# 5

# An MOO Strategy and Evaluation of Beacon Configurations

With the demonstration of interplay among cost, coverage and accuracy, we choose total beacon count and average beacon density as two contradictory objectives for multi objective Pareto assessment [Wingo, 1979]. Pareto analysis [Baumgartner et al., 2004; Deb et al., 2002] among the two contradictory objectives results in non-dominant solutions for decision variables as represented in Figure 5.1. This presents an opportunity to put forth system's capability and utilize solutions as per the requirement.

In addition to the rationale mentioned in Section 4.4, count an density are linear objectives with respect to beacon configuration. This presents an advantage for the computational performance of optimization. In addition to this, we present the choice of constraints and basis for evaluating resulting beacon configurations as the following.

- **Objectives**: Total beacon count, average beacons for each location estimation.
- **Constraints**: LoS connectivity to atleast 3 beacons $(k \geq 3)$, sensing range $(R)$ , GDoP $(g)$.
- **Configuration assessment by**: Output RMSE, percentage of CDL Coverage

Realization of aforementioned is essentially a chain of mathematical tools leading to configuring an optimization problem. This Optimization-Tool-Chain (OTC) is described step-by-step in the next section.

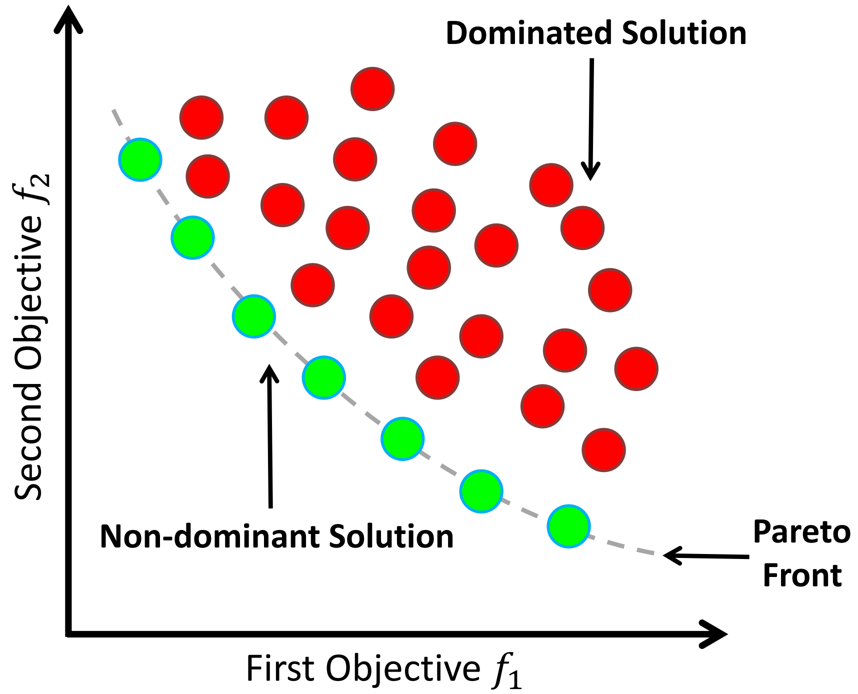



**Figure 5.1:** A visual representation of pareto front resulting from the contradictory i.e. min-max optimization of two objective functions $f_1, f_2$. The points circled green are inferior to no other candidate solutions with $f_1, f_2$ taken collectively unlike the solutions circled red.

## 5.1 Problem Formulation

This section contains the formulation of a multi objective BPP divided into three steps as the following:

### 5.1.1 Step-1: Formulating Design Elements

With reference to proposed 3D point cloud design as presented in Chapter 3, this step describes mathematical formulation for matrix and vectors related to CDLs, CBLs and OPCs in the following:

1. **Device**:
   - Assuming $p$ CDLs are there in the environment.
   - A $p \times 3$ matrix $D = \{(x_d^j, y_d^j, z_d^j), \forall j = 1..p\}$ holds the point cloud of CDLs.
2. **Beacon**:

- Assuming $q$ CBLs are there in the environment.
- We define a $q \times 1$ binary ***status*** vector $b = \{b_j \in (0,1), \forall j = 1..q\}$. This is essentially a decision variable intended to store a value 1 at the indices corresponding to selected beacon locations by optimizer.
- A $q \times 3$ matrix $B = \{(x_b^j, y_b^j, z_b^j), \forall j = 1..q\}$ holds the point cloud of CBLs.

3. **Obstacle**:

- Assuming $s$ obstacles are there in the environment.
- Each of the $s$ obstacles are represented by $n_1, n_2, ..., n_s$ coordinates.
- This gives $s$ matrices for each OPCs collectively defined as $O = \{o_i = \{(x_o^j, y_o^j, z_o^j), \forall j = 1..n_i\}, \forall i = 1..s\}$. Here set $O$ is stores the point cloud for each of the $s$ OPCs $o_1, o_2, ..., o_s$.

### 5.1.2 Step-2: Formulating Metadata

With reference to last subsection, next comes the population of beacon to device LoS connectivity and GDoP information, as explained in the following:

1. **Beacon to Device Connectivity**:

- For all combinations of $p$ CDLs and $q$ CBLs, we define matrix $\psi$ storing connectivity information as the following:

$$\psi_{i,j} = \begin{cases} 1 & \text{if LoS exists between } (D_i, B_j) \text{ and } ||D_i, B_j|| \leq R \\ 0 & \text{otherwise} \end{cases}, \forall i = 1..p, \forall j = 1..q$$

Here, LoS calculation is done according to the PCOC algorithm as described in Chapter 4. Also, $||\cdot||$ represents the euclidean distance operator. This matrix will be used as a coefficient in constraint to ensure 3-coverage. Figure 5.2 presents an example view of the entries in the matrix $\psi$.

***q* Beacons** ⟹

***p* Devices** ⟱

| | | | |
|---|---|---|---|
| 0 | 1 | 1 | ... |
| 1 | 0 | 0 | ... |
| 0 | 0 | 1 | ... |
| ... | ... | ... | ... |

**Figure 5.2:** A view of sample binary entries in the matrix $\psi$ with row and column correspondence to devices and beacons respectively.

2. **GDoP matrix for all Beacon combinations**:

   - The objective of this matrix is to store the resulting GDoP value over each of the $p$ CDL corresponding to all $k$ combinations out of $q$ CBLs i.e $C_{qk} = \binom{n}{k}$. To recall, $k$ is chosen to be 3 for present implementation as a minimum requirement for 3D localization.

   - Here, the range observations are assumed to be mutually independent with zero mean Gaussian noise. Moreover, in order to create beacon configurations that can tolerate varying levels of noise, the standard deviation in range measurements is assumed to be a percentage ($\rho$) of length of signal propagation. This is in accordance to the analysis in Chapter 3.

   - For a particular $\rho$ value, a $p \times C_{qk}$ matrix $\Lambda$ is populated to store the GDoP values for all the $p$ devices and corresponding beacon combinations out of $C_{qk}$. Figure5.3 presents an example view of the entries in the matrix $\Lambda$.

**$C_{qk}$ combinations** ⟹

***p* Devices** ⟱

| | | | |
|---|---|---|---|
| 1.5 | 3.2 | 2.3 | ... |
| 2.1 | 1.9 | 3.5 | ... |
| 3 | 1.3 | 1.7 | ... |
| ... | ... | ... | ... |

**Figure 5.3:** A view of sample binary entries in the matrix $\Lambda$ with row and column correspondence to devices and $C_{qk}$ combinations respectively.

