Abstract

A q-deformed physical system in quantum group structure is an exploration of the possible deformation in the well-known physical phenomena models. The q-deviation may help to observe the changes in physical behaviour of the system. Deformation of a function introduces an additional parameter q into the function's definition in such a way that the original function can be recovered under the limit $q \to 1$. There exists multiple deformations for the same function and some deformations are inspired by [Heine, 1846; Tsallis, 1988]. The concept of q-deformation on nonlinear logistic map was studied and showed that the deformed logistic map exhibits coexisting attractors [Jaganathan and Sinha, 2005]. The coexistence of attractor is known as the Allee effect in population dynamics and has been deeply studied for unimodal maps [Schreiber, 2003]. Later, q-deformations have been studied on the nonlinear maps—the logistic map [Banerjee and Parthasarathy, 2011; Cánovas and Muñoz-Guillermo, 2019] and Gaussian map [Patidar and Sud, 2009; Cánovas and Muñoz-Guillermo, 2020]. The deformation on the two-dimensional Hénon map was analyzed numerically and it was shown that the q-deformation of the Hénon map suppresses chaos as compared to the canonical Hénon map for a specific values of deformed parameter [Patidar *et al.*, 2011]. The q-deformation of the nonlinear maps can be helpful in modelling of several phenomena that are not exactly modelled with the canonical maps, but could benefit from the q-deformed variant in quantum computing applications. The newly introduced parameter q of deformed nonlinear map can be varied according to the requirement that allow us to fit a wide range of functional forms that are identical in nature.

To study the q-deformed nonlinear maps for higher dimensional system, one can consider two-dimensional Hénon-like maps. The reason is that these maps are good models for creating chaos in real time applications. The Hénon-like maps are given by $H(x,y) = (f(x) - by, x)$, where f is a unimodal map and b is small perturbation. The appropriately defined renormalizations $R^n H$ of Hénon-like maps H converge exponentially to the one-dimensional renormalization fixed point f_* [De Carvalho *et al.*, 2005]. The universality features could coexist with unbounded geometry, which happens due to the lack of rigidity. In [Lyubich and Martens, 2011], it was shown that infinitely renormalizable Hénon-like maps form a curve in the parameter space, which is parametrized by the average Jacobian, and all infinitely renormalizable Hénon-like maps near the Feigenbaum point are topologically distinct.

In the first part of the thesis, we apply Heine deformation on one-dimensional maps and analyze the dynamics of these newly deformed maps. In particular, we consider deformed logistic map and deformed Gaussian map and discuss the basic dynamics like periodic attractors and transition from periodicity to chaotic attractor. We compute the topological entropy by using two different methods and finally we show that there exist a region of physically observable chaos, which are separated by the region where the chaos are not physically observable.

We describe the deformation schemes inspired by Heine and Tsallis in reference of q-deformed physical systems related to the quantum group structures and the statistical mechanics. We discuss the dynamics of deformed unimodal maps in the reference of strong chaotic properties. We show that, there exists a set of parameter values with positive measures, for which these deformed maps exhibit stochastically stable chaos, in the sense of [Baladi and Viana, 1996]. The deformed maps have a chaotic behaviour for a large space of deformed parameters than the canonical maps which are intended to be used in cryptography. Further, we show that the q-deformation scheme applied on both sides of the difference equation of logistic map is topologically conjugate to the canonical logistic map and therefore there is no dynamical changes by this q-deformation. We propose an

improved version of q-deformation scheme and apply on logistic map to describe the dynamical changes. Parrondo's paradox is illustrated by assuming the chaotic region as a gain. Finally we show that in the neighbourhood of particular parameter value, q-logistic map has stochastically stable chaos.

In the second part of the thesis, we study the dynamics of two-dimensional deformed maps. In particular, we apply Tsallis deformation on Hénon-like map to construct the q-Hénon map. We evaluate the basic properties and stability of the fixed points of the system. We propose an algorithm for constructing a curve with parameter (a,b) such that the q-Hénon map on this curve has an attracting period 2^n-cycles. These curves are denoted by $\gamma_{2^n,\varepsilon}$ for each ε associated to the deformed parameter q. As $n \to \infty$, the curves converges to the Feigenbaum map $\gamma_{2^\infty,\varepsilon}$, at which the q-Hénon map undergoes a transition from periodic to chaotic behaviour. For $\varepsilon > 0$, the phase transition in the q-Hénon maps occurs much earlier than the canonical Hénon-like maps, which explains the Parrondo's paradoxical behaviour. We trace the unstable manifold of the fixed points to describe the location of periodic attractors. Next, we define the hetroclinic web and describe the hetroclinic bifurcation on each curve $\gamma_{2^\infty,\varepsilon}$ for various ε values. Further, we show that all q-Hénon maps are infinitely renormalizable and having Cantor set as an attractor, before the hetroclinic bifurcation on the curve $\gamma_{2^\infty,\varepsilon}$. Finally, we show that the basin of attraction of the q-Hénon maps do not have an escape region for each $\varepsilon \in (0, \varepsilon^*)$, which is an interesting dynamical behaviour. This property illustrates the similarity of q-Hénon map with Lorenz system in which all trajectories are bounded.