2
Equation of state model: Relativistic Mean Field Theory

As already discussed in chapter-1, the matter interior to NSs may vary from sub-saturation to as much as 10 times nuclear saturation density (n_0). So the challenge is to develop a model which in addition to describing the high density matter behavior also explains the matter properties at around n_0. The various aspects of Lorentz covariance, electromagnetic gauge invariance as well as microscopic causality within a many-body system have to be brought into consideration due to the pronounced relativistic effects (at zero temperature limit, kinetic energy corresponding to the Fermi energy ~ 40 MeV which is equivalent to $v \approx 0.3c$) in dense matter regimes. The interaction between quarks via the exchange of gluons is described by the quantum chromodynamics (QCD) theory and it is the correct framework to explain dense matter systems. Unfortunately, this can not be realized explicitly at such large density scales. The dense matter models can be categorized into several classes as: microscopic models which includes the Brueckner-Hartree-Fock (BHF) (non-relativististic) and its relativistic counterpart Dirac-Brueckner-Hartree-Fock (DBHF) theories as well as variational many-body approach, effective field theory models such as density functional theory (DFT) and χ-perturbation theory models and lastly the phenomenological theory models comprising the effective two-body interactions (non-relativistic) and relativistic mean-field (RMF) model for relativistic scenario. Even though the microscopic models are parameter free, but they are based on the nucleon-nucleon scattering data which is at present available to roughly $\sim 2-3$ times nuclear saturation density. Thus, in order to study dense matter beyond such densities which is present interior to NSs one needs to implement an effective theory where instead of considering quarks at the fundamental level, we assume hadrons in which quarks are confined.

Johnson and Teller [1955] proposed the non-relativistic quantized field theoretical illustration of nuclear matter based on hadronic degrees of freedom known as quantum hadron dynamics (QHD). But it has to be duly noted that this theory can be considered as an effective (or, phenomenological) one due to the fact that hadrons are composite particles. The relativistic effects on the model by Johnson and Teller [1955] was incorporated by Duerr [1956] and further justified by Walecka [1974]. In QHD framework, the baryon-meson couplings constrain the nuclear models. These couplings are estimated based on the different nuclear saturation properties, namely saturation density, binding energy, symmetry energy and compression modulus. Various works have been accomplished to ascertain the coupling parameters for QHD based on non-linear scalar (density-indepedent) (refer to sec.-2.2) and density-dependent (refer to sec.-2.4) nature of the coupling models. In this work, we incorporate the *mean-field approximation* (MFA) model to describe the dense matter systems as presented in the subsequent sections.

2.1 Quantum Hadron Dynamics

QHD-I, also known as $\sigma - \omega$ model is the simplest parameter set to describe hadronic matter interactions. In this model, the long and intermediate range attractive interactions between the nucleons are mediated via the exchange of neutral isoscalar-scalar σ-meson and that of the short range repulsive interactions are via exchange of neutral isoscalar-vector ω-meson. In short, it can be stated that the nucleon-nucleon interaction is illustrated by baryon-, scalar meson- and vector meson-fields. The Lagrangian density of this system is given by [Walecka, 1974] (We use natural units $\hbar = c = 1$ throughout)

$$\begin{aligned}\mathcal{L} = &\sum_{N\equiv n,p} \bar{\psi}_N[\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu,\end{aligned} \tag{2.1}$$

where γ_μ represent the gamma, or Dirac matrices which manifest the Dirac equation in a compact form, σ, ω^μ denote the isoscalar-scalar and isovector-vector meson fields respectively with $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$. m_σ, m_ω represent the meson masses and m_N stands for the bare nucleon mass. The scalar and vector meson couplings to nucleons are symbolized as $g_{\sigma N}$, $g_{\omega N}$ respectively.