### 5.1.3 Complete Multi Objective Optimization (MOO) Problem

Now, we formally define the MOO in Equation 5.1 using the design elements and metadata as derived in the previous subsections.

$$
\begin{aligned}
&\text{minimize} && \sum_{j=1}^{q} b_j \\
&\text{maximize} && \left( \sum_{i=1}^{p} \sum_{j=1}^{q} \psi_{i,j} b_j \right) / p \\
&\text{subject to} && \sum_{j=1}^{q} \psi_{i,j} b_j \geq k,\; i = 1 \ldots p \\
&\text{and} && \forall i \in p[\exists j \in C_{qk}(\Lambda_{i,j} \leq g)]
\end{aligned}
\tag{5.1}
$$

Here, the first minimization objective is the expression for summation of elements in vector $b$ representing the minimization of total beacon count. The second maximization objective is the expression of average beacon density for $p$ CDLs. Among constraints, the first expression enforces the requirement of minimum $k$ $(= 3)$ beacon density and second ensures the availability of atleast one beacon configuration satisfying the GDoP threshold $g$, for each of the $p$ CDLs.

## 5.2 Proposed Strategy: Optimization Tool Chain (OTC)

Summarizing the problem formulation described in the last section, the proposed 5-step OTC is shown in Figure 5.4. The steps in OTC are: (i) designing indoor coordinate clouds for CDLs, CBLs and OPCs, (ii) populating connectivity information for all CDL-CBL pairs by in-range LoS estimation, (iii) calculating GDoP values for all available $k$ CBL combinations for each CDL, (iv) multi objective optimization, (v) assessment of resulting configurations for output RMSE and coverage percentage. The next section describes the simulations for optimization and assessment with respective parameter choices.

