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Summary and future scope

6.1 SUMMARY

In this thesis work, we explored the dynamics of deformed one-dimensional maps and deformed two-dimensional maps using different types of deformations. We have considered well known models in population dynamics: the logistic map, the Gaussian map and the Ricker map for one-dimensional maps, and Hénon-like maps for two-dimensional maps. In the type-1 deformed q-logistic map, we observed that the parameter value a_∞, at which the phase transition takes place from simple to chaotic region occurs much earlier than the parameter value a_∞ of canonical logistic map.

In case of q-logistic map of type-2 deformation, we observed that there is a delay in the phase transition. This means that the parameter a_∞ corresponding to type-2 logistic map is obtained after the a_∞ of canonical logistic map. This behaviour is different from the q-logistic map of type-1 and q-logistic map of type-3 deformation. In Section 2.2, we conclude that the composition of two simple maps, namely the deformation map $[x]_{q_1}$ and the logistic map $f_a(x)$ (where $a < a_\infty$), which is q-logistic map of type-1 has a chaotic dynamics, this leads to the Parrando's paradox.

The existence of Li and Yorke chaos showed by computing topological entropy and highlights the region where chaos are unobservable. In this region, all the trajectories seems to converge at the fixed point $x^* = 0$ numerically, but this was not happened. We proved that there exists a invariant Cantor set Γ and therefore the orbits of points in this set do not converge to x^*. In the q-Gaussian map of type-1, we showed the existence of Li-Yorke chaos by computing topological entropy using the Lap number method and noticed that the region of unobservable chaos are also present.

We have explored the stochastically stable chaos in the family of q-logistic of type-3, q-Gaussian map of type-1 and q-Ricker map of type-3. These deformed maps exhibits coexistence of attractors, the fixed point on boundary of invariant interval coexists with either the periodic attractor, or the chaotic attractor, or the Cantor-like attractor. Therefore these maps have at most one metric attractor, either a Cantor attractor of measure zero, or a finite union of intervals. Further, we showed that these deformed maps possess strong chaotic properties with positive frequency in parameter space, which are stable under stochastic perturbation. This concludes that the deformed maps exhibits chaotic behaviour for a large space of deformed parameter than the canonical maps.

We have applied the Tsallis deformation (type-3 deformation) on Hénon-like map and analyzed the dynamical properties of the q-Hénon map $\mathscr{H}_{a,b,\varepsilon}$. We proposed a method for computing the parameters (a,b), at which q-Hénon map has super stable periodic points of period-2^n for a given deformed parameter ε. Then we have constructed the "most attracting curve" on the parameter plane, which is denoted by $\gamma_{2^n,\varepsilon}$. As $n \to \infty$, the curves $\gamma_{2^n,\varepsilon}$ converges to the Feigenbaum map $\gamma_{2^\infty,\varepsilon}$, where the transition takes place from simple to chaotic behaviour. We observed that as the deformed parameter ε increases, the q-Hénon system become chaotic, in the wide range of parameter space a. When $\varepsilon > 0$, the Parrondo's paradox is encountered as the phase transition in q-Hénon map attains earlier than the canonical map. We also described the location of periodic attractor by tracing the stable and unstable manifolds of fixed points. The concept of heteroclinic web was discussed, which is based on the structure of stable and unstable manifolds of saddle periodic points. We have computed the heteroclinic bifurcations using the heteroclinic web by changing the b value on each curve $\gamma_{2^n,\varepsilon}$. Further, we showed that all q-Hénon maps are infinitely renormalizable and having Cantor set as an attractor for each $\varepsilon \in \mathscr{D}_{\varepsilon}^{*}$. Finally, we analyzed the basin of attraction of q-Hénon map and found that there is no escape region for each $\varepsilon \in (0, \varepsilon^{*})$. This is an interesting property that demonstrate the similarity of q-Hénon map with Lorenz system, in which all trajectories are bounded.

This refined study play a crucial role in further research on q-Hénon maps, which includes the geometry of the Cantor attractor and study of line fields on the Cantor attractor of infinitely renormalizable q-Hénon maps.

6.2 FUTURE SCOPE

The above study leads to the following research problems.

1. The dynamical study of low-dimensional deformed maps, where the deformation is obtained by the composition of type-1 and type-2 deformations or type-2 and type-3 deformations or type-1 and type-3 deformations.

2. The construction of analytical renormaliztion fixed point in the space of infinitely renormalizable q-Hénon maps, in the restricted domain of deformed parameters.

3. The study of geometry of the Cantor attractor of infinitely renormalizable q-Hénon maps.

4. The study of geometry of line-fields on the Cantor attractor of infinitely renormalizable q-Hénon maps.