Now, implementing the Euler-Lagrange equation given by

$$\frac{\partial\mathcal{L}}{\partial\zeta} - \frac{\partial}{\partial x^\mu}\left[\frac{\partial\mathcal{L}}{\partial(\partial\zeta/\partial x^\mu)}\right] = 0, \tag{2.2}$$

where ζ represents the different fields in the system, we obtain the field equations as

$$\begin{aligned}\text{Scalar field:}\quad &(\partial_\mu\partial^\mu + m_\sigma^2)\sigma = g_{\sigma N}\bar{\psi}_N\psi_N, \\ \text{Vector field:}\quad &(\partial_\mu\partial^\mu + m_\omega^2)\omega_\mu - \partial_\mu\partial^\nu\omega_\nu = g_{\omega N}\bar{\psi}_N\gamma_\mu\psi_N, \\ \text{Nucleon field:}\quad &[\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N = 0.\end{aligned} \tag{2.3}$$

Due to the complexity in solving the coupled, non-linear equations presented in eq.-(2.3), the MFA model was introduced. In this approximation, the system is considered to be in a uniform state with nucleons interacting via exchange of mean meson field values ($\sigma \longrightarrow \langle\sigma\rangle$, $\omega \longrightarrow \langle\omega_\mu\rangle$). The nucleon (or, baryon) operators in the equation of motion of meson fields can be replaced by their ground state expectation values modifying eq.-(2.3) as

$$\begin{aligned}\text{Scalar field:}\quad &\sigma = \sum_N \frac{1}{m_\sigma^2} g_{\sigma N}\langle\bar{\psi}_N\psi_N\rangle, \\ \text{Vector field:}\quad &\omega_0 = \sum_N \frac{1}{m_\omega^2} g_{\omega N}\langle\bar{\psi}_N\gamma^0\psi_N\rangle,\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}\text{Scalar density:}\quad &n_N^s = \langle\bar{\psi}_N\psi_N\rangle = \frac{2}{\pi^2}\sum_N\int_0^{k_{F_N}} \frac{m_N - g_{\sigma N}\sigma}{\sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}}k^2 dk, \\ \text{Vector (or, baryon) density:}\quad &n_N = \langle\bar{\psi}_N\gamma^0\psi_N\rangle = 4\sum_N\int_0^{k_{F_N}} \frac{dk}{(2\pi)^3}.\end{aligned}$$

(2.5)

Here, k_{F_N} denotes the Fermi momentum of the nucleons. In MFA, contribution of only the time components of vector meson field ω_0 is brought into picture.

Next, in order to evaluate the energy density and matter pressure of the system, we need to implement the energy-momentum tensor given by

$$T^{\mu\nu} = \frac{\partial \zeta}{\partial x_\nu} \frac{\partial \mathcal{L}}{\partial(\partial \zeta / \partial x^\mu)} - \eta^{\mu\nu} \mathcal{L}, \tag{2.6}$$

with $\eta_{\mu\nu}$ being the Lorentz transformation matrix. The energy density of the system is given by the first element of the $T^{\mu\nu}$ matrix. *i.e.* $\varepsilon = \langle T^{00} \rangle$ and the matter pressure is given by the condition, $P = (1/3)\langle T^{ii} \rangle$ where $i = 1, 2, 3$.

Using eq.-(2.1) alongside the field equations and energy-momentum tensor, we get the expressions for energy density

$$\varepsilon = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{2}{\pi^2} \sum_N \int_0^{k_{F_N}} \sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2} k^2 dk \tag{2.7}$$

and for the matter pressure

$$P = -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{2}{3\pi^2} \sum_N \int_0^{k_{F_N}} \frac{k^2}{\sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}} k^2 dk. \tag{2.8}$$

The first and second terms on the right hand side of eqs.-(2.7), (2.8) are the mass contributions from the scalar and vector fields respectively. The last term in the expressions for energy density and matter pressure is the contribution from a relativistic Fermi gas of nucleons (or, baryons) of effective mass $m_N^*(\sigma) = m_N - g_{\sigma N}\sigma$.

So, the nuclear matter equation of state is evaluated by self-consistent solutions of the energy density, $\varepsilon(n_N)$ and matter pressure, $P(n_N)$ expressions.