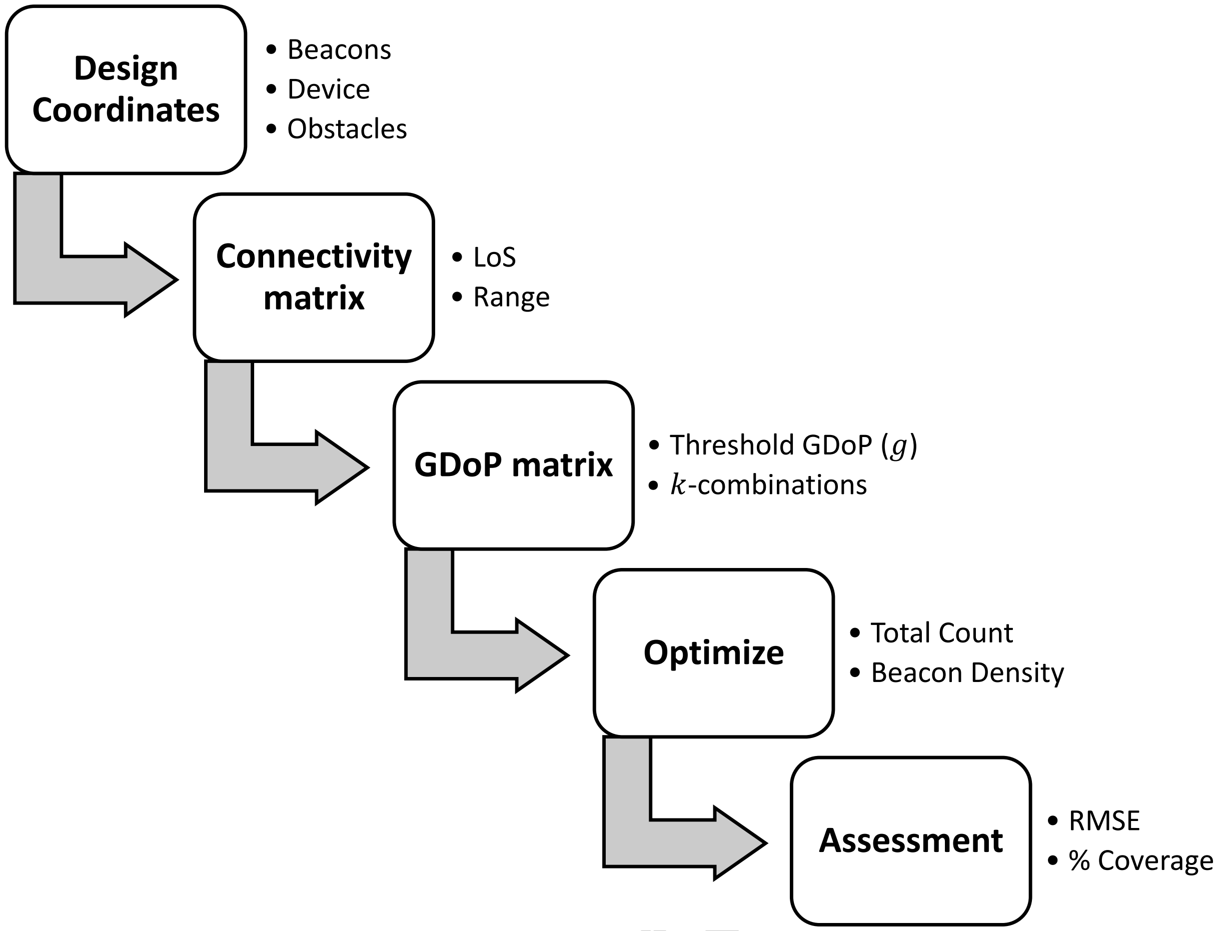


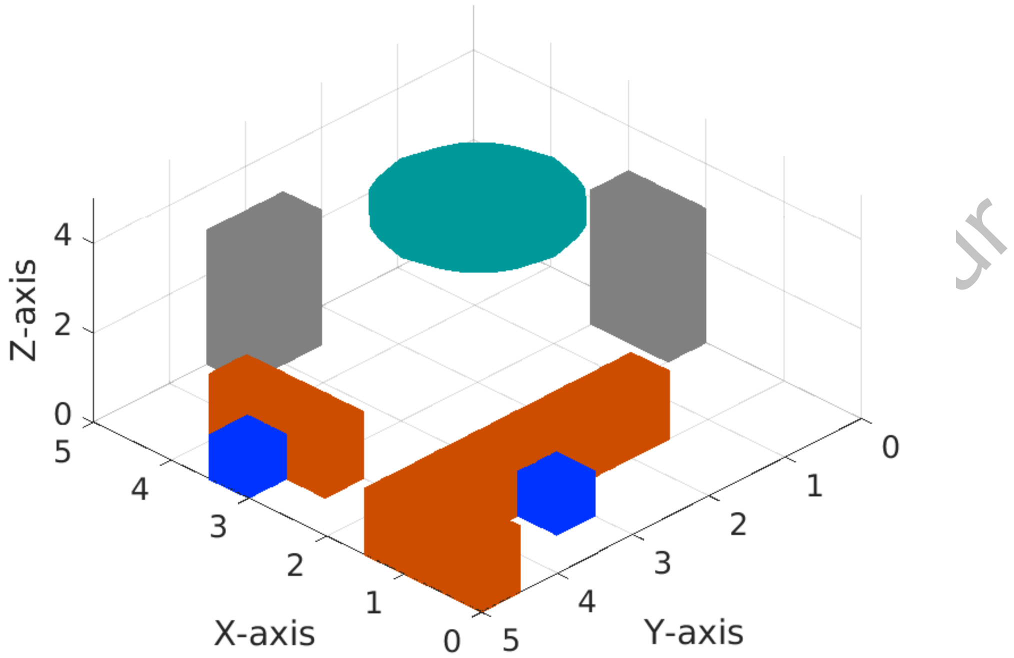

**Figure 5.4:** Proposed Multi Objective Optimization Tool Chain (OTC)

## 5.3 Generating Pareto Optimal Beacon Configurations

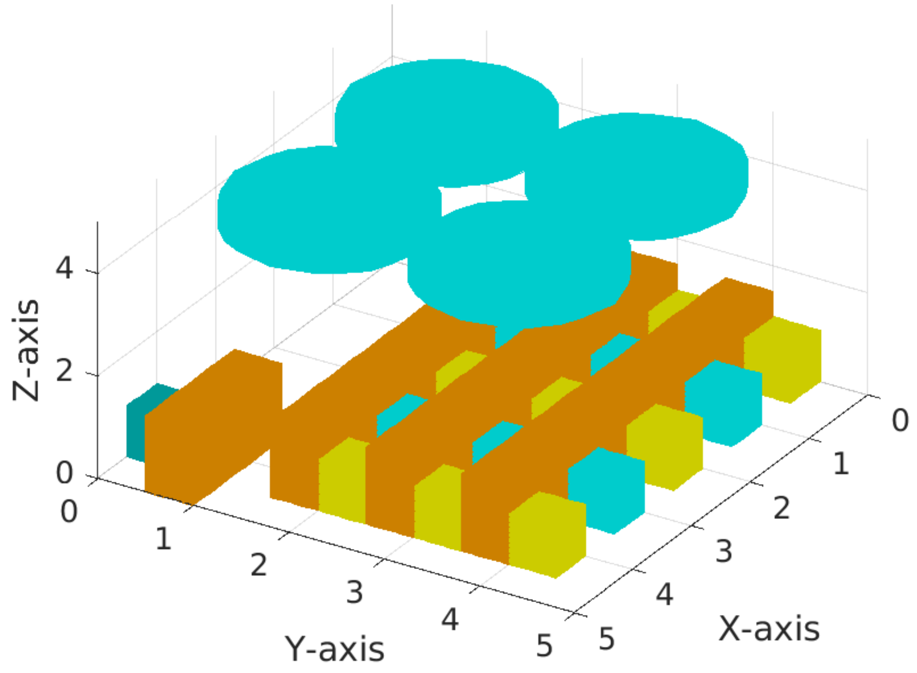
In the OTC sequence, the first simulation step is to generate the set of non-dominant beacon configurations by MOO. In the next subsections, we define the values of simulation parameters essential to the optimization.

### 5.3.1 Indoor Designs

For simulations, cubic indoor designs with dimensions $length(l) = width(w) = height(h) = 5\ units$ are synthesized. For each design, coordinate clouds for the classes CDL, CBL and OPC were generated using MATLAB. The choice of cubic design avoids the inherent presence of any dimension induced biases. Figure 5.5 presents the choice of two indoor designs with varying obstacle placement density. To have a clear understanding of the indoor, only obstacle placement is shown here. These designs are synthesized referencing typically an office space and a classroom as shown in Figure 5.5a and Figure 5.5b. Designs in Figure 5.5 are convex hull wrappings of OPC




(a) Design 1: An office space




(b) Design 2: A classroom

**Figure 5.5:** Obstacle placement in two indoor designs as chosen for simulations.

coordinates as generate in MATLAB. To elaborate for example, in Figure 5.5a, the blue objects represent sitting tables, the orange objects represent tables with varying sizes, the grey cuboids represents almirahs and the dark-cyan cylindrical object represents space occupied by a ceiling fan.

### 5.3.2 Simulation Parameters

With reference to the (i) qualitative assessment of design and analytical elements as described in Chapter 3 and (ii) problem formulation context mentioned earlier in this section, we present the choice of simulation parameters as the following:

1. Room dimension: $l = w = h = 5\, units$.
2. DGS: $\Delta x_d = \Delta y_d = \Delta z_d = 1\, unit$
3. A value of $k = 3$ is chosen to ensure the minimum requirement for 3D positioning.
4. BGS: $\Delta x_b = \Delta y_b = \Delta z_d = 0.5\, unit$
5. $R = \{3, 4, 5\}$: R is chosen to vary from $\lceil l/2 \rceil$ to $l$ in unit increments.
6. OGS: $\Delta x_o = \Delta y_o = \Delta z_o = 0.1\, unit$
7. Percentage Noise: $\rho = \{10, 20, 30, 40, 50\}$
8. GDoP Threshold: $g = \{1, 2, 5\}$

### 5.3.3 MOO: Obtaining Pareto Solutions

The objectivity of this research remains with the successful formulation of MOO for 3D point cloud indoor design and evaluation of localization performance of resulting beacon configurations. Hence, we choose to solve the MOO by an evolutionary Non-dominated Sorting Genetic Algorithm (NSGA)-II [Deb et al., 2002; Deb, 2001]. NSGA-II is frequently applied to two objective optimization problems [Campos Ciro et al., 2016; Deb and Jain, 2014] and proven better in computational performance to other multi objective genetic algorithms for the same as it adds elitism and faster non-dominated sorting method. The process of obtaining Pareto solutions is divided into first finding a seed solution and second feeding it to the NSGA-II, as explained in the following.

#### Seed Solution by Single Objective Optimization (SOO)

Before proceeding to the MOO, we attempt to find a seed solution by minimizing only the first objective in Equation 5.1 i.e. total beacon count, using Mixed Integer Linear Programming

(MILP) as shown in Equation 5.2. This is due to the reason that, the solution space of the total MOO in Equation 5.1, is going to be the subset of this Single Objective Optimization (SOO), making this a valid starting domain to work as a seed.

$$\begin{aligned} &\text{minimize} \quad \sum_{j=1}^{q} b_j \\ &\text{subject to} \quad \sum_{j=1}^{q} \psi_{i,j} b_j \geq k,\ i = 1 \ldots p \end{aligned} \tag{5.2}$$

## Pareto Solutions

Using the resulting binary status $b_j$ of each beacon as a chromosome for the initial population in NSGA-II, we attempt the MOO with parameter selection as shown in Table 5.1 for the Genetic Algorithm.

**Table 5.1:** Parameter selection for NSGA-II implementation

| NSGA-II parameters | Values |
|---|---|
| Chromosome Size | $q$ |
| Crossover Fraction | 0.8 |
| Maximum Generation Count | $200q$ |
| Migration Fraction | 0.2 |
| Pareto Fraction | 0.35 |
| Population Size | 200 |

Figures 5.6, 5.7 and 5.8 present the pareto solutions obtained for the indoor design in Figure 5.5a while Figures 5.9, 5.10 and 5.11 present the pareto solutions obtained for the indoor design in Figure 5.5b. Here, the x and y axes represent the Pareto optimal objective values for total beacon count and average beacon density. For the ease of explanation and understanding, the resulting solutions for percentage noise values of $\rho = 10, 20, 30$ have only been shown here. Complete repository of the solutions is available at Thesis-Results[1].

Intuitively, as shown in the output plots in Figures 5.6 to 5.11, either the requirement to tolerate high observation noise or output low GDoP reflects in increasing the overall beacon count in comparison to low noise and high GDoP scenarios. To assess the capabilities of above

