3
Modulation Classification using Curve Fitting to Constellation

3.1 INTRODUCTION

In this chapter, modulation domain classification is done using the graphical structure of the constellation diagram. To extract the constellation, some of the essential parameters carrier frequency, symbol rate, carrier frequency offset, and phase offset are estimated. The extracted constellation is further used for modulation classification based on the curve fitting. ASK has symbol points on a line that is separated from PSK and QAM using linear regression and PSK and QAM are classified using circle fitting to the constellation [Yadav *et al.*, 2017].

3.2 CARRIER FREQUENCY OFFSET ESTIMATION AND CORRECTION

Consider that the received down-converted signal is having a Δf frequency offset and $\Delta\phi$ phase offset which are required to be estimated and corrected for the extraction of constellation points [Sethi and Ray, 2013]. CFO of Δf amount gives the rotation of $n\Delta f\Delta t$ to n^{th} constellation point, which makes points to be distributed on a circle of radius equal to the euclidean distance of symbol point as shown in Figure 3.1, where Δt is the sampling interval. To estimate the CFO, received down-converted signal $r(t)$ is passed through a eighth order nonlinearity [Sethi and Ray, 2013].

$$R(f) = |FFT(r(t)^8)| \tag{3.1}$$

A peak is observed in $R(f)$ at the index value corresponding to the $8\Delta f$. As 8^{th} order nonlinearity has been used, the frequency corresponding to the highest valued peak must be divided by 8 to get the actual CFO value. Figure 3.2, shows graph of $R(f)$ having CFO of 60kHz. Once CFO estimation is done, the signal will be corrected from the CFO error by introducing a frequency shift of $-\Delta f$ as given in the below expression

$$y(t) = s(t)e^{-j(\Delta f t)} = [I(t) + jQ(t)][\cos(\Delta f t) + j\sin(\Delta f t)]$$
$$= I(t)\cos(\Delta f t) + jQ(t)\cos(\Delta f t) + jI(t)\sin(\Delta f t) - Q(t)\sin(\Delta f t)$$

$$= I(t)\cos(\Delta f t) - Q(t)\sin(\Delta f t) + j[Q(t)\cos(\Delta f t) + I(t)\sin(\Delta f t)] \tag{3.2}$$

3.3 SYMBOL RATE ESTIMATION

Symbol rate has to be estimated for the extraction of accurate constellation points, which is done by using [Sethi and Ray, 2013]. The received In-phase and Quadrature-phase components

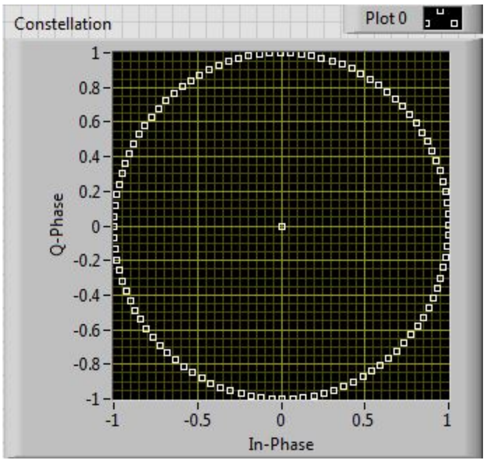


Figure 3.1 : Effect of carrier frequency offset on constellation

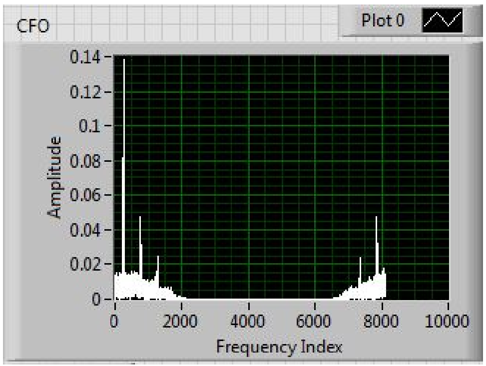


Figure 3.2 : Graph of $R(f)$ for CFO of 60kHz

after frequency correction are squared and added. Fourier transform of the resultant signal shows up one peak corresponding to the DC value at the zero index. Two other peaks are identified corresponding to the symbol rate as shown in Figure 3.3. The index value of the one peak which is near to the zero index is identified at k, and the symbol rate can be calculated as

$$R_s = \frac{kf_s}{N} \tag{3.3}$$

Where, f_s and N are sampling frequency(Hz) and FFT length, respectively. Signal is considered with no phase synchronization error.