2.2 Non-Linear scalar coupling model

Fitting the baryon-meson coupling constants in the QHD-I model it was found that even though it reproduced the nuclear matter saturation at Fermi wavenumber of 1.30 fm^{-1} and binding energy per nucleon of -15.75 MeV, this parameter set over-estimates the value of compression modulus of nuclear matter. Boguta and Bodmer [1977] proposed that to evaluate the value compressibility factor accurately at saturation density, the introduction of scalar self-couplings into the nuclear matter Lagrangian density is necessary and given by

$$\mathrm{U}(\sigma) = \frac{1}{3} b m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4} c (g_{\sigma N}\sigma)^4, \tag{2.9}$$

where b, c represent the dimensionless coupling coefficients and m_N denotes the bare nucleon mass. Incorporating eq.-(2.9) into eq.-(2.1), we get

$$\begin{aligned} \mathcal{L} = &\sum_{N \equiv n,p} \bar{\psi}_N [\gamma_\mu (i\partial^\mu - g_{\omega N}\omega^\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \mathrm{U}(\sigma), \end{aligned} \tag{2.10}$$

The modified equations obtained by plugging in eq.-(2.9) are given as

$$
\begin{aligned}
\text{Scalar field:} \quad & \sigma = -\frac{1}{m_\sigma^2}\frac{\partial \mathrm{U}(\sigma)}{\partial \sigma} + \sum_N \frac{1}{m_\sigma^2} g_{\sigma N}\langle \bar{\psi}_N \psi_N \rangle, \\
\text{Energy density:} \quad & \varepsilon = \mathrm{U}(\sigma) + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 \\
& + \frac{2}{\pi^2}\sum_N \int_0^{k_{F_N}} \sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}\, k^2 dk, \\
\text{Matter pressure:} \quad & P = -\mathrm{U}(\sigma) - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 \\
& + \frac{2}{3\pi^2}\sum_N \int_0^{k_{F_N}} \frac{k^2}{\sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}} k^2 dk.
\end{aligned}
\tag{2.11}
$$

It may be noted that the additional two coupling coefficients allow for two nuclear properties viz. compression modulus (K) and Dirac effective nucleon mass (m^*) at nuclear saturation density.

2.3 Isovector-vector coupling

In order to provide an account of another nuclear saturation parameter namely, symmetry energy (E_{sym}) the incorporation of the charged isovector-vector $\boldsymbol{\rho}$-meson is necessary. These isovector mesons couple differently with neutrons and protons due to their corresponding isospin. The introduction of $\boldsymbol{\rho}$-meson to describing the isospin force in dense matter system is termed as QHD-II and the Lagrangian density is given by modification in eq.-(2.10) as

$$
\begin{aligned}
\mathcal{L} = & \sum_{N\equiv n,p} \bar{\psi}_N[\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\boldsymbol{\tau}_N\cdot\boldsymbol{\rho}_\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\
& - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \mathrm{U}(\sigma) - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu,
\end{aligned}
\tag{2.12}
$$

where $\boldsymbol{\rho}_\mu$ denotes isovector meson field with $\boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$ and $\boldsymbol{\tau}_j$ represents the isospin projection of the jth-particle (in this case nucleons) present in the dense matter.

Now, implementing the Euler-Lagrange equation on eq.-(2.12), we get the modified field equations as

$$
\begin{aligned}
\text{Isovector-vector field:} \quad & (\partial_\mu\partial^\mu + m_\rho^2)\boldsymbol{\rho}_\mu - \partial_\mu\partial^\nu\boldsymbol{\rho}_\nu = g_{\rho N}\bar{\psi}_N\gamma_\mu\boldsymbol{\tau}\psi_N, \\
\text{Nucleon field:} \quad & [\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N = 0,
\end{aligned}
\tag{2.13}
$$

Table 2.1: Parameter values of the non-linear scalar RMF coupling models considered in our work. Here, $g_2 = bm_N g_{\sigma N}^3$ and $g_3 = cg_{\sigma N}^4$. The nucleon mass, m_N is considered to be ~ 939 MeV in this work.

Non-Linear RMF Model	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_2 (fm^{-1})	g_3	m_σ (MeV)	m_ω (MeV)	m_ρ (MeV)
GM1	9.5708	10.5964	8.1957	12.2817	−8.9780	550	783	770
GM2	8.4305	8.7115	8.5415	9.9066	67.084	550	783	770
GM3	8.7821	8.7119	8.5415	27.8810	−14.401	550	783	770
NL3	10.2170	12.8680	8.9480	10.4310	−28.8850	508.194	782.50	763
NL3-II	10.2020	12.8540	8.9600	10.3910	−28.9390	507.680	781.869	763
NL-SH	10.4444	12.9450	8.7660	6.9099	−15.8337	526.059	783	763
NL-RA1	10.3623	12.9211	8.8117	10.0599	−27.5565	515.70	783	763
NL3*	10.0944	12.8065	9.1496	10.8093	−30.1486	502.574	782.60	763
GMT	9.9400	12.2981	9.2756	10.5745	−24.1907	511.198	783	770