[1]https://github.com/ravitheking/Thesis-Results.git

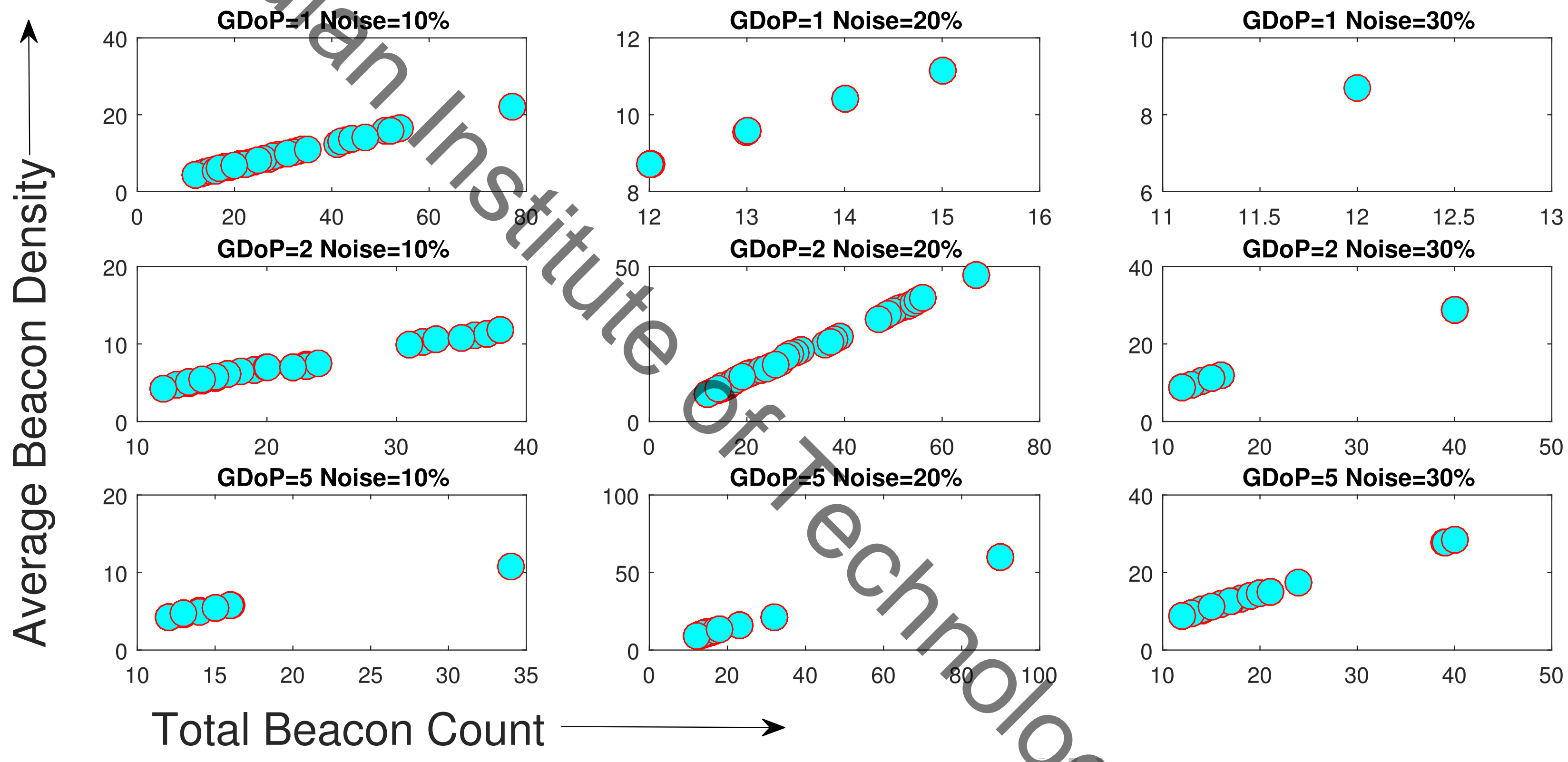



**Figure 5.6:** Non-dominant solutions with $R = 3$ for indoor design in Figure 5.5a

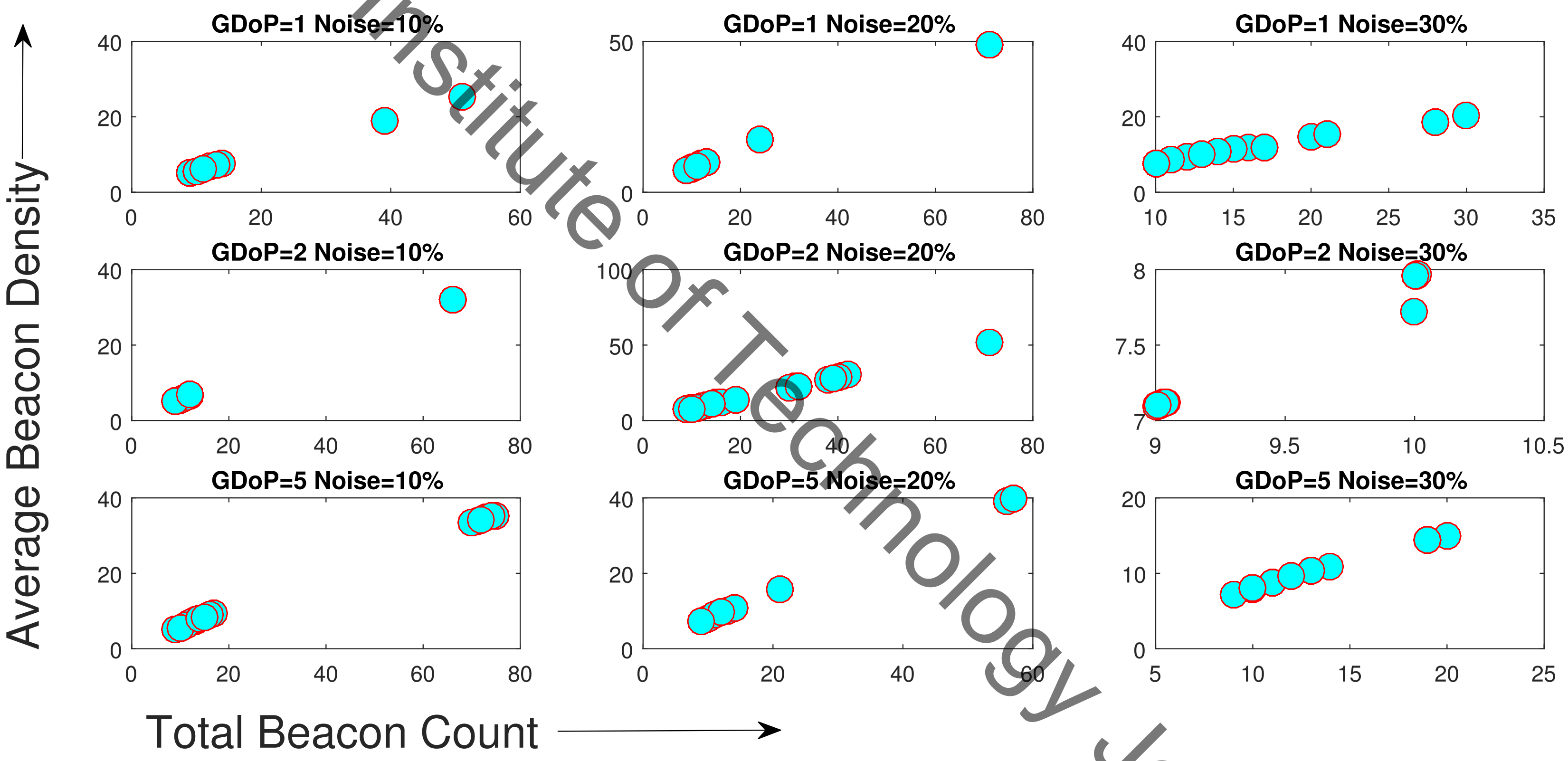



**Figure 5.7:** Non-dominant solutions with $R = 4$ for indoor design in Figure 5.5a

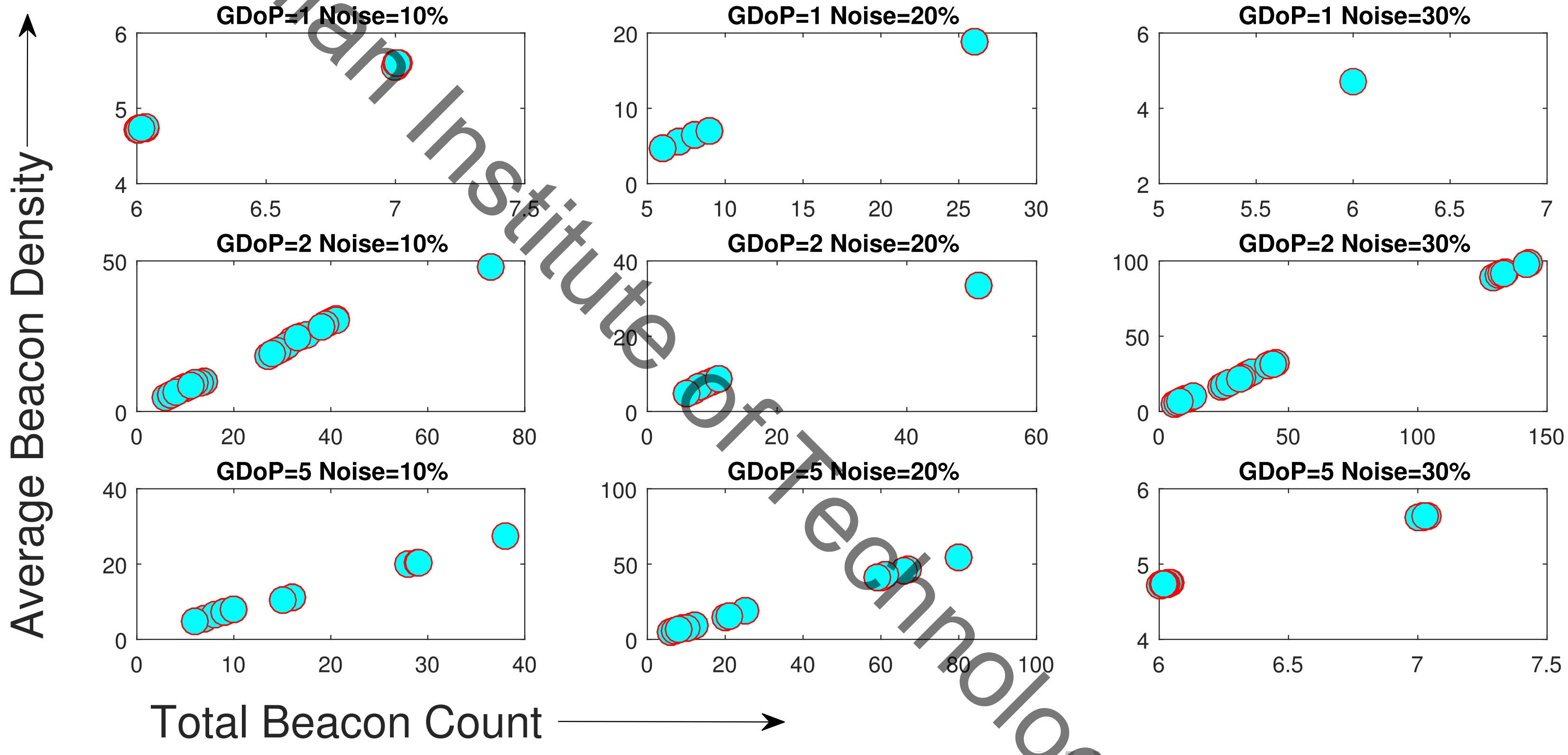



**Figure 5.8:** Non-dominant solutions with $R = 5$ for indoor design in Figure 5.5a

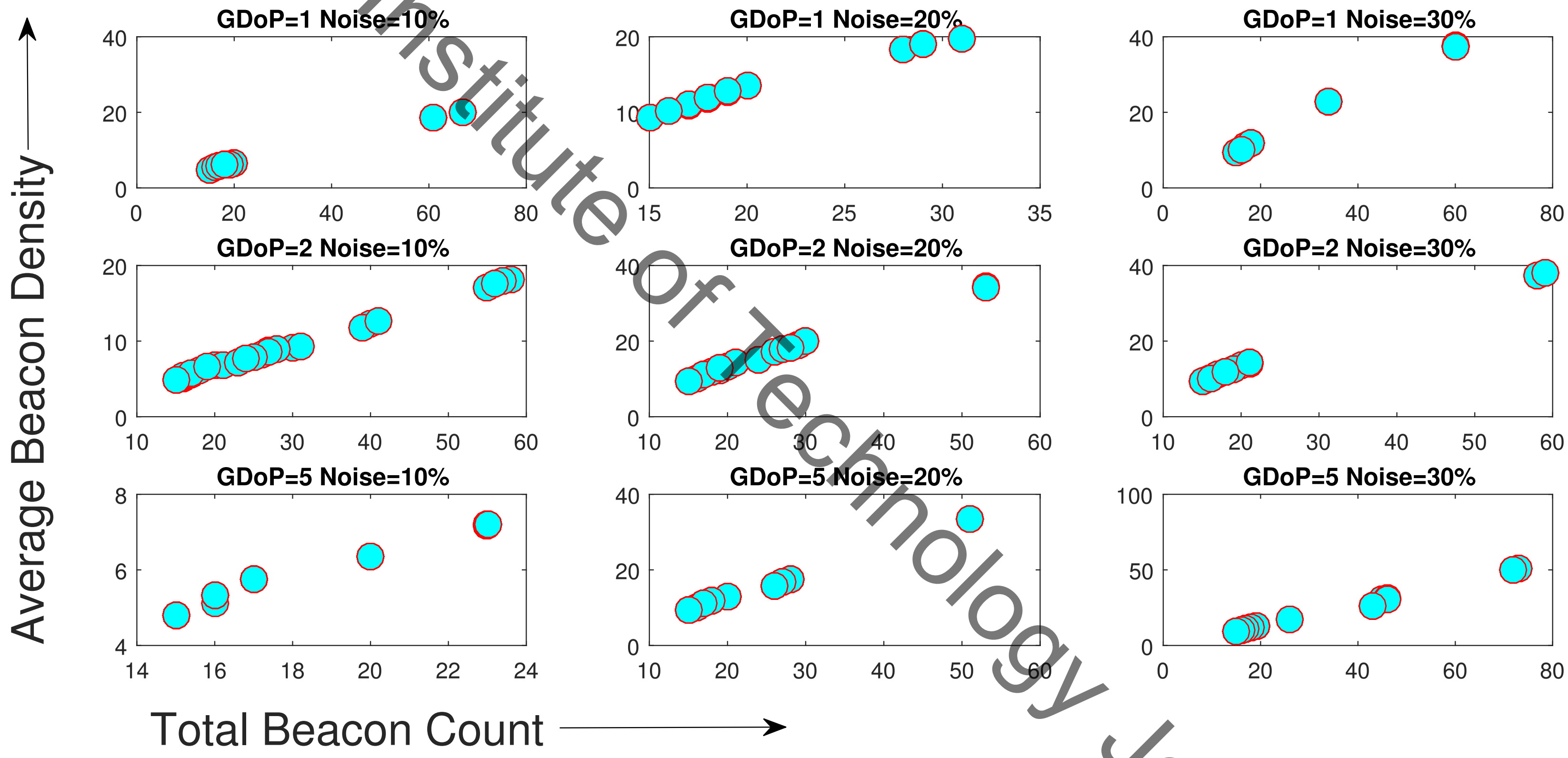



**Figure 5.9:** Non-dominant solutions with $R = 3$ for indoor design in Figure 5.5b

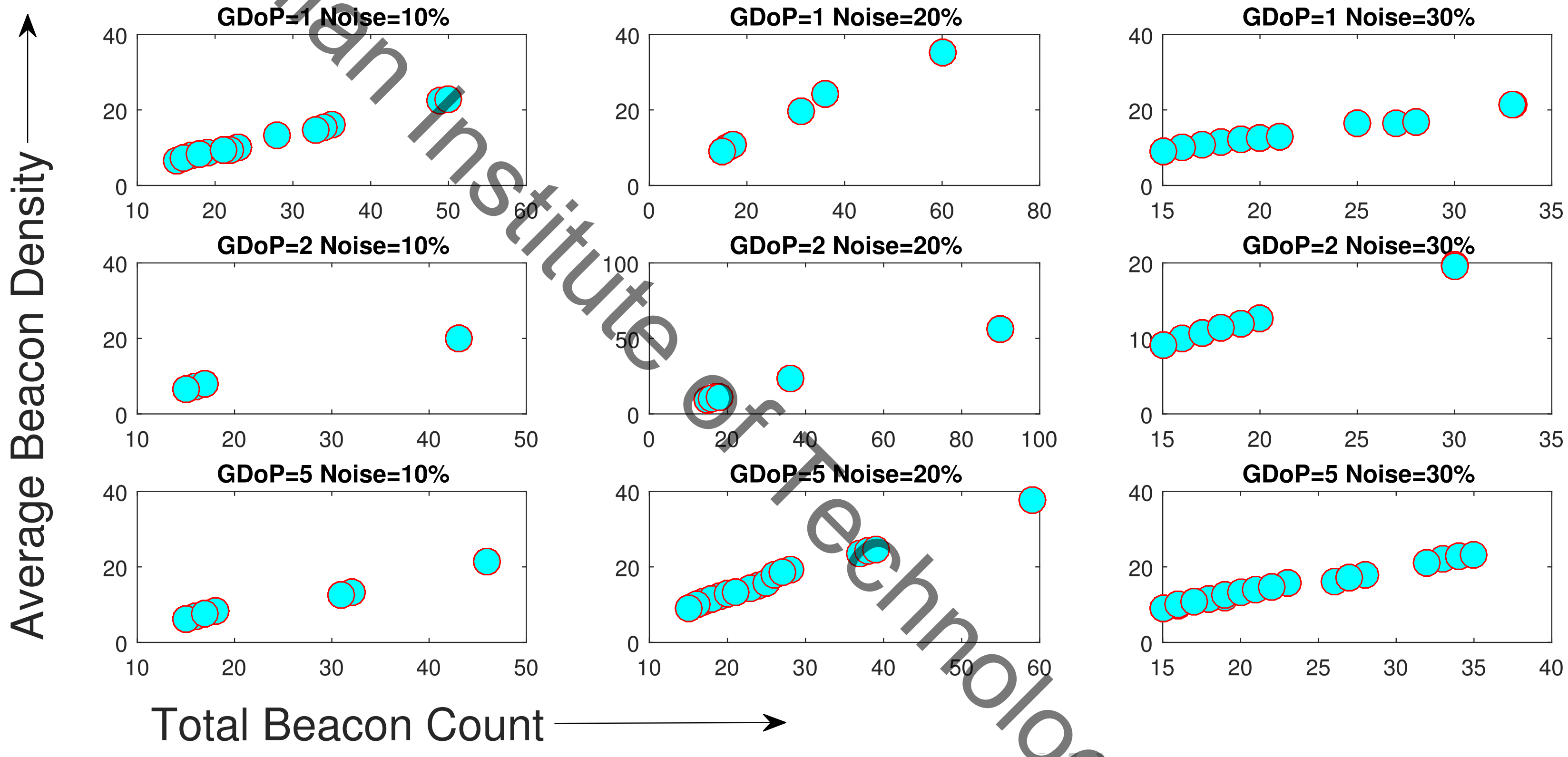