3.4 PROPOSED MODULATION CLASSIFICATION ALGORITHM

The modulation classification algorithm is divided into two steps: in the first step ASK is separated from PSK and QAM based on linear regression error, and in the second step PSK and QAM are classified using error in circle fitting to the constellation. ASK has the lowest linear regression and PSK has the lowest circle fitting error.

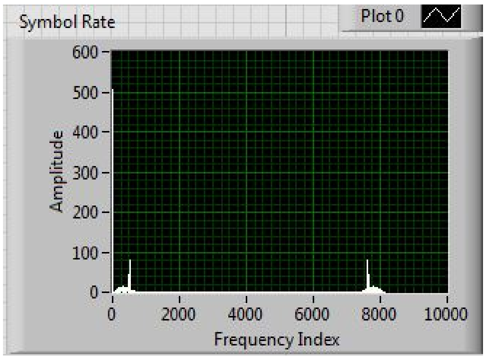


Figure 3.3 : Symbol rate estimation

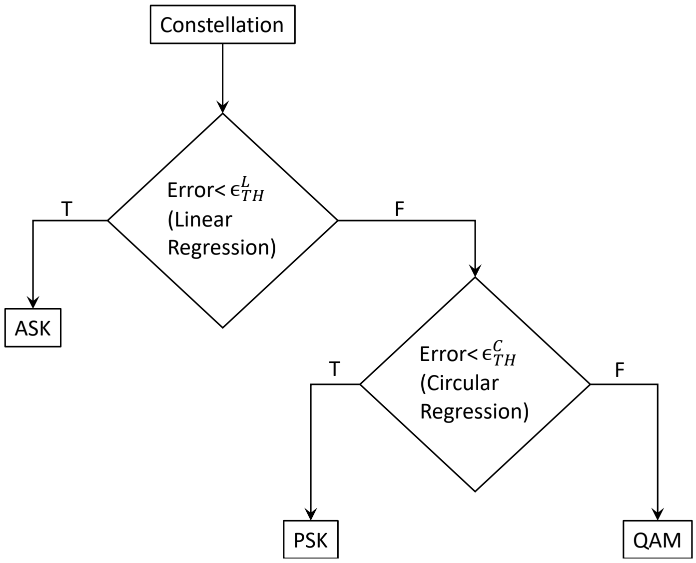


Figure 3.4 : Proposed Modulation Classification Algorithm

3.4.1 Linear Regression

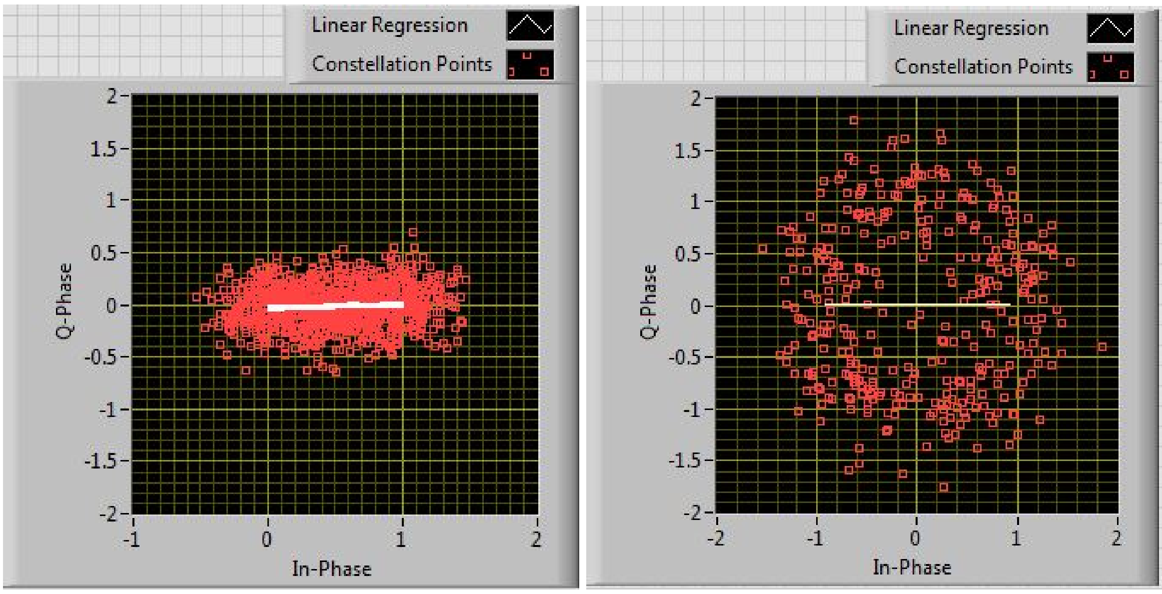
Consider that a straight line with slope β and y-intercept α is regressed to a objective constellation $\{(x_k, y_k), k = 1, 2, \cdots N\}$, to make lowest mean square error (MSE) $U(\alpha, \beta)$. Our aim is to find the coefficients α and β for which the line