followed by the expressions for $\boldsymbol{\rho}$-meson field, energy density and matter pressure as

$$\begin{aligned}
\text{Isovector-vector field:} \quad & \rho_{03} = \sum_N \frac{1}{m_\rho^2} g_{\rho N} \langle \bar{\psi}_N \gamma^0 \boldsymbol{\tau}_{N3} \psi_N \rangle, \\
\text{Energy density:} \quad & \varepsilon = \mathrm{U}(\sigma) + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\
& + \frac{2}{\pi^2} \sum_N \int_0^{k_{F_N}} \sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}\, k^2 dk, \qquad (2.14) \\
\text{Matter pressure:} \quad & P = -\mathrm{U}(\sigma) - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 \\
& + \frac{2}{3\pi^2} \sum_N \int_0^{k_{F_N}} \frac{k^2}{\sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}} k^2 dk.
\end{aligned}$$

The conservation of charge implies the survival of only third component in isospin-space of the isovector-vector $\boldsymbol{\rho}_{\mu\nu}$ meson field.

The introduction of $\boldsymbol{\rho}$-meson brings into picture an additional restoring energy which favors isospin symmetry. The computation of symmetry energy coefficient and its relation with isovector-vector baryon-meson coupling will be discussed in subsequent chapters.

In the non-linear scalar coupling sector, a good number of coupling parametrizations have been developed so far. Some of the non-linear scalar coupling parametrizations namely GM1, GM2, GM3 [Glendenning and Moszkowski, 1991], NL3, NL3-II [Lalazissis et al., 1997], NL-SH [Sharma et al., 1993], NL-RA1 [Rashdan, 2001], NL3* [Lalazissis et al., 2009] and GMT [Pal et al., 2000a] implemented in this thesis work with their corresponding parameter values are provided in table-(2.1). The nuclear saturation properties are tabulated in table-(2.2). Based on these coupling parameter sets various properties of dense matter and NS observables are studied which will be discussed in subsequent chapters. It is noteworthy to mention that the symmetry energy coefficients of non-linear scalar coupling sets other than GM1, GM2 and GM3 are quite high and lie beyond the admissible range as obtained from various experimental

Table 2.2: The nuclear properties of the RMF models at respective n_0.

RMF Model		n_0 (fm^{-3})	$-E_0$ (MeV)	K_0 (MeV)	E_{sym} (MeV)	L_{sym} (MeV)	m_N^*/m_N
	GM1	0.153	16.30	300.00	32.50	93.857	0.700
	GM2	0.153	16.30	300.00	32.50	89.289	0.780
	GM3	0.153	16.30	240.00	32.50	89.627	0.780
	NL3	0.148	16.29	271.76	37.40	118.317	0.600
Non-	NL3-II	0.149	16.28	272.15	37.70	119.563	0.590
Linear	NL-SH	0.146	16.346	355.36	36.10	113.654	0.600
	NL-RA1	0.1466	16.15	285.00	36.10	115.305	0.600
	NL3*	0.150	16.31	258.27	38.68	122.71	0.594
	GMT	0.145	16.30	281.00	36.90	112.796	0.634
	DD1	0.1487	16.021	240.00	31.60	55.949	0.565
	DD2	0.149065	16.02	242.70	32.73	54.966	0.5625
	DD-ME1	0.152	16.20	244.50	33.10	55.370	0.578
	DD-ME2	0.152	16.14	250.89	32.30	51.253	0.572
Density-	PKDD	0.149552	16.267	262.181	36.79	90.139	0.5712
Dependent	TW99	0.153	16.247	240.00	33.39	55.309	0.555
	DDV	0.151	16.097	240.00	33.589	71.463	0.586
	DDF	0.1469	16.024	223.10	31.60	55.919	0.556
	DD-MEX	0.152	16.14	267.059	32.269	49.576	0.556

findings and their consequent implications. Further details on this aspect will be addressed in subsequent chapters.