**Figure 5.10:** Non-dominant solutions with $R = 4$ for indoor design in Figure 5.5b

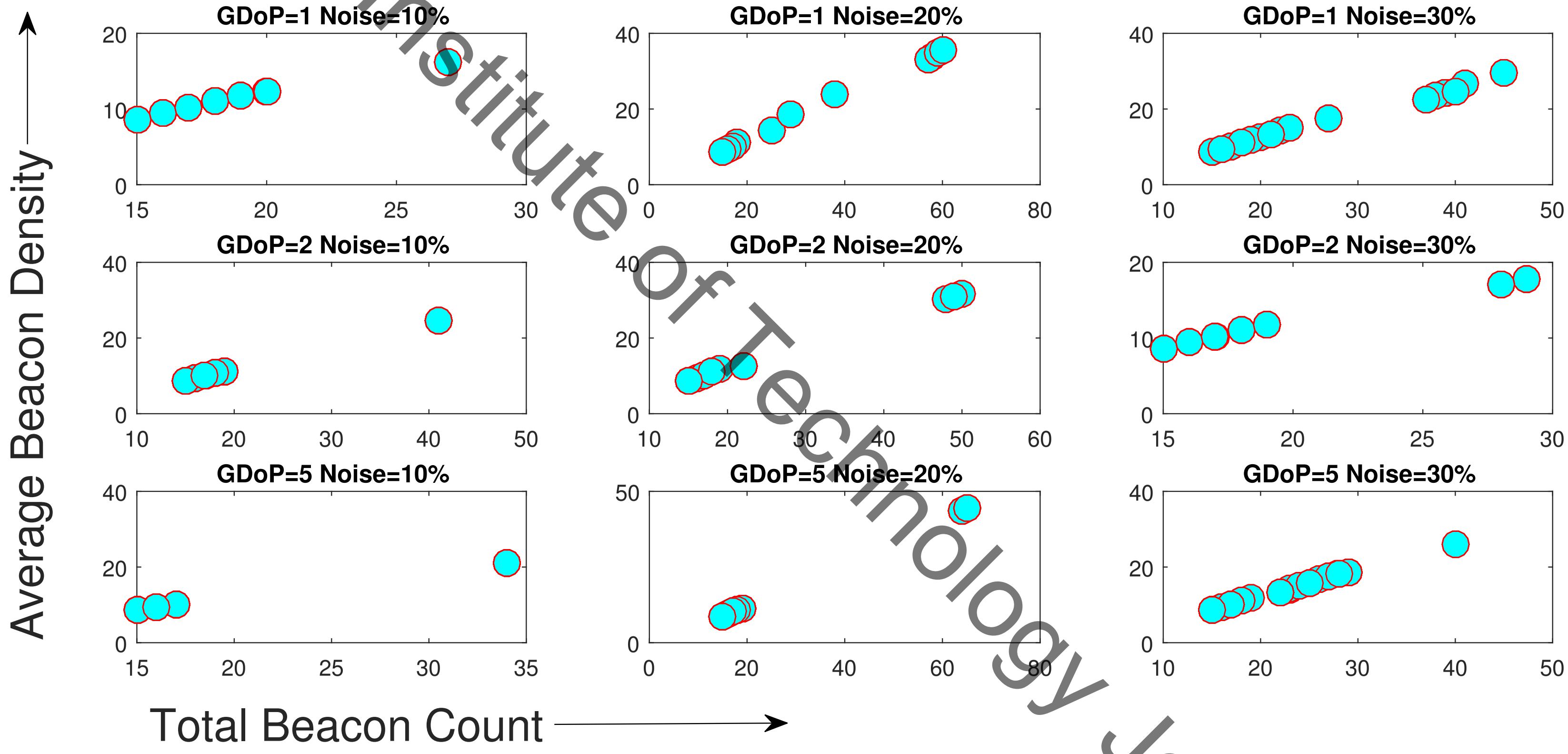



**Figure 5.11:** Non-dominant solutions with $R = 5$ for indoor design in Figure 5.5b

configurations, in the next section, we evaluate their performance for localization and present an unbiased ranking scheme to select the optimal candidate.

## 5.4 Recommender System: *Evaluation and Ranking*

This section presents a two step evaluation of optimal beacon configurations as generated in the last section. The first step calculates the resulting RMSE and percentage of Coverage for each configuration over a uniform distributed CDL coordinate cloud. The second step presents a normalization and scaling based mechanism to rank the configurations for their output RMSE and percentage Coverage.

### 5.4.1 Performance Evaluation: Network Simulation

We now present the results of evaluation of Pareto solutions over a 2.4 GHz wireless network, as simulated in MATLAB. The following steps describe the process of evaluating each beacon configuration for localization.