$$y = \alpha + \beta x, \tag{3.4}$$

has minimum value of the objective function $U(\alpha, \beta)$.
Find $\min\limits_{\alpha,\beta} U(\alpha, \beta)$, for

$$U(\alpha, \beta) = \sum_{k=1}^{N} {\varepsilon_k}^2 = \sum_{k=1}^{N} (y_k - \alpha - \beta x_k)^2 \tag{3.5}$$

By solving this problem, α and β can be given by the standard results, i.e.



(a) (b)

Figure 3.5 : Linear regression at 7dB SNR in (a) 4ASK and (b) 8PSK

$$\beta = \frac{\sum_{k=1}^{N}(x_k - \bar{x})(y_k - \bar{y})}{\sum_{k}^{N}(x_k - \bar{x})^2} \tag{3.6}$$

$$\alpha = \bar{y} - \beta\bar{x} \tag{3.7}$$

MSE for a set of constellation points can be given by

$$MSE = \sum_{k=1}^{N}(y_k - \alpha - \beta x_k)^2 \tag{3.8}$$

If the MSE is less than the threshold value ${\varepsilon^L}_{TH}$ then the modulation scheme is identified as ASK otherwise PSK or QAM. This threshold value is calculated by a large number of Monte-Carlo simulations with a 0dB to 10dB SNR range. Fitting of line into 4ASK and 8PSK has shown in Figure 3.5(a) and Figure 3.5(b) at 7dB SNR.

3.4.2 Circle Fitting

Modulation class between PSK and QAM can be identified on the bases of residual MSE after circle fitting [Umbach and Jones, 2003] to the constellation points set $\{(x_k, y_k), k = 1, 2, \cdots N\}$. Circle $(x-a)^2 + (y-b)^2 = r^2$ can be formed that fits the best to constellation points set. We need to minimize the objective function $U(a,b)$ for the optimum value of center (a,b).

Find $\min_{a,b} U(a,b)$, where

$$U(a,b) = \sum_{k=1}^{N-1} \sum_{l=k+1}^{N} \Big\{ a(x_l - x_k) + b(y_l - y_k) - 0.5\big(({x_k}^2 - {x_l}^2) + ({y_k}^2 - {y_l}^2) \big) \Big\}^2 \tag{3.9}$$

By solving this optimization problem

$$a = \frac{DC - BE}{AC - B^2} \tag{3.10}$$

$$b = \frac{AE - BD}{AC - B^2} \tag{3.11}$$

Where,

$$A = N \sum_{k=1}^{N} {x_k}^2 - (\sum_{k=1}^{N} x_k)^2 \tag{3.12}$$

$$B = N \sum_{k=1}^{N} x_k y_k - (\sum_{k=1}^{N} x_k)(\sum_{k=1}^{N} y_k) \tag{3.13}$$

$$C = N \sum_{k=1}^{N} {y_k}^2 - (\sum_{k=1}^{N} y_k)^2 \tag{3.14}$$

$$D = 0.5 \Big\{ N \sum_{k=1}^{N} x_k {y_k}^2 - (\sum_{k=1}^{N} x_k)(\sum_{k=1}^{N} {y_k}^2) + N \sum_{k=1}^{N} {x_k}^3 - (\sum_{k=1}^{N} x_k)(\sum_{k=1}^{N} {x_k}^2) \Big\} \tag{3.15}$$