2.4 Density-dependent coupling model

Initially proposed by Fuchs et al. [1995], in this coupling model the nucleon(baryon)-meson coupling constants are considered to be varying with matter density. Following this approach, the non-linear self-interactions of scalar mesons are substituted by density dependent meson coupling values. Typel and Wolter [1999] suggested that this coupling model provides better logical and reasonable descriptions of matter behavior at high density and charge asymmetry regimes when compared with Dirac-Brueckner theory of nuclear matter.

In this section, we briefly describe the density-dependent coupling approach within RMF theory framework considering matter to be composed of nucleons and their interactions coordinated via the exchange of isoscalar-scalar σ, isoscalar-vector ω and isovector-vector $\boldsymbol{\rho}$ mesons. The Lagrangian density of this dense matter system is given by [Typel and Wolter, 1999]

$$\mathcal{L} = \sum_{N\equiv n,p} \bar{\psi}_N[\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\boldsymbol{\tau}_N \cdot \boldsymbol{\rho}_\mu) - (m_N - g_{\sigma N}\sigma)]\psi_N + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2)$$
$$- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu, \tag{2.15}$$

where g_{iN} denotes the density-dependent nucleon-meson coupling parameter with $i = \sigma$, ω, $\boldsymbol{\rho}$ and the other notations are similar as in case of eqs.-(2.1), (2.12). The density-dependent isoscalar meson couplings to nucleons are provided by the relation

$$g_{iN}(n) = g_{iN}(n_0)f_i(x) \qquad \text{for } i = \sigma, \omega \tag{2.16}$$

where, $x = n/n_0$, with n being the vector (or, baryon) number density and

$$f_i(x) = a_i\frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}. \tag{2.17}$$

The function defined in eq.-(2.17) obtains similar results as that of various Dirac-Brueckner calculations of symmetric nuclear matter. Based on the conditions, $f_i(1) = 1$, $f_\sigma^{''}(1) = f_\omega^{''}(1)$ and $f_i^{''}(0) = 0$, the number of independent parameters to determine the seven parameter values in eq.-(2.17) is reduced to three. And, de Jong and Lenske [1998] on the basis of Dirac-Brueckner calculations of asymmetric nuclear matter suggested an exponential dependence for $\boldsymbol{\rho}$-meson coupling given by

$$g_{\rho N}(n) = g_{\rho N}(n_0)\exp[-a_\rho(x-1)]. \tag{2.18}$$

The independent parameters are calibrated in such a way so as to reproduce the properties of symmetric, asymmetric nuclear matter and various nuclear saturation properties. One can refer to Typel and Wolter [1999] for further discussion regarding the estimation of coefficient values in eqs.-(2.17), (2.18).

The meson field equations in the density-dependent coupling model are given by

$$\begin{aligned}
&\text{Isoscalar-scalar field:} \quad \sigma = \sum_N \frac{1}{m_\sigma^2} g_{\sigma N}\langle\bar{\psi}_N\psi_N\rangle,\\
&\text{Isoscalar-vector field:} \quad \omega_0 = \sum_N \frac{1}{m_\omega^2} g_{\omega N}\langle\bar{\psi}_N\gamma^0\psi_N\rangle,\\
&\text{Isovector-vector field:} \quad \rho_{03} = \sum_N \frac{1}{m_\rho^2} g_{\rho N}\langle\bar{\psi}_N\gamma^0\boldsymbol{\tau}_{N3}\psi_N\rangle.
\end{aligned} \tag{2.19}$$

Due to the density-dependent nature of nucleon-meson couplings, additional rearrangement contributions in the self-energies of meson fields are introduced in the Dirac equation. The Dirac equation in density-dependent coupling model is given by

$$[\gamma_\mu(i\partial^\mu - g_{\omega N}\omega^\mu - g_{\rho N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu - \Sigma^{r(\mu)}) - (m_N - g_{\sigma N}\sigma)]\psi_N = 0, \tag{2.20}$$

where

$$\Sigma_0^r = \sum_N \left[\frac{\partial g_{\omega N}}{\partial n}\omega_0 n_N - \frac{\partial g_{\sigma N}}{\partial n}\sigma n_N^s + \frac{\partial g_{\rho N}}{\partial n}\rho_{03}\boldsymbol{\tau}_{N3}n_N\right] \tag{2.21}$$

represents the time component of rearrangement term and n_N, n_N^s denote the vector, scalar number densities respectively as defined in eq.-(2.5). The chemical potentials of nucleons are modified as