1. **Step-1:** For each indoor design, create a set of high density CDL cloud $C$ having $DGS = 0.1$. In contrast to the earlier choice for CDL design, this is relatively 10 times denser choice. Select of set of of 1000 uniformly distributed CDL coordinates $C_{1000}$ out of $C$.

2. **Step-2:** For each non-dominant beacon configuration $B$ resulting from the MOO, create an euclidean-distance-weighted adjacency matrix $W_B$ corresponding to $C_{1000}$.

3. **Step-3:** Table 5.2 presents physical layer parameters used for simulating the localization. Here, based on the actual distances as stored in $W_B$, the received power is generated by reverse path loss modelling. Then, a normal random fraction of receiver noise is added to it for synthesizing path loss. Assuming the Free Space propagation [Friis, 1946], this path loss gets modelled for distance estimates which are then used as noisy range measurements for localization.

4. **Step-4:** To solve the non-linear system of range equations resulting from the previous step,

**Table 5.2:** WLAN simulation parameters for Physical layer modelling

| | |
|---|---|
| Transmitter ($T_x$) power | 20 dBm |
| Antenna Gain ($G$) | 1 |
| Modulation | OFDM |
| Carrier Frequency (GHz) | 0.9, 2.4, 5 |
| Receiver Noise | 5 dBm |
| Temperature (Kelvin) | 300 |
| Sample Rate (per second) | 5 |
| Path Loss Model | Free Space |

the *Trust-Region-Dogleg* procedure [Powell, 1970] is adopted due to its robustness.

The above process is simulated using MATLAB software. The performance evaluation using RMSE and percentage coverage is carried out over the 90 configurations as generated from the MOO in Section 5.3.3. This gives a total of 180 plots for assessment. Due to the constraint of space, we present and discuss the performance of configurations prepared for a GDoP tolerance of 2 and simulated with carrier frequency 2.4 GHz.

The localization accuracy by RMSE and coverage as a percentage of points in $C_{1000}$ is shown in Figure 5.12 and 5.13. The complete repository of the performance assessment is available at Thesis-Results[2].

As can be seen in Figure 5.12 and 5.13, although scenarios with $R = 3$ achieve low $(< 1)$ RMSE with low beacon count $(< 30)$ but at the same time acquire lower $(< 90\%)$ coverage percentage in comparison to $R = \{4, 5\}$ as well. For low beacon count $(< 20)$, the achievable coverage with $R = 3$ drops from $> 90\%$ to $80 - 90\%$ with an increase in obstacle density from Design 5.5a to Design 5.5b as seen in Figure 5.12 and 5.13 respectively. Configurations with prepared with $R = 4$ tend to display a consistent low $(< 1)$ RMSE and high $(> 90\%)$ coverage in almost all the scenarios. On the other hand, configurations with $R = 5$, promise a coverage of $> 95\%$ for all the cases at a consistent cost of highest RMSE among other configurations. This is in accordance to the fact that, allowing a higher sensing range for beacons tend to induce higher error propagation in RSS path loss modelling.

[2]https://github.com/ravitheking/Thesis-Results.git

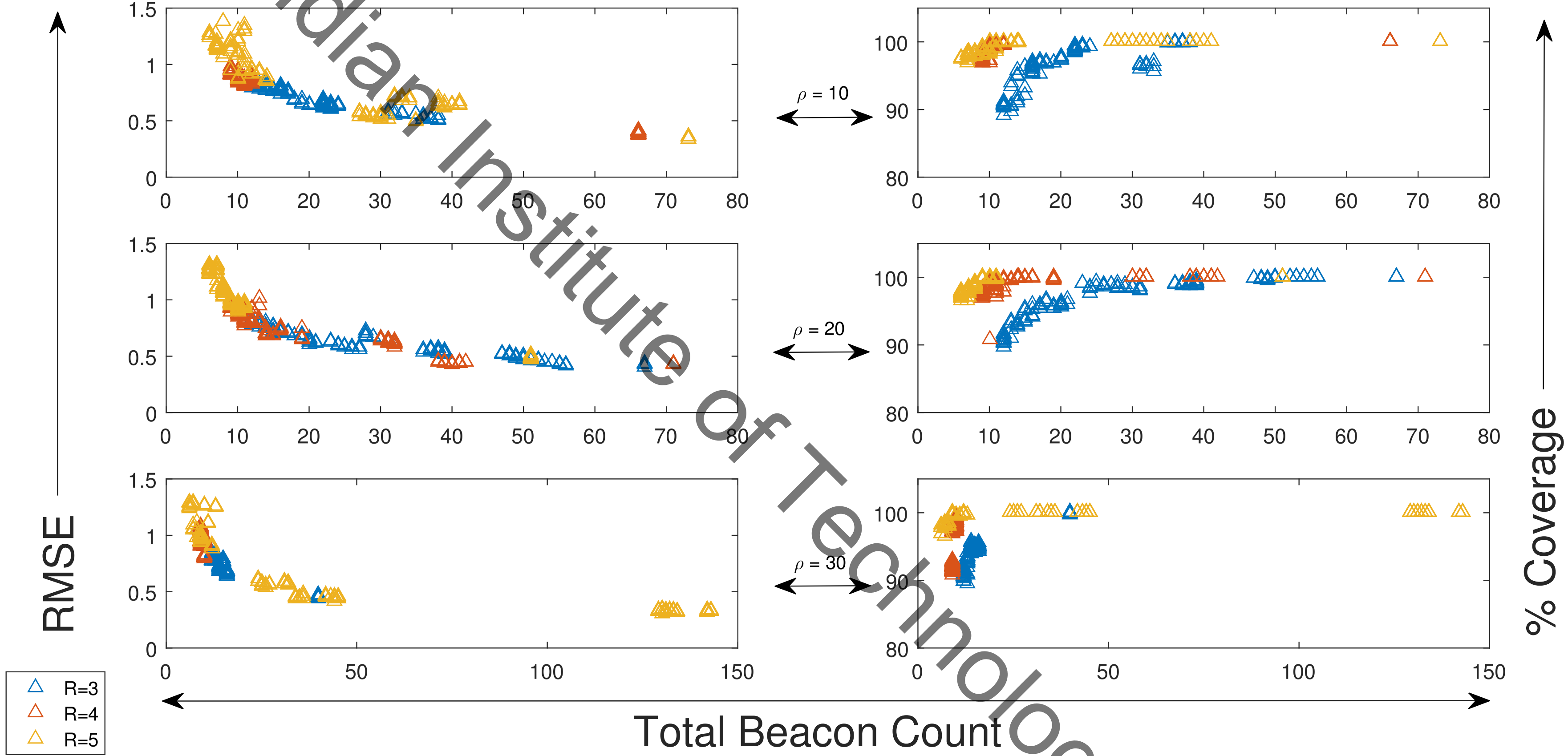



**Figure 5.12:** Accuracy (left) and coverage (right) performance of the beacon configurations with GDoP tolerance 2, for design in Figure 5.5a