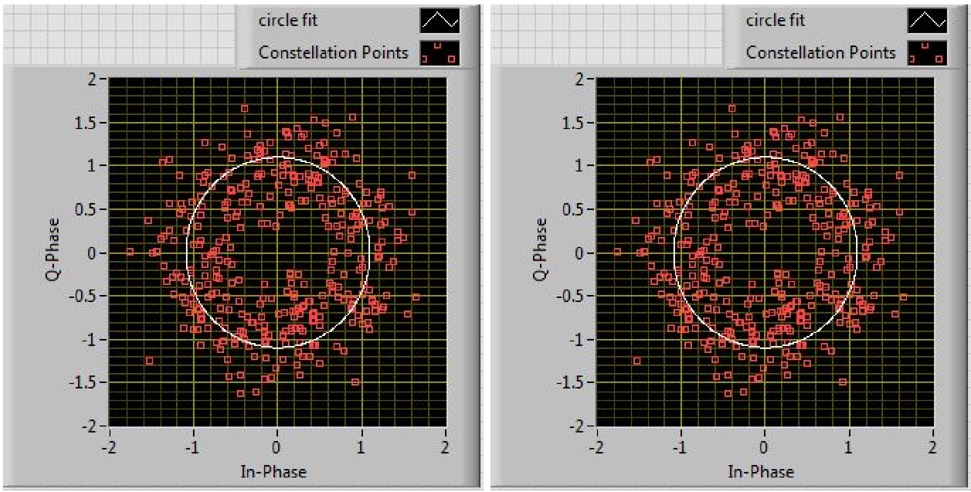
$$E = 0.5 \Big\{ N \sum_{k=1}^{N} y_k {x_k}^2 - (\sum_{k=1}^{N} y_k)(\sum_{k=1}^{N} {x_k}^2) + N \sum_{k=1}^{N} {y_k}^3 - (\sum_{k=1}^{N} y_k)(\sum_{k=1}^{N} {y_k}^2) \Big\} \tag{3.16}$$

and radius of the circle is square root of average of squares, which can be written as

$$r = \sqrt{\frac{1}{N} \Big\{ \sum_{k=1}^{N} (x_k - a)^2 + (y_k - b)^2 \Big\}} \tag{3.17}$$

Equation (3.9) - (3.19) are referred from Umbach and Jones [2003], which gives optimum way to fit circle to random data with minimum MSE and gives parameters (radius, center) of the circle. Once circle fitted to the constellation, MSE is calculated by

$$MSE = \sum_{k=1}^{N} (d_k - r)^2 \tag{3.18}$$



(a) (b)

Figure 3.6 : Circle fitting at 7dB SNR in (a) 8PSK and (b) 8QAM

Where d_k is the euclidean distance of point (x_k, y_k) from center of the circle.

$$d_k = \sqrt{(x_k - a)^2 + (y_k - b)^2} \tag{3.19}$$

The modulation scheme is identified as PSK if MSE is less than $\varepsilon^{C}{}_{TH}$ as MSE is always less in PSK case than QAM. Circle fitting to 8PSK and 8QAM at 7dB SNR has shown in Figure 3.6(a) and Figure 3.6(b), respectively.

Table 3.1 : Confusion matrix for ASK (7dB)

	ASK	PSK	QAM
4ASK	100	0	0
8ASK	100	0	0
16ASK	100	0	0

Table 3.2 : Confusion matrix for PSK (7dB)

	ASK	PSK	QAM
QPSK	0	89	11
8PSK	0	92	8
16PSK	0	89	11

3.5 SIMULATION RESULTS

Modulated signals of 1000 symbols for each modulation scheme of ASK, PSK, and QAM are generated using LabVIEW 2015. All signals are having a symbol rate of 1 MHz and AWGN noise with an SNR of 7dB and above. As per the first step of hierarchy given in Figure3.4,

Table 3.3 : Confusion matrix for QAM (7dB)

	ASK	PSK	QAM
8QAM	0	0	100
16QAM	0	0	100
32QAM	0	0	100
64QAM	0	0	100
128QAM	0	0	100

ASK is separated from the rest by comparing MSE of linear regression with the threshold value ${\varepsilon^L}_{TH} = 0.155$ and decision have taken in favor to ASK for less MSE. Similarly, MSE in circle fitting is compared with threshold value ${\varepsilon^C}_{TH} = 0.085$ for classification of PSK and QAM. Numerical values of thresholds are obtained by a large number of Monte-Carlo simulations in a range of SNR values. Confusion matrix of modulation classification are shown in table 3.1 - 3.3 for all three modulation classes.

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