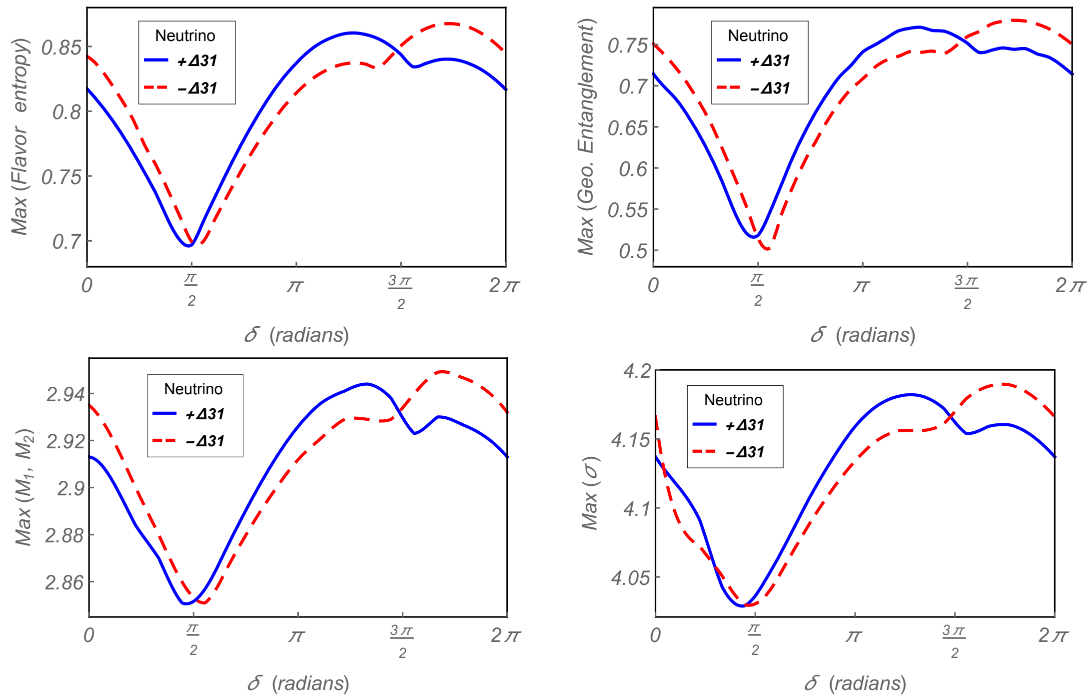

Figure 4.3: T2K: Showing Flavour entropy (first), Geometric entanglement (second), Mermin parameter (M_1, M_2) (third) and Sevtlichny parameter (σ) (fourth) parameters, as function of the CP violating phase δ. The energy is taken between $0.1 - 1$ GeV and the baseline is 295 km.

effects and CP violation, we are restricting L/E within the experimentally allowed range; consequently the various nonclassical measures do not reach their maximum allowed values. Mermin inequalities are violated for all values of δ which means that if one of the three parties is traced out, still there will be residual nonlocality in the system. Violation of the Svetlichny inequality reflects the nonlocal correlation between every subsystem of the tripartite system. To achieve significant violation of correlation measures one should use neutrino-beam if the sign of Δm^2_{31} is positive (normal mass ordering), while antineutrino-beam should be used in case of negative sign of Δm^2_{31} (inverted mass ordering).

From the definitions of flavour entropy (Eq. (4.8)) and geometric entanglement (Eq. (4.9)), it is clear that these are measurable quantities since these can be expressed in terms of survival and oscillation probabilities making them suitable for experimental verification. Expressing the Mermin and Svetlichny parameters in terms of these measurable quantities is a nontrivial task. However, guided by the previous work [18], where the measures of quantum correlations viz. Bell-CHSH inequality, teleportation fidelity and geometric discord were expressed in terms of survival and transition probabilities for two flavour neutrino-system, it could be possible to perform such an exercise for Mermin and Svetlichny parameters in the context of three flavour neutrino oscillations.

The parameter δ which appears in the complex PMNS mixing matrix causes CP and T-violation in neutrino sector and the determination of its value, along with the issue of determining the neutrino mass ordering, is still an open challenge for oscillation experiments. In neutrino oscillation scenario, CP-violation can be observed as a non-zero value of $\Delta CP = P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ and the T-violation is observed in terms of $\Delta T = P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$. Under the assumption of CPT-conservation, ΔCP and ΔT are equal, while different values of ΔCP and ΔT can signify pronounced CPT-violation. Hence, the separate observations of ΔCP and ΔT are crucial.

However, one should notice that in this thesis we have focused mainly on the problem of disentangling the neutrino mass ordering effects. Our results exhibit that the determination of neutrino mass ordering by the observation of correlation measures can be obtained regardless the value of δ parameter. This is true for DUNE experimental setup where the matter effect is quite significant. For NOνA (with slightly smaller matter effect) one can still find the δ-ranges that allow the discrimination between normal and inverted mass ordering. In case of T2K, such ranges of δ could not be found.

Moreover, the correlation measures studied here are found to be sensitive to the δ-parameter that provides a scope of finding the value of this parameter from the observation of these correlation quantities. However, it can be a non-trivial task to determine the δ-value in the context of current oscillation experimental setups, where neutrinos undergo significant matter density potentials, due to the presence of matter induced extrinsic CP and T-violating effects [211–213].