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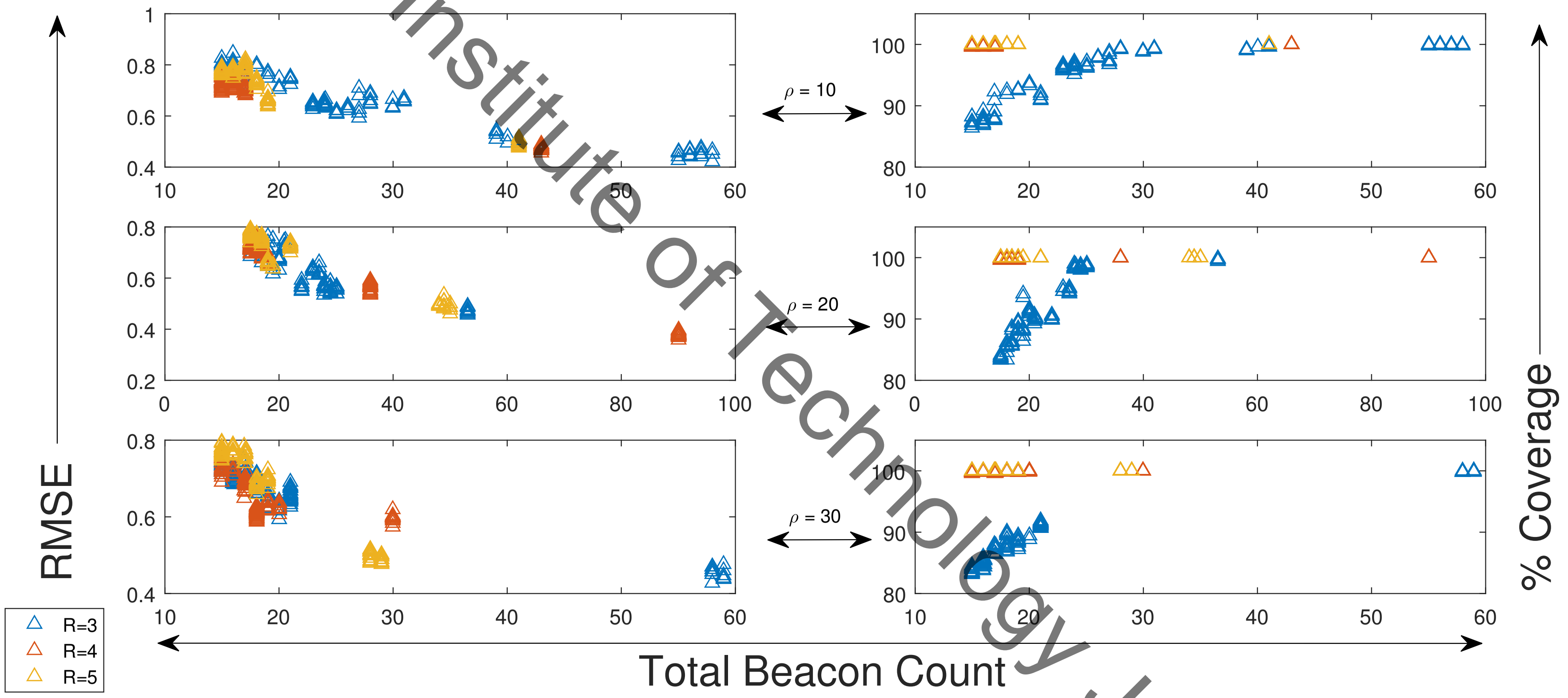


**Figure 5.13:** Accuracy (left) and coverage (right) performance of the beacon configurations with GDoP tolerance 2, for design in Figure 5.5b

Thus, a simultaneous variation in accuracy and coverage, with non-dominant beacon configurations, makes it difficult to declare a definite winner. It also makes it essential for the requirement of a comparative analysis to exist and present a range of configurations with respective trade-off to the user.

### 5.4.2 Ranking

In order to finalize a single solution among the contesting configurations, as generated from the MOO in previous subsection, we propose a rank based performance assessment technique. At first, corresponding to a configuration with $t$ solutions, a linear mapping of the resulting coverage $C$ and error $E$ values over the interval $[1,2]$ is performed. As shown in Equation 5.3, while $max$ and $min$ functions represent the calculated maximum and minimum values of the corresponding vector array, $N_C^i$ and $N_E^i$ store the normalized values of Coverage and RMSE for the $i^{th}$ solution among the $t$ in total.

$$N_C^i = \frac{C^i - max(C)}{max(C) - min(C)} + 2$$
$$N_E^i = \frac{E^i - max(E)}{max(E) - min(E)} + 2 \tag{5.3}$$

In order to compare the resulting solutions by assessing overall performance, a metric $P^i$ as shown in Equation 5.4 is devised. The direct proportionality of $N_C^i$ and reciprocity of $N_E^i$ to $P^i$ is supported by the fact that, a better solution should have higher coverage and lower error.

$$P^i = N_C^i / N_E^i \tag{5.4}$$

To rank the MOO solutions, the resulting values of $P^i$ are linearly scaled to the interval $[1, t]$ using Equation 5.5.

$$P^i = \left[\frac{t - max(P)}{max(P) - min(P)}(P^i - max(P))\right] + 1 \tag{5.5}$$

Table 5.3 presents the statistics of recommended configurations with rank value of 1, out of the performance of all configurations as explained in the last subsection referring to plots in Figure 5.12 and 5.13.

**Table 5.3:** Parameter settings and performance of Rank-1 configurations

| Parameter Setting | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Design | g | R | $\rho$ | Beacon Count | Avg. Density | %Coverage | RMSE |
| 1 | 1 | 2 | 3 | 10 | 19 | 6.7 | 97.00 | 0.652 |
| 2 | 1 | 2 | 4 | 10 | 12 | 6.7 | 99.90 | 0.861 |
| 3 | 1 | 2 | 5 | 10 | 41 | 30.5 | 100.00 | 0.642 |
| 4 | 1 | 2 | 3 | 20 | 29 | 21.7 | 99.00 | 0.670 |
| 5 | 1 | 2 | 4 | 20 | 71 | 51.6 | 100.00 | 0.422 |
| 6 | 1 | 2 | 5 | 20 | 10 | 8.0 | 99.80 | 0.896 |
| 7 | 1 | 2 | 3 | 30 | 15 | 11.2 | 95.30 | 0.754 |
| 8 | 1 | 2 | 4 | 30 | 10 | 8.0 | 99.30 | 0.801 |
| 9 | 1 | 2 | 5 | 30 | 45 | 32.5 | 100.00 | 0.439 |
| 10 | 2 | 2 | 3 | 10 | 58 | 18.2 | 99.90 | 0.421 |
| 11 | 2 | 2 | 4 | 10 | 17 | 7.8 | 99.90 | 0.701 |
| 12 | 2 | 2 | 5 | 10 | 15 | 15.0 | 88.20 | 0.829 |
| 13 | 2 | 2 | 3 | 20 | 21 | 14.2 | 90.10 | 0.714 |
| 14 | 2 | 2 | 4 | 20 | 36 | 23.8 | 100.00 | 0.536 |
| 15 | 2 | 2 | 5 | 20 | 15 | 15.0 | 83.50 | 0.787 |
| 16 | 2 | 2 | 3 | 30 | 21 | 14.2 | 91.40 | 0.625 |
| 17 | 2 | 2 | 4 | 30 | 18 | 11.5 | 100.00 | 0.586 |
| 18 | 2 | 2 | 5 | 30 | 15 | 15.0 | 83.50 | 0.726 |

As can be seen in Table 5.3, a clear trade-off between the Total Beacon Count and Average Beacon Density exists, constrained by GDoP and noise tolerance of the configurations, that keeps the RMSE to sub-meter levels and coverage above $80\%$. This establishes the capability of proposed approach, that uses point cloud for approximating indoor designs, for delivering noise tolerant beacon configurations.



## 5.5 Summary

This chapter presented an OTC to assess the capability of an indoor environment for obtaining a suitable beacon configuration that delivers high accuracy and coverage for localization. We adopted a multi objective approach for simultaneous min-max Pareto optimization over the requirement of total beacon count and average beacon density respectively. The configurations were designed over varying sensing threshold and noise values, constrained by the GDoP for error propagation in localization. For each parameter setting, the resulting Pareto solutions i.e. beacon configurations were evaluated by network simulations for output RMSE and percentage Coverage achieved. Based on this evaluation, a ranking mechanism is developed to select the best candidate configuration for an unbiased priority between coverage and RMSE.