$$\mu_N = \sqrt{k_{F_N}^2 + (m_N - g_{\sigma N}\sigma)^2} + g_{\omega N}\omega_0 + g_{\rho N}\boldsymbol{\tau}_{N3}\rho_{03} + \Sigma_0^r \tag{2.22}$$

which results in introduction of rearrangement term in the expression for matter pressure explicitly. This rearrangement term maintains the thermodynamic consistency of the dense matter system. The energy density and matter pressure are then given by

$$\begin{aligned}
\text{Energy density:} \quad \varepsilon &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \frac{2}{\pi^2}\sum_N \int_0^{k_{F_N}} \sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}k^2 dk, \\
\text{Matter pressure:} \quad P &= -\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \frac{2}{3\pi^2}\sum_N \int_0^{k_{F_N}} \frac{k^2}{\sqrt{k^2 + (m_N - g_{\sigma N}\sigma)^2}}k^2 dk + n\Sigma_0^r.
\end{aligned} \tag{2.23}$$

The various density-dependent coupling models considered in this work are DD1 [Typel, 2005], DD2 [Typel et al., 2010], DD-ME1 [Nikšić et al., 2002], DD-ME2 [Lalazissis et al., 2005], PKDD [Long et al., 2004], TW99 [Typel and Wolter, 1999], DDV [Typel and Alvear Terrero, 2020], DDF [Klähn et al., 2006] and DD-MEX [Taninah et al., 2020]. The nuclear saturation properties corresponding to each parametrization is provided in table-(2.2). And the respective parameter and coefficient values are tabulated in table-(2.3).

Additionally, to describe NS matter, the weak β-equilibrium between the particles (in this case, nucleons) given by $\mu_n - \mu_p = \mu_e = \mu_\mu$ and the electric charge neutrality condition $n_p = n_e + n_\mu$ need to be satisfied [Glendenning, 1996].

In the next chapters, we will discuss the implementation of various mentioned baryon-meson coupling parametrizations within the framework of non-linear scalar as well as density-dependent coupling models with viable inclusion of exotic particle degrees of freedom.

Table 2.3: The parameter and coefficient values for the density-dependent coupling parametrizations, at n_0.

Density-dependent RMF Model	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	m_N (MeV)	m_σ (MeV)	m_ω (MeV)	m_ρ (MeV)
DD1	10.685257	13.312280	7.278046	939	547.204590	783	763
DD2	10.686681	13.342362	7.25388	939.56536	546.212459	783	763
DD-ME1	10.44340	12.89390	7.61060	939.90	549.255	783	763
DD-ME2	10.53960	13.01890	7.36720	939.90	550.1238	783	763
PKDD	10.73850	13.14760	8.59960	939.5731	555.5112	783	763
TW99	10.72854	13.29015	7.32196	939	550	783	763
DDV	10.13696	12.77045	7.84833	939.5654	537.6001	783	763
DDF	11.0240	13.5750	7.290	939.56536	555	783	763
DD-MEX	10.706722	13.338846	7.23804	939.56536	547.332728	783	763

	a_σ	a_ω	a_ρ	b_σ	b_ω	c_σ	c_ω	d_σ	d_ω
DD1	1.371545	1.385567	0.4987	0.644063	0.521724	1.034552	0.869983	0.567627	0.618991
DD2	1.35763	1.369718	0.518903	0.634442	0.496475	1.005358	0.817753	0.57581	0.638452
DD-ME1	1.3854	1.3879	0.5008	0.9781	0.8525	1.5342	1.3566	0.4661	0.4957
DD-ME2	1.3881	1.3892	0.5647	1.0943	0.9240	1.7057	1.4620	0.4421	0.4775
PKDD	1.327423	1.342170	0.183305	0.435126	0.371167	0.691666	0.611397	0.694210	0.738376
TW99	1.365469	1.402488	0.5150	0.226061	0.172577	0.409704	0.344293	0.901995	0.983955
DDV	1.20990	1.23750	0.33260	0.21290	0.03910	0.3080	0.07240	1.04030	2.14570
DDF	1.48670	1.54490	0.447930	0.19560	0.183810	0.42817	0.43969	0.88233	0.87070
DD-MEX	1.3970	1.39260	0.62022	1.334964	1.01910	2.067122	1.605966	0.40160	0